

# Numerical Simulation of Rotating Mixing in Cylindrical Vessel with Couple of Mix Rotating Stirrers

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Received: January 16 2014

Accepted: March 4 2014

## ABSTRACT

This article is the result of numerical simulation of rotating of mixing fluid in cylindrical vessel induced by mix-rotating. The predicted solutions compared against previously simulated numerical results and found with good agreement. Two-dimensional incompressible complex flow of power law model fluid is relevant to the mixing industry. The numerical method adopted a finite element semi-implicit time-stepping Taylor-Galerkin/pressure-correction fractional stepping scheme, posed in a cylindrical coordinate system. The flow replicates the behaviour of actual industrial dough mixing.

Key Words: Exact solution; Newtonian fluids; Similarity solution; Undetermined Coefficient method; Method of stretching variable; Hartmann number; Shrinking sheet; Magnetohydrodynamic (MHD) flow and Coefficient of friction.

**Key Words:** Finite Element Method, TGPC scheme, Mixing Flows, Mix Rotating Stirrers, Power Law Model Fluids.

## I INTRODUCTION

This study is focusing on the optimization of mixer design to minimize the power consumption and maximise the amount of work done on the dough mixing process in the mining industries. This problem is generally related with processing industries [1], such as in the chemical process applications, powder mixing processes [2], granular mixing, mixing of paper pulp in paper industry, particularly with mixing of dough in a food processing industry and many other industrial processes [3-4]. Many mixing processes faced the complicating factors during mixing. The use of the complex fluids which exhibit very complicated behaviour throughout the mixing process. In fact, these complications have been motivated to analyse the different rotational directions of motion of stirrers and velocities in this study.

For industrial mixers, the dough partially fills the geometry, is driven around a bowl by stirring rods [6]. A cylindrical vessel with two stationary and rotated stirrers fixed with ends of the lid fitted with the top of the vessel concentrically is considered in this study as computational domain. The stability, accuracy and convergence of a numerical method are significant issues for the robustness of the numerical algorithm [7-8], when applied to a challenging system of fundamental governing system of equations.

Great consideration of researcher and mathematicians in the field of computational fluid dynamics has been focused on finite element method (FEM) due to its flexibility to adjust the complex computational domain and its accuracy. The success of the finite element method in solving a wide range of nonlinear problems in virtually every phase of science and engineering has been phenomenal. In previous studies a stream function and vorticity formulation is used, here, in the present study the primitive variables formulation is employed [5, 9-10].

Two-dimensional Navier-Stokes equations in cylindrical coordinates are considered in this study. The consideration is given a time marching semi implicit Taylor-Galerkin/Pressure-Correction (TGPC) multi stages scheme adopted [13, 14]. The flow is modelled as incompressible via so called TGPC finite element scheme as introduced by the pioneers of this algorithm [5]. The effects of inertia, impact of rotational velocity for Newtonian fluids are clearly marked. The predicted solutions are displayed through contours plots and isobars, these are plotted from non-dimensional minimum value (marked by oval shape) to maximum value (marked by square shape), over a

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range. The Reynolds numbers are  $Re = 8.0$ ,  $Re = 0.8$  and  $Re = 0.08$  corresponding to zero shear viscosities  $\mu = 1.05$  Pas,  $\mu = 10.5$  Pas and  $\mu = 105.0$  Pas respectively, a range of material properties is covered from those for model fluids, to model dough, to actual dough respectively [11–12].

## II PROBLEM SPECIFICATION

Stirring is one of the most important operations in process technology [15]. The case of cylindrical container with a pair of contra rotating stirrers fixed at the top of the container by a lid eccentrically is considered. Three cases of velocity of rotating stirrers are analysed i.e., half, same and double ( $v_0 = 0.5, 1.0$  and  $2.0$ ) rotational velocity. The outer cylindrical container rotates in counter clockwise and stirrers in clockwise direction. In industries, outer container is fixed physically and stirrers rotate around its own centres as well as around the centre of container. Due to mathematical complexities in the problem and to preserve the originality of physical problem instead of the rotation of stirrers around centre of vessel, the rotation of container is considered.

Cylindrical Container  $v_\square = a$

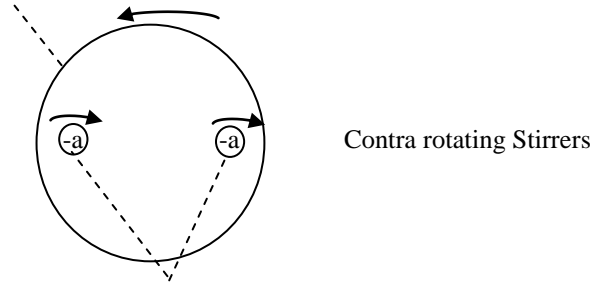


Figure 01: Two dimensional computational domain of eccentric rotating container.

Eccentric rotating cylindrical flow domain of interest involved is shown in figure 01. For the analysis of computational domain, the discretisation adopts triangular finite elements. For the simulation, a two-dimensional cylindrical coordinate frame of reference is taken over the domain.

## III GOVERNING SYSTEM OF EQUATIONS AND NUMERICAL SCHEME

Incompressible Newtonian rotating flow is considered in this research work. In the absence of body force, the system of equations can be represented through the conservation of mass and the conservation of momentum

transport equations as  $\nabla \cdot \mathbf{u} = 0$  and  $\rho \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \boldsymbol{\sigma} - \rho \mathbf{u} \cdot \nabla \mathbf{u}$  respectively over the spatial domain  $\Omega$ .

The details of numerical scheme adopted here, reader is referred to [12]. For a well-posed problem, it is necessary to define initial and boundary conditions. Simulations starts from rest and on both container and stirrers follow moving wall conditions ( $v_r = 0$  and  $v_\square = a$ ). A pressure datum is specified as zero on outer rotating vessel wall [6]. For the time marching process, the pre described level of tolerance for which the steady state convergence is achieved to be of the order of  $10^{-6}$  and  $\Delta t$  is fixed as  $10^{-3}$ . The mesh density is highly affected to the accuracy of the numerical predictions. The study on mesh independency was conducted earlier [6], where mesh M2 is realised for optimal solution of various flow characteristics, also adopted in this study [4]. In experiment, dough mixing the rotational velocity of outer cylinder is taken 75 rpm, which relate to Reynolds number equal to 0.08.

## IV NUMERICAL RESULTS AND DISCUSSIONS

Predicted solutions are analysed through contours of flow structure to demonstrate the effects of inertia ( $Re = 0.08, 0.8$  and  $8.0$ ) with respect to rotational velocities (half, same and double) on stirrers against the velocity of outer container. These numerical results illustrate typical solution contours of streamlines (shown in figure–02) and pressure (shown in figure–03) for Newtonian fluids. At half speed of stirrers; six vortices formed throughout the geometry and minimum value of recirculation is noticed at container wall which is zero and maximum value in upper and lower regions is 2.62734 at  $Re = 8.0$ . As inertia increases, centre of vortices moves counter clock wise away from horizontal line at half speed. At the same speed, all six vortices are formed but a noticeable new feature emerges, that is, the vortices twist clockwise direction when inertial level is increased and maximum values are noted at centre of vortices formed in upper, lower and nip of stirrers in narrow gap and minimum value at centre of vortices is formed on the vertical axis line. Similar fashion of recirculation regions is observed at the double speed of the stirrers. The magnitude and smooth shape is noticed in formation, but as the speed of the stirrers increases from half to double speed, the gap between vortices reduces and central vortices appear together.

In the case of contra rotating stirrers, the fluid gets condensed on the entry to the constricted region within stirrers and container and hence the maximum pressure arises. When the fluid exit from the gap, the flow expands, here the minimum value of pressure is noticed due to release of energy. The pressure differential at all comparable parameter

values for contra rotating instances influence relate across the geometry at  $Re = 0.08$  and at  $Re = 0.8$ , symmetric pressure isobars appear with equal magnitude in non-dimensional positive and negative extreme on both the sides of the stirrers in the narrow gap at half and same speed and there is replicated pattern about each stirrer with respect to upstream and downstream (pre and post nip gap) flow. When the inertia reaches at 8.0 in the case of double speed of stirrers, change in the position of positive maxima is noticed i.e., 6.93863 at the container wall before the fluid entering in the narrow gap and similar fashion is analysed at same speed.

Initially, the graph of work done and power consumption is high to drive the flow in the container. Afterwards, graph of power consumption decreases rapidly and reaches at steady state some level asymptotic solution for contra rotating stirrers at the low speed but for high speed, this passion steadily decreases. Whilst, work done increases in same fashion up to steady state. Numerical results are tabulated in Table.

Table: Streamlines and Pressure of Newtonian fluids for contra rotating stirrers; inertial levels 0.08, 0.8 and 8.0; speed of stirrers is 0.5, 1 and 2.

Variables	Speed of Stirrer	Re = 0.08		Re = 0.8		Re = 8.0	
		Minima	Maxima	Minima	Maxima	Minima	Maxima
Streamlines	Half	0	2.66273	0	2.66225	0	2.62743
	Same	-0.93386	2.02191	-0.9386	2.01946	-1.49938	1.81904
	Double	-5.019	1.58694	-5.0849	1.58592	-7.41413	1.71834
Pressure	Half	-5.26616	5.24019	-5.38462	5.14477	-6.71983	4.5179
	Same	-7.18218	7.13253	-7.40446	6.92244	-10.0362	4.98229
	Double	-11.0943	10.9296	-11.8754	10.2863	-24.6213	6.93863

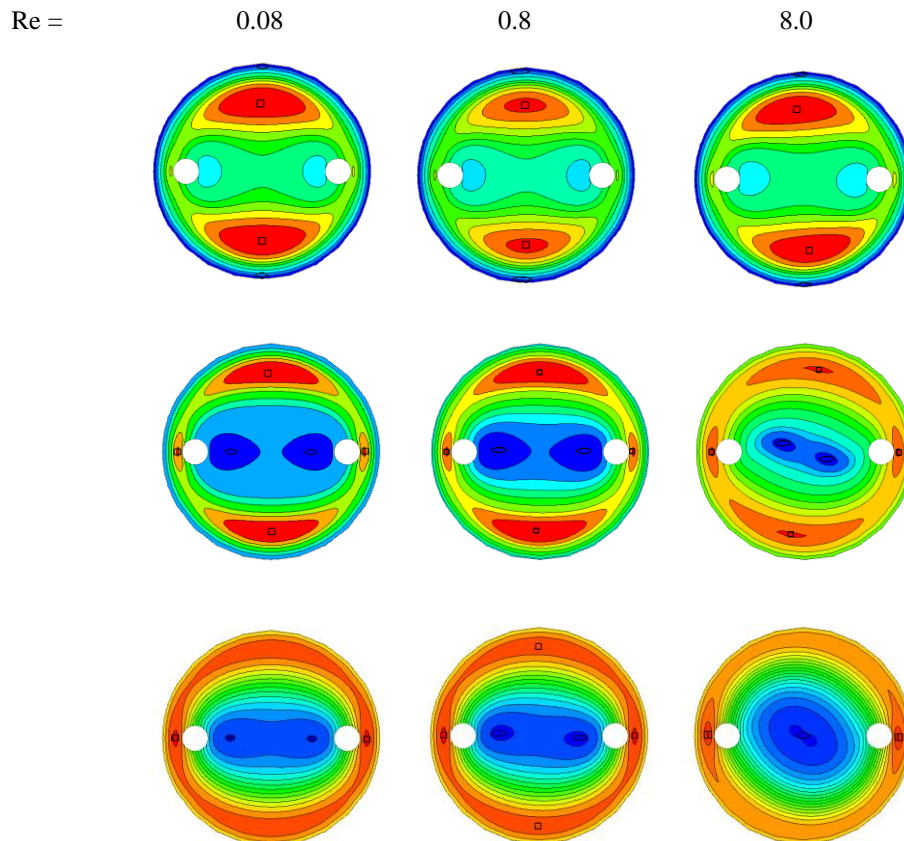


Figure 02: Flow structure patterns for contra rotating stirrers; at inertial levels 0.08, 0.8 and 8.0 (from left to right); speed of stirrers is 0.5, 1 and 2 (from top to bottom)

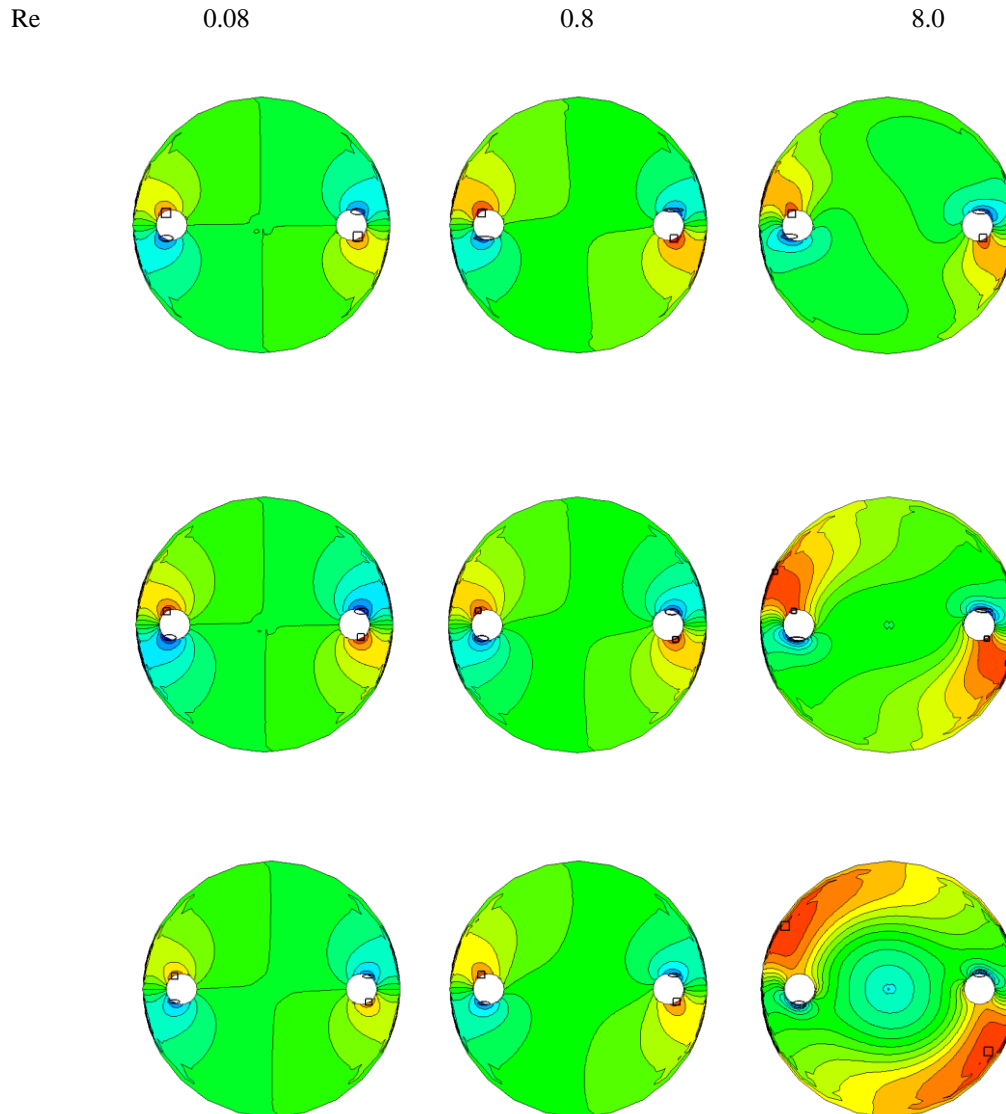


Figure 03: Pressure isobars for contra rotating stirrers; inertial levels 0.08, 0.8 and 8.0 (from left to right); speed of stirrers is 0.5, 1 and 2 (from top to bottom).

## V CONCLUSION

We have successfully demonstrated the use of a numerical flow solver for Newtonian fluids as a predictive tool for dough kneading. We have been able to provide physically realistic simulations for these complex rotating flows. Contra rotating stirrer solutions display certain aspects of symmetry. The streamlines and pressure isobars are localized in extrema in the neighbourhood of the stirrers. Maxima rate of work done noticed at the narrowest part of the nip-gap between stirrers and container. However, the power consumption is higher for this contra rotating case. Increase of container rotation speed, raises inertia, twists vortex patterns. The verse scenario to the container rotating demonstrates the actual industrial situation.

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