

Deep Learning in Mathematics through STG Method

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ABSTRACT

At present era, traditional methods of teaching, especially in the field of mathematics education, do not respond all educational needs, and intelligent educational methods can help students in depth and easy understanding mathematical concepts. But there are some problems such as lack of necessary facilities in some schools in remote areas, apathy of some teachers on utilization of new methods in mathematics education, that practically make using this methods impossible. The existence of all of the required conditions does not mean the independence of modern skills from the old ones. Even some traditional methods can be implemented with the help of new tools. In this research, STG instruction skills have been implemented on two different groups of students at two different bases in two different provinces in Iran, and each of them are compared through inferential statistical methods with a base that have been trained in traditional education. Therefore using statistical analysis shows that instruction of mathematical concepts through STG method modify the misconceptions and difficulty of mathematical problems for students.

KEY WORDS: Mathematics, Socratic Method, Team working method, Generalization method.

1. INTRODUCTION

It is often difficult for students to understand mathematical concepts and principles. In their mind, mathematics is a useless knowledge and there is not any need for training it in the life. When they hear about the application of mathematics, they expect to take them some benefits! The absence of mathematics in their every day life such as radio, television and newspapers and the logical difficulties in its implications, change the mathematics to a useless and harsh science for them. But when in mathematical classes some non-mathematical examples come to speak, or in text books come to write, deep understanding and good relationships between concepts under consideration will be glared in the students [16]. Mathematics is a rich source of concepts that students usually encounter difficulties in the understanding of them. Mathematical thinking is a dynamical process which increases the student's ability to understand complex concepts [9] or is a process which increases the field of application of abstract concepts [21]. Mathematics is reasoning and reasoning begins from the assumptions and leads to results [1]. Thus, in the view of some philosophers, mathematics is a method of reasoning [8]. Most students have the ability to understand mathematical concepts, but in the argument stage which some connections between different concepts would be established, they are faced with defects. In this case the students have different reactions. Some of them are impotent when face with such situations, but if they have a second chance later, they will succeed without any great effort. A number of them can solve the problems for the first time, but when encounter the same problems for the next stage, they will be incapable of solving them, and at last there are some ones who can resolve the problem with several techniques. Regardless the differences between students, improved methods of teaching mathematics are among the first cases which must engage by mathematical societies of countries. In most schools the teacher writes the definitions and theorems on the blackboard to prove them consecutively, and the destitute student, who usually, incapable of understanding them, sees himself in a world of wandering and despair [11]. According to the subjects planted in [15] reforming the methods of education of mathematics is among the basic needs of every country that wishes to progress. The notes which are remained after the death of Carl Weierstrass, the great German mathematician and instructor, show the importance of mathematics education. In the different places of this notes which is written by a patient German scientist, there is a gap that has been written there in German "here a joke" [13]. Some of the traditional methods of teaching are general but others are only used in mathematics education. In this work, the researcher having tried to introduce three educational methods which are suitable in mathematics. These methods are gregarious and suitable for working in the classroom. In the present research we experience these three methods simultaneously in teaching mathematics, and then compare this combined method with the traditional instruction method which now is established in most schools by using simple devices such as chalk and board [19].

2. REQUIRED KINDS OF EDUCATION METHODS

There are three general methods in education which all of those, based on the human intelligence. Socratic Method, Team working Method and at last Generalization Method can cause the ability and opportunity for teachers and trainers to awake the dormant talents of students (see Fig.1). Especially, in the Socratic Method, the thought navigation of the student is increased, while the aim of Generalization Method is increasing the ability of applying the results to different contexts

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and mathematical models [3] and at last the Team Working method uses some psychological characteristics of the students in teaching. In this method, the students should discuss with their more clever classmates whom is determined by their teacher, about the problems which they are not permitted to ask the teacher about them directly. A number of researchers have debated the important role of these skills of education in science [5].

2.1 Socratic Method

Students generally are incapable of understanding abstract concepts. In this case there are several methods to help them. Each of these methods is an appropriate device for one age level and has its own characteristics. Analogical reasoning is suitable for primary school aged [18]. Students learn more about the world which they live in and then able to increasing the domain of their knowledge to reasoning about the relationships between the items [4]. The Socratic Method is one of the ways that is appropriate for all ages [12]. As the Longman Advanced American Dictionary explains, Socrates (469-399 B.C.) was a Greek philosopher from Athens, who was the teacher of Plato and is known for developing a method of examining ideas according to a system of questions and answers, the method that teacher does not give information directly, but instead asks the students a series of questions, as a way of directing the students to improve their thinking and knowledge. Socrates believed that among the factors that could cause the students to learning is the sharing their passions for knowing. In his method, the subject was divided into small sections, and then the teacher planned some questions each of which corresponding to one of those small sections and its answer is just the same small part in the lesson, then asked them from one student to another respectively [11]. The researcher used this method many times in the classroom and has receipted its very good results. A noteworthy point is considering educational notes in planning the questions. Matching the scientific ability of students with the range of difficulty of questions is one of the most basic points which if fails to follow then eliminates the student's confidence. The logical questions which asks by the teacher, resolution of questions, giving students adequate time for response, partnering all of the students according to their academic ability, intelligent assistance to disabled students which has difficulty in answering the questions, among the features that can make this method coincided with success. Here we will prove the following problems through Socratic Method:

Problem 1. The teacher asks student that solves the equation $x^4 - 5x^2 + 6 = 0$.

Solve: This problem is difficult for the students who know the decomposition of quadratics only. The aim of the teacher is to guide the students to the right path, because his (her) ability of decomposition of a quadratic is known for the teacher and hence choosing the auxiliary unknown $y = x^2$ is proposed.

The help of the teacher should not be so that the utilities of thinking and reasoning of students are failed to act. The teacher should not be arrested him any more than what is required. His method should be such that the student feels that the main job is done by him. Designing the auxiliary problems is a major function of the human brain. Designing the problem which is useful for solving another one, is a thin achievements of human wisdom and his conscience [14].

Problem 2. The teacher asks student to prove that the sum of inner anglers of a triangle ABC is 180.

Solve: This problem is usually hard for the students who know the law of bevel line (see Fig. 2). So the teacher will draw the line $X'AX$ parallel to BC . Then he asks the student: why $\angle BCA = \angle CAX$? Similarly why $\angle CBA = \angle BAX$? And here, the light of exploration will be emerged!

2.2 Team Working Method

Today information technology has a great role in the development of educational objectives in developed countries. Rapid exchanges that take place, provides the availability of information around the country and make available for students an equal education. Internet has increasingly been used to provide significant resources for all of the students in various branches of Science. According to [17] the students of mathematics also use internet to learn Mathematics and to gain knowledge about the development of this branch of science, as well as their teachers which internet is a significant resource of concepts for them. In USA, a large scale experiment introducing and evaluating intelligent tutoring in an urban high school setting. They built an intelligent tutor, called PAT that supports this curriculum and has been made a regular part of ninth grade Algebra in three Pittsburgh schools. The results of this intelligent system have shown that comments of teachers that working in the computer lab with PAT, activate the students whom are confronted with difficulties in the normal classrooms [6]. Also the researchers which are performed a research in smart schools field in Malaysia believed that the smart school education program is a systematically designed program that integrates teaching and learning with IT applications, include computer based teaching and web-based learning. The aim of these smart schools is based on teaching-learning principles, and practice to develop and produce future generations who are technologically literate, who can think critically and creatively and are able to manage and apply knowledge effectively as well as innovatively [20]. But unfortunately such cases are not true for poor or developing countries. The limited facilities at most schools and low interest of the teachers of mathematics in using the Internet for mathematical purposes are the problems that disrupts the effectiveness of this approach [12]. Therefore, a method that can produce the same goals, although in a limited range is necessary. In fact if the deficiencies of the

educational programs, teachers and books are ignored and the precision and accuracy to carry out them are supposed without any defect, yet there is not a good opportunity for students which are not usually at a same scientific level, to thinking in the classrooms. So one of the methods that can be used in these cases, is using of the opportunities that students are not in the classroom. The teacher, according to the number of his bright students, divided all of the students into groups of three, four or five persons, and appoints each of his bright students as a header of one of those groups. Then the necessary materials which are needed to train each of those groups describes by the teacher to the header. The header must work with the students of his (her) group according to the teacher's training manual. Decreasing the differences in academic level of students is the minimum result of this method, and if done with the precision and subtlety, this difference can lead to an acceptable level [11].

2.3 Generalization Method

To some believes of mathematicians, problems are the heart of mathematics and to some others, those are like the veins that carry blood to the body of mathematics. Moreover, from a point of view, mathematics is indeed solving the problems which are one of the human mental phenomenons [13]. On the other hand, as in [14] teaching mathematics is an art such as active sports like skiing, swimming and playing piano. This art can be taught only by imitation of a good model. This art is no magic key to unlock all doors but there are guidelines that will familiarize us with some useful doors. In the depths of human soul, a greater tension lies which is seeking to find some magical methods to open each site. This tendency in most of ours is secret and some legends and writings are created about them by some philosophers in their literary works. Descartes wanted to seek a useful method for all problems [2]. Leibniz also planned to find a general method where can explained all of the intellectual truths by calculations and symbols [2]. But these attempts were abortive such as the change some low-value metals into gold. There are some hopes which are still remaining. In fact, unattainable ideals were not so useless, for example who has achieved so far to the sun? But who can claim that the sun is useless?

In this method, the teacher or one of the students raises a theorem, and then students try to justify it with the teacher guidance or propose a generalized form of the theorem with the same assumptions and then offer its proof. Perhaps most of the great mathematicians have benefited of this way in their works. Major part of mathematics is a consecutive generalization, but the clues to this generalization are not clear, because of their owners' skills. In fact, Mathematics is thinking [9] and so students should also learn the ways of thinking, not just a set of concepts, definitions and theorems. A motive force is needed to stimulate the student to thinking, which could be a matter of geometry, number theory, or any other branch of mathematics.

Here's an example that happened to the researcher in the classroom and was discussed by the students:

Example 3: The second power of one, five and six, are limited to the same numbers respectively! Are there any numbers with two digits which the second powers of those tend to the same digits? How about the numbers with three digits?

Solve: A student who knew the rule of computing the square of the numbers with two digits which tend to five, offered the number $\overline{25}$ as a solution with two digits. For three digits the third student assumed that $a = \overline{x25}$ be such that $a^2 = \overline{...x25}$ and by a simple calculation showed $x = 6$ and $a = 625$.

In fact, those students show that the Diophantine equation $c_n^2 - c_n = a_n \times 10^n$ has a solution for $n = 1, 2$. Now the following problem immediately comes to mind,

Problem 4: Is the following Diophantine equation $c_n^2 - c_n = a_n \times 10^n$ has a solution for all $n \in \mathbb{N}$?

Discussion and solve: At this stage the sudden appearance of this equation for current students is no wonder. Because they have a prospect in their minds. But the solution in this case does not so simply. For solving this problem we use a note in the above students' dialogue! Note that the second solution came from the first and the third from the second. So we consider the equation $c_{n+1}^2 - c_{n+1} = a_{n+1} \times 10^{n+1}$ for which c_{n+1} is a $n+1$ digits sequence. The assumption $c_{n+1} = \overline{xc_n}$ and two previous equations with a simple calculation show that

$$10^n x^2 + (2c_n - 1)x - 10a_{n+1} + a_n = 0$$

If c_n tends to 5, then a necessary and sufficient condition for the existence of a solution for the last equation, is the existence of solution for

$$(2c_n - 1)x + a_n = 10u$$

But this is a simple result of the theory of linear Diophantine equations, since $\gcd(2c_n - 1, 10) = 1$ [10]. Now since a_n, c_n have been identified by mathematical induction, x is obtained by solving the last equation.

3. Statistical analysis

In 2012 we did a statistical analysis to study the effects of Socratic-Team working-Generalization triple method (STG) in progress of mathematics education. In the present research two classes CM_1 and CM'_1 of first year high school in Markazi province and two classes CH_2 and CH'_2 of second year high school in the Hamadan province of Iran were selected as statistical societies. Traditional teaching in classes CM_1 and CH_2 and STG training in the classes CM'_1 and CH'_2 was conducted by a unique tutor. For this study, four Mathematical tests were used. In the two first mathematical tests, the prior knowledge of students was evaluated. The questions of third and fourth tests which has been designed by specialists to assess the mathematical skills of students after applying the methods of traditional and STG education. The Table 1 is regulated after collecting data using descriptive statistics.

In Table 1, it is shown that marks' means of traditional and STG instruction groups of first year high school in Markazi province are ($M=13.1$, $M'=13.6$) respectively. Also the marks' means of the students of second year high school in Hamadan province are ($M=12.5$, $M'=14.1$). Therefore the marks' means of STG instruction groups are higher than marks' means of traditional instruction groups in two provinces. In addition, a similar result is established for the variance of the groups. We will estimate a confidence interval for mean difference of 95 percent of the first class of students and such an interval for the students of grade two. Here, $N_1, N_2 > 30$ and the t-student distribution is consistent with the normal distribution approximately. The variance of the whole society is unknown. If S_p , L and U be defined as in [7], then the details of calculating of 0.95 confidence intervals of two provinces have been recorded in Table 2.

Fig1.Categories of Education

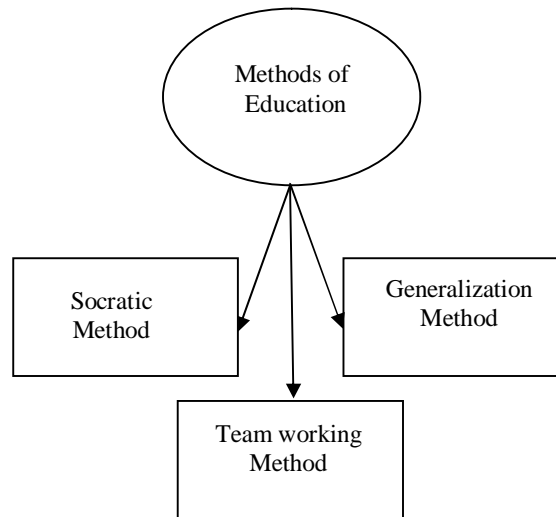


Fig 2. An example of Socratic Method

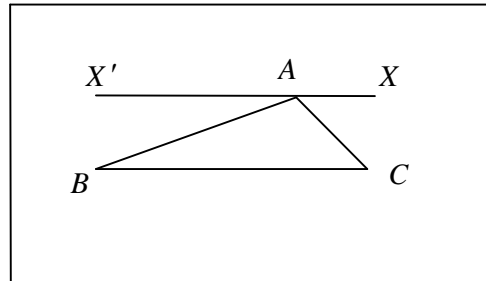


Table 1 .Descriptive statistics of traditional and STG instruction groups in math

	CM_1	CM'_1	CH_2	CH'_2
N	36	40	40	38
Mean	13.1	13.6	12.5	14.1
Variance	0.9	0.5	0.8	0.5

Table 2. The comparison of means of Traditional and STG instructions groups in Markazi and Hamadan Provinces

	CM_1	CM_1'	S_p	L	U	95% Confidence Interval of the Difference	
Markazi Province	13.1	13.6	0.738	0.219	0.738	Lower bound	Upper bound
						0.219	0.738
	CH_2	CH_2'	S_p	L	U	95% Confidence Interval of the Difference	
Hamadan Province	12.5	14.1	0.670	1.299	1.901	Lower bound	Upper bound
						1.299	1.901

4. DISCUSSION AND CONCLUSION

STG instruction method can be used to elevate the learning and teaching processes. The results of this research have demonstrated that using this instruction methods play a basic role in deep and better learning in mathematics at high schools. In fact, STG method is a combination of three methods each of those extends a dimension of deep teaching and learning in mathematics. Team working method is a good cooperative for teacher in teaching mathematics and has the ability to increase the level of deep teaching and learning of students with the aid of their classmates, while the Socratic and Generalization methods use the mind of student in content creation. In this method, as good as the students' awareness raise, the differences in their mathematical knowledge also decrease.

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