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MHD FLOW OF MICROPOLAR FLUIDS OVER A STRETCHING SHEET

[1]Dr. Sajjad Hussain, [2, a] Dr. Farooq Ahmad

[1]Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University, Multan, Pakistan
[2,a] Punjab Higher Education Department, Principal, Government Degree College Darya Khan, (Bhakkar), Pakistan

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ABSTRACT

MHD flow of micropolar fluids over a stretching sheet is studied and the numerical results have been obtained. The highly non-linear governing partial differential equations of fluid motion have been converted into ordinary differential form by using similarity transformations. The resulting equations are then solved numerically using Simpson's (1/3) rule and Successive Over Relaxation (SOR) method. Richardson's extrapolation to the limit is used to improve the results. The effects of Magnetic parameter M^2 and non-dimensional material constants C_1, C_2 and C_3 (related to micropolar nature of the fluids) are examined on velocity and microrotation profiles. Comparison of the results for Newtonian fluids and micropolar fluids is presented.

AMS Subject Classification: **76M20**

Key Words: **Stretching surface, MHD flow, Similarity transformations, Richardson's Extrapolation**

I INTRODUCTION

The study of fluid flow problems over stretching surfaces has generated considerable interest because of its applications in several technological processes, examples may be found in continuous casting, glass fiber production, metal extrusion, hot rolling, textiles and wire drawing. Hiemenz [1] studied the steady two-dimensional boundary layer flow near the forward stagnation point on an infinite wall using similarity transformation. This solution is later improved by Howrath [2]. Crane [3] obtained a closed form solution for the stretching sheet whose velocity is proportional to the distance from the slit. The three-dimensional and axisymmetric stretching surface was studied by Wang [4]. Shafique et al. [5] presented numerical solution of uniform suction/blowing effect on Newtonian fluid flow due to a stretching cylinder. Magnetic effect in fluid dynamics is considered important because of its role in many industrial applications. Mamaloukas [6] studied the steady, laminar flow of a Newtonian electrically conducting fluid over a stretching sheet. Chiam[7] analyzed hydromagnetic flow over a surface stretching with a power-law velocity. Mamaloukas et al. [8] examined MHD flow of a Newtonian fluid over a stretching sheet.

Eringen [9] formulated and presented the theory of micropolar fluids. Eringen’s micropolar model includes the classical Navier-Stokes equations as a special case. Micropolar fluids theory includes the flow of low concentration suspension, liquids crystals, animal blood, colloidal fluids, lubrication, turbulent shear flow etc. These fluids are interesting in themselves and important from practical point of view. Several researchers have made extensively theoretical and experimental investigations for micropolar fluid flows along with magnetic field, suction/injection, stagnation point flow and stretching sheet. Kasiviswanathan and Gandhi [10] obtained a class of exact solutions for the MHD flow of a micropolar fluid confined between two infinite, insulated, parallel, non-coaxially rotating disks. The effect of the magnetic field on the flow of a micropolar fluid past a continuously moving plate has been studied by Seddeek [11]. Kamal and Hussain [12] studied the three dimensional micropolar fluid motion caused by the stretching surface. Shafique and Rashid [13] obtained numerical solution for three dimensional micropolar fluid flows due to stretching flat surface. Sajjad and Kamal [14] obtained numerical solution of micropolar fluid flow over a stretchable disk. Gorla et al. [15] examined the simultaneous occurrence of buoyancy and magnetic forces in the flow of an electrically conducting micropolar fluid along a hot vertical plate in the presence of a strong cross magnetic field. Kumar [16] analyzed the effect of heat and mass transfer in the hydromagnetic micropolar fluid flow along a stretching sheet. Srinivasacharya and Shiferaw[17] have presented numerical solution of the steady conducting micropolar fluid through porous annulus under the influence of an

[a] **Corresponding Author:** Farooq Ahmad, Punjab Higher Education Department, Principal, Government Degree College Darya Khan, (Bhakkar), Pakistan. Email: farooqgujar@gmail.com & Farooq.Ahmad@gcdaryakhan.edu.pk

applied uniform magnetic field. Further, an important investigation for flow due to stretching sheet were made by Andersson [18]. Chakraborty and Mazumdar [19] found an approximate solution of MHD flow of a Newtonian fluid over a stretching sheet.

In this work, MHD flow of micropolar fluids over a stretching sheet is examined and numerical solutions have been obtained using SOR method and Simpson's (1/3) rule. The results have been obtained and presented for velocity and microrotation for different values of magnetic parameter M^2. The numerical results are improved by using Richardson's extrapolation to the limit. This numerical scheme is very easy, straightforward and efficient.

II MATHEMATICAL ANALYSIS

The fluid flow is assumed to be steady, two dimensional and incompressible. Fluid is electrically conducting and a uniform magnetic field of strength B_0 is applied in the positive y-direction normal to the sheet. The flow is confined in the region $y > 0$ and sheet coincides with the plane y=0. The Cartesian coordinates system is being used.

Under these assumptions the basic equations of motion Eringen [9] become:

$$\nabla.\underline{V} = 0, \tag{1}$$

$$-(\mu+\kappa)\nabla\times(\nabla\times\underline{\mathbf{V}})+\kappa(\nabla\times\underline{\upsilon})-\nabla p+\underline{J}\times\underline{B}=\rho\left(\underline{\mathbf{V}}.\nabla\right)\underline{\mathrm{V}}, \tag{2}$$

$$-\gamma(\nabla\times\nabla\times\underline{\upsilon})+\kappa(\nabla\times\underline{\mathbf{V}})-2\kappa\underline{\upsilon}=\rho\, j\left(\underline{\mathbf{V}}.\nabla\right)\underline{\upsilon}\ , \tag{3}$$

where ρ is the density, p is pressure, $\underline{V} = V(u,v,0)$ the velocity vector, $\underline{\upsilon} = \upsilon(0,0,\upsilon_3)$, the micro-rotation or spin, μ is dynamic viscosity coefficient, j the micro-inertia, γ and κ are material constants.

The boundary conditions are

$$\begin{aligned} &y = 0, u = cx, v = 0, P = P_w, \underline{\upsilon} = 0 \\ &y \to \infty, u = 0, \underline{\upsilon} = 0, \end{aligned} \tag{4}$$

where c >0 is stretching constants.

$$u = cxf'(\eta), v = -\sqrt{c\nu}\,xf(\eta),\ \upsilon_3 = cx\sqrt{\frac{c}{\nu}}L(\eta),\ P = P_w - \frac{1}{2}c\mu g(\eta) \tag{5}$$

$\eta = \sqrt{\frac{c}{\nu}}y$ is dimensionless variable.

The continuity equation (1) is satisfied identically. The equations (2) and (3) are given below in dimensionless form.

$$(1+C_1)f''' + ff'' - f'^2 - M^2 f' - C_1 L' = 0 \tag{6}$$

$$\frac{1}{2}g' - (1+C_1)f'' - ff' + C_1 L = 0 \tag{7}$$

$$L'' + C_2 f'' + C_2 L = C_3(Lf' - fL') \tag{8}$$

with the corresponding boundary conditions :

$$\left.\begin{aligned} &\eta = 0,\ f = 0,\ f' = 1, g = 0, L = 0 \\ &\eta \to \infty,\ f' = 0, L = 0, \end{aligned}\right\} \tag{9}$$

where prime denotes differentiation with respect to η, $M^2 = \frac{\sigma B_0^2}{\rho c}$ is magnetic parameter and $C_1 = \frac{\kappa}{\mu},\ C_2 = \frac{\kappa\nu}{\gamma c},\ C_3 = \frac{\rho j \nu}{\gamma}$ are dimensionless material constants. When the microtation vector $\underline{\upsilon}$ and κ are made zero the problem reduces to Newtonian fluid flow.

III. FINITE DIFFERENCE EQUATIONS

In order to obtain the numerical solution of nonlinear ordinary differential equation (6), let

$$f' = q \tag{10}$$

$$(1+C_1)q'' + fq' - q^2 - M^2 q - C_1 L' = 0 \tag{11}$$

$$L'' + C_2 q' - C_2 L = C_3 (Lf' - fL'). \tag{12}$$

The boundary conditions (8) become:

$$\left.\begin{aligned} &\eta = 0,\ f = 0,\ q = 1,\ g = 0, L = 0 \\ &\eta \to \infty,\ q = 0,\ L = 0 \end{aligned}\right\} \tag{13}$$

By using the central difference approximation for derivatives involved in equations (11) and (12) at a typical point $\eta = \eta_n$ of the interval [0, b], where b is sufficiently large. we obtain

$$\begin{aligned} &[2(1+C_1) + hf_n]q_{n+1} - [4(1+C_1) + 2h^2(M^2 + q_n)]q_n \\ &+ [2(1+C_1) - hf_n]q_{n-1} - C_1 h(L_{n+1} - L_{n-1}) = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &(2 + C_3 hf_n)L_{n+1} - (4 - 4C_2 h^2 + 2C_3 h^2 q_n)L_n \\ &+ (2 - C_3 hf_n)L_{n-1} + hC_2(q_{n+1} - q_{n-1}) = 0 \end{aligned} \tag{15}$$

,

where h denotes a grid size. Also the symbols used denote $q_n = q(\eta_n)$ and $f_n = f(\eta_n)$.

IV RESULTS AND DISCUSSION

The equation (10) is integrated using Simpson's (1/3) rule Gerald [20] with the formula given by Milne [21] and the finite difference equations (14) and (15) are solved by using SOR method Smith [22] subject to the appropriate boundary conditions.

The numerical solutions of $f' = q$ and microrotation L are of order of accuracy $O(h^2)$ due to second order finite differences approximations to the derivatives. However, the solution of f is accurate to the order of accuracy $O(h^5)$ and the higher order accuracy $O(h^6)$ for the solution of f' and L is achieved by using Richardson's extrapolation to the limit Burden [23].

Numerical results have been computed for several values of the magnetic parameter M^2, the calculations are made on three different grid sizes namely *h*, *h*/2 and *h*/4 to check the accuracy of the numerical results. Three different sets of material constants C_1, C_2 and C_3 have been chosen arbitrarily to make better understanding of micropolar fluids behavior.

Table 1.

Case	C_1	C_2	C_3
I	0.1	0.5	1.5
II	1.5	2.5	3.5
II	3.0	4.0	5.0

The micropolar fluids have four additional viscosity coefficients along with the usual viscosity of the Newtonian fluids. The material constants C_1, C_2 and C_3 given in table 1 are related to these viscosities and play role for the micromotion of the fluids.

Table 2 presents the results for skin friction coefficient - $f''(0)$. The comparison of the results for Newtonian fluids, micropolar fluids and the previous results is given to elaborate the validity of present results. The results are in good comparison. The magnitude of - $f''(0)$ is lesser for micropolar fluids than for Newtonian fluids. Fig.1 and fig.2 depict the pattern of velocity components f' and f respectively for different values of M^2. Both the velocity components decrease with increase in the values of M^2. Fig.3 demonstrates the dimensionless microrotation *L*. It is observed that the microrotation increases initially but decreases afterward under the effect of increasing values of M^2. Comparison of micropolar fluids and Newtonian fluids is shown in fig.4. The velocity component f' is less in magnitude for Newtonian fluids than for micropolar fluids. Fig.5 shows the characteristics of f' for three different cases of the material constants given in table 1. The increase in the magnitudes of these constants increase the magnitude of f' for fixed value of M^2.

Table 1. Numerical results for skin friction coefficient - $f''(0)$ for different values of M^2.

M^2	Anderson[18]	Chakarborty and Mazumdar[19]	Present numerical results (Newtonian fluids)	Micropolar fluids
0	1.	0.988	1.0052	1.0013251
1	1.414	1.403	1.4134	1.3588726
2	1.732	1.722	1.7228	1.645147

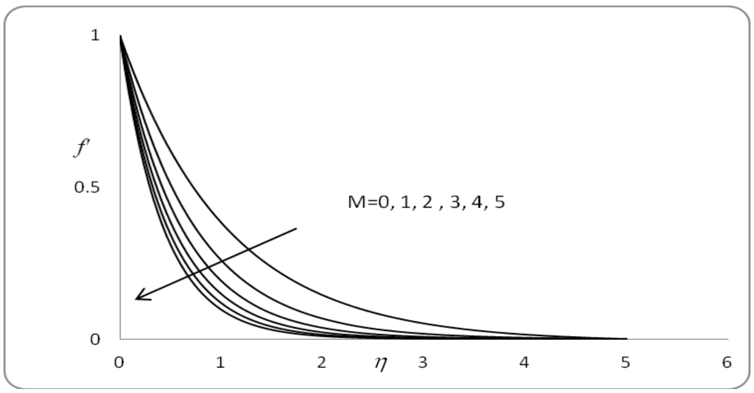

Fig.1 Graph of f' for different values of M^2

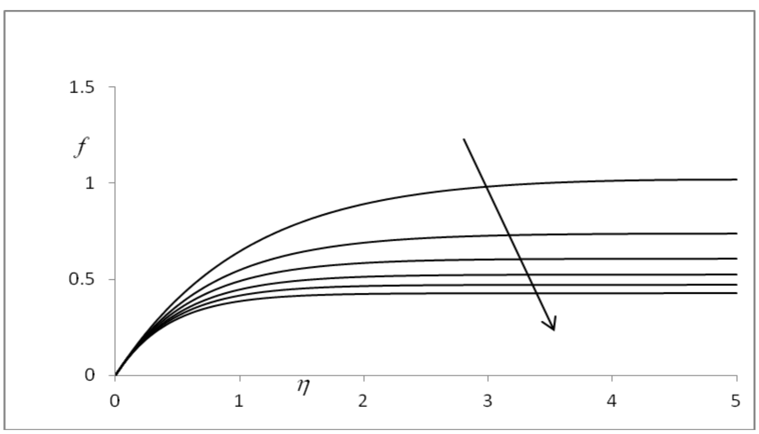

Fig.2. Graph of *f* for different values of M^2 = 0, 1, 2, 3, 4, 5 from top to bottom.

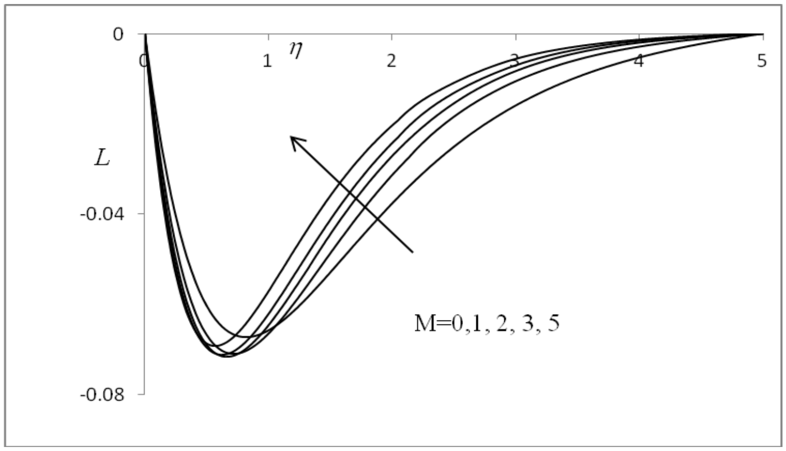

Fig.3 Graph of microrotation *L* for different values of M^2.

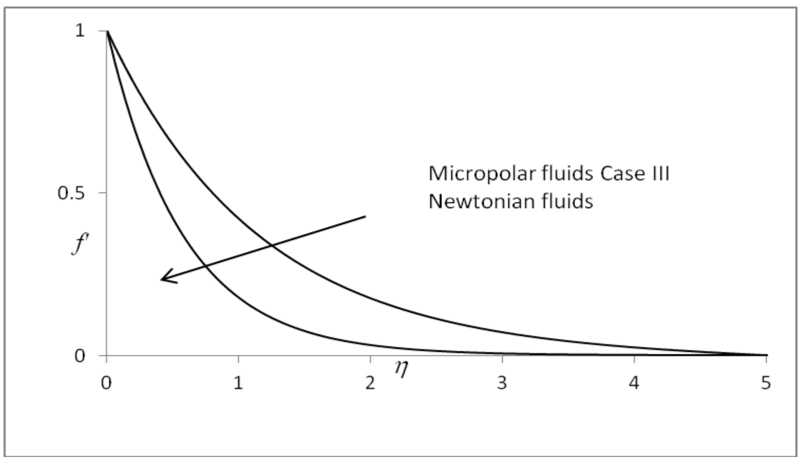

Fig.4. Graph of f' for comparison of Micropolar and Newtonian fluids when M^2=2.

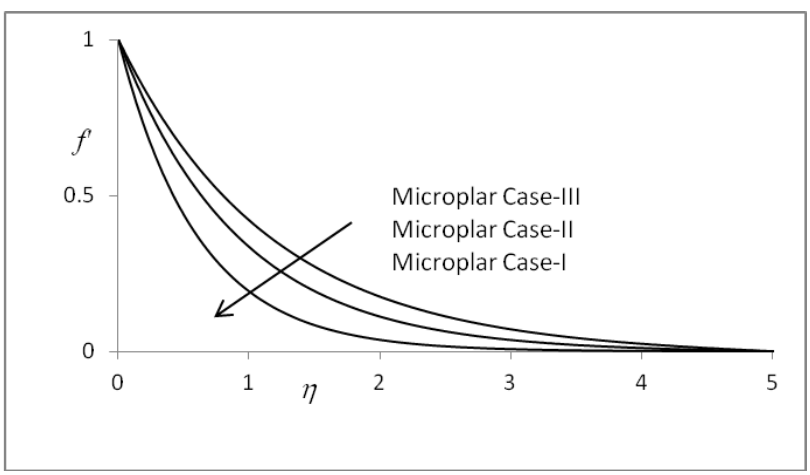

FIG.5. GRAPH OF f' FOR COMPARISON OF MICROPOLAR AND NEWTONIAN FLUIDS WHEN M^2=2.

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