

# Analysis Stainless Steel and Nickel for Vibration Two Type FGM Cylindrical Shell

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## ABSTRACT

Study on the vibration of cylindrical shell made of a functionally gradient material (FGM) composed of stainless steel and nickel is presented. The effects of the FGM configuration are studied by studying the frequencies of two FGM cylindrical shells. Type I FGM cylindrical shell has Nickel on its inner surface and stainless steel on its outer surface and Type II FGM cylindrical shell has stainless steel on its inner surface. The study is carried out based on third order shear deformation shell theory. The objective is to study the natural frequencies, the influence of constituent volume fractions and the effects of configurations of the constituent materials on the frequencies. The properties are graded in the thickness direction according to the volume fraction power-law distribution. The governing equations are obtained using energy functional with the Rayleigh-Ritz method. Results are presented on the frequencies for a Type I, II FGM cylindrical shell.

**KEY WORDS:** Stainless Steel, Nickel, Vibration, FGM.

## INTRODUCTION

The study of the vibration of cylindrical shells is an important aspect in the successful applications of the cylindrical shells. The study of the free vibrations of cylindrical shells has been carried out extensively. Among those who have studied the vibrations of cylindrical shells include Arnold and Warburton [1], Ludwig and Krieg [2], Chung [3], Soedel [4], Forsberg [5], Bhimaraddi [6], Soldatos and Hajigeoriou [7], Bert and Kumar [8]. The concept of functionally gradient materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan, [9], [10]. as a means of preparing thermal barrier materials. Since then FGMs have attracted much interest as heat-Shielding materials. FGMs are made by combining different materials using power metallurgy methods [11].

They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc. and this results in gradients in the mechanical and thermal properties [12] and [13].

Studies on FGMs have been extensive but are largely confined to analysis of thermal stress and deformation [14], [15] and [16]. Najafizadeh and Isvandzibaei presented the vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support [17]. The advantage of on FGMs is that desired mechanical properties can be tailored and this holds enormous application potential for FGMs.

In this paper a study on the vibration of cylindrical shells composed of functionally gradient material (FGM) is presented. The functionally gradient material considered is composed of stainless steel and nickel where the volume fractions follow a power–law distribution. The objective is to study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies for two kind of FGM cylindrical shell. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. The displacement fields employ consist of some beam eigenfunctions of vibrations that guarantee satisfaction of edge boundary conditions.

## **1- FUNCTIONALLY GRADED MATERIAL**

For the cylindrical shell made of FGM the material properties such as the modulus of elasticity E, Poisson ratio v and the mass density  $\rho$  are assumed to be functions of the volume fraction of the constituent materials when the coordinate axis across the shell thickness is denoted by z and

\*Corresponding Author: Mohammad Reza Isvandzibaei is Ph.D. Student in Mechanical Engineering. Email: esvandzebaei@yahoo.com measured from the shell's middle plane. The functional relationships between E, v and  $\rho$  with z for a stainless steel and nickel FGM shell are assumed as

$$E = (E_1 - E_2) \left(\frac{2Z + h}{2h}\right)^N + E_2$$
 (1)

$$v = (v_1 - v_2) \left(\frac{2Z + h}{2h}\right)^N + v_2$$
 (2)

$$\rho = (\rho_1 - \rho_2) \left( \frac{2Z + h}{2h} \right)^N + \rho_2$$
 (3)

The strain-displacement relationships for a thin shell [18].

$$\epsilon_{11} = \frac{1}{A_1(1 + \frac{\alpha_3}{R_1})} \left[ \frac{\partial U_1}{\partial \alpha_1} + \frac{U_2}{A_2} \frac{\partial A_1}{\partial \alpha_2} + U_3 \frac{A_1}{R_1} \right]$$
(4)

$$\in_{22} \frac{1}{A_2(1+\frac{\alpha_3}{R_2})} \left[ \frac{\partial U_2}{\partial \alpha_2} + \frac{U_1}{A_1} \frac{\partial A_2}{\partial \alpha_1} + U_3 \frac{A_2}{R_2} \right]$$
(5)

$$\epsilon_{33} = \frac{\partial U_3}{\partial \alpha_3} \tag{6}$$

$$\epsilon_{12} = \frac{A_1(1+\frac{\alpha_3}{R_1})}{A_2(1+\frac{\alpha_3}{R_2})} \frac{\partial}{\partial \alpha_2} (\frac{U_1}{A_1(1+\frac{\alpha_3}{R_1})}) + \frac{A_2(1+\frac{\alpha_3}{R_2})}{A_1(1+\frac{\alpha_3}{R_1})} \frac{\partial}{\partial \alpha_1} (\frac{U_2}{A_2(1+\frac{\alpha_3}{R_2})})$$
(7)

$$\epsilon_{13} = A_1 \left(1 + \frac{\alpha_3}{R_1}\right) \frac{\partial}{\partial \alpha_3} \left(\frac{U_1}{A_1 \left(1 + \frac{\alpha_3}{R_1}\right)}\right) + \frac{1}{A_1 \left(1 + \frac{\alpha_3}{R_1}\right)} \frac{\partial U_3}{\partial \alpha_1}$$
(8)

$$\in_{23} = A_2(1 + \frac{\alpha_3}{R_2})\frac{\partial}{\partial\alpha_3}\left(\frac{U_2}{A_2(1 + \frac{\alpha_3}{R_2})}\right) + \frac{1}{A_2(1 + \frac{\alpha_3}{R_2})}\frac{\partial U_3}{\partial\alpha_2}$$
(9)

where  $A_1$  and  $A_2$  are the fundamental form parameters or Lame parameters,  $U_1$ ,  $U_2$  and  $U_3$  are the displacement at any point  $(\alpha_1, \alpha_2, \alpha_3)$ ,  $R_1$  and  $R_2$  are the radius of curvature related to  $\alpha_1, \alpha_2$  and  $\alpha_3$  respectively. The third- order theory of Reddy used in the present study is based on the following displacement field:

$$U_{1} = u_{1}(\alpha_{1},\alpha_{2}) + \alpha_{3}\phi_{1}(\alpha_{1},\alpha_{2}) + \alpha_{3}^{2}\psi_{1}(\alpha_{1},\alpha_{2}) + \alpha_{3}^{3}\beta_{1}(\alpha_{1},\alpha_{2})$$

$$U_{2} = u_{2}(\alpha_{1},\alpha_{2}) + \alpha_{3}\phi_{2}(\alpha_{1},\alpha_{2}) + \alpha_{3}^{2}\psi_{2}(\alpha_{1},\alpha_{2}) + \alpha_{3}^{3}\beta_{2}(\alpha_{1},\alpha_{2})$$

$$U_{3} = u_{3}(\alpha_{1},\alpha_{2})$$
(10)

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to  $\epsilon_{13} = \epsilon_{23} = 0$  at  $Z = \pm \frac{h}{2}$  Thus for third order theory

$$\begin{cases} U_1 = u_1(\alpha_1, \alpha_2) + \alpha_3 \phi_1(\alpha_1, \alpha_2) - C_1 \alpha_3^3 \left(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1 \partial \alpha_1}\right) \\ U_2 = u_2(\alpha_1, \alpha_2) + \alpha_3 \phi_2(\alpha_1, \alpha_2) - C_1 \alpha_3^3 \left(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2 \partial \alpha_2}\right) \\ U_3 = u_3(\alpha_1, \alpha_2) \end{cases}$$
(11)

where  $C_1 = \frac{4}{3h^2}$ . Substituting Eq. (11) into nonlinear strain-displacement relation (4) - (9), it can be obtained for the third-order theory of Reddy

$$\begin{cases} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{cases} = \begin{cases} \epsilon_{11}^{0} \\ \epsilon_{22}^{0} \\ \epsilon_{12}^{0} \end{cases} + \alpha_{3} \begin{cases} k_{11} \\ k_{22} \\ k_{12} \end{cases} + \alpha_{3}^{3} \begin{cases} k'_{11} \\ k'_{22} \\ k'_{12} \end{cases}$$
(12)

$$\begin{cases} \epsilon_{13} \\ \epsilon_{23} \end{cases} = \begin{cases} \gamma_{13}^{0} \\ \gamma_{23}^{0} \end{cases} + \alpha_{3}^{2} \begin{cases} \gamma_{13}^{2} \\ \gamma_{23}^{2} \end{cases} + \alpha_{3}^{3} \begin{cases} \gamma_{13}^{3} \\ \gamma_{23}^{2} \end{cases}$$
(13)

#### **2-FORMULATION**

Consider a cylindrical shell is shown in Fig.1. R is the radius, L is the length and h is the thickness. The reference surface is chosen to be the middle surface of the cylindrical shell where an orthogonal coordinate system  $x, \theta, z$  is fixed. The deformations of the shell with reference to this coordinate system are denoted by  $U_1, U_2$  and  $U_3$  in the  $x, \theta$  and z directions, respectively.



Fig. 1. Geometry of a FGM cylindrical shell

For a thin cylindrical shell, plane stress condition can be assumed. The constitutive relation for a thin cylindrical shell is consequently given by the tow-dimensional Hook's law as

$$\{\sigma\} = [Q]\{\varepsilon\}$$
(14)

where,  $\{\sigma\}$  is the stress vector,  $\{\varepsilon\}$  is the strain vector and [Q] is the reduced stiffness matrix. The stress vector for plane stress condition is

$$\{\sigma\}^{T} = \{\sigma_{11} \ \sigma_{22} \ \sigma_{12} \ \sigma_{13} \ \sigma_{23}\}$$
(15)

where  $\sigma_{11}$  is the stress in x direction,  $\sigma_{22}$  the stress in the  $\theta$  direction and  $\sigma_{12}$  is the shear stress on the  $x\theta$  plane and  $\sigma_{13}$  is the shear stress on the xz plane and  $\sigma_{23}$  is the shear stress on the  $\theta z$  plane. The strain vector is defined as

$$\{\boldsymbol{\varepsilon}_{1}^{T} = [\boldsymbol{\varepsilon}_{11} \, \boldsymbol{\varepsilon}_{22} \, \boldsymbol{\varepsilon}_{12} \, \boldsymbol{\varepsilon}_{13} \, \boldsymbol{\varepsilon}_{23}] \tag{16}$$

where  $\varepsilon_{11}$  is the strain in x direction,  $\varepsilon_{22}$  the strain in the  $\theta$  direction and  $\varepsilon_{12}$  is the shear strain on the  $x\theta$  plane and  $\varepsilon_{13}$  is the shear strain on the xz plane and  $\varepsilon_{23}$  is the shear strain on the  $\theta z$ plane. The reduced stiffness [Q] matrix is given as

$$\begin{bmatrix} Q \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}$$

(17)

For an isotropic cylindrical shell the reduced stiffness  $Q_{ij}$  (*i*, j=1, 2 and 6) are defined as

$$Q_{11} = Q_{22} = \frac{E}{1 - v^2} \tag{18}$$

$$Q_{12} = \frac{v E}{1 - v^2}$$
(19)

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} \tag{20}$$

where E is the Young's modulus and v is Poisson's ratio. For a thin cylindrical shell the force and moment results are defined as

$$\begin{cases} N_{11} \\ N_{22} \\ N_{22} \\ N_{22} \\ N_{22} \\ N_{22} \\ N_{22} \\ \sigma_{22} \\ \sigma_{22} \\ \sigma_{22} \\ \sigma_{22} \\ \sigma_{22} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{23}$$

$$\begin{cases} P_{11} \\ P_{22} \\ P_{12} \end{cases} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{cases} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{cases} \alpha_{3}^{3} d\alpha_{3} , \begin{cases} P_{13} \\ P_{23} \end{cases} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{13} \\ P_{23} \end{cases} \alpha_{3}^{3} d\alpha_{3}$$

$$(22)$$

$$\begin{cases} Q_{13} \\ Q_{23} \end{cases} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \sigma_{13} \\ \sigma_{23} \end{cases} d\alpha_{3} , \begin{cases} R_{13} \\ R_{23} \end{cases} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \sigma_{13} \\ \sigma_{23} \end{cases} \alpha_{3}^{2} d\alpha_{3}$$

$$(23)$$

The constitutive equation is obtained as

$$\{N\} = [S] \{\varepsilon\}$$
(24)

where  $\{N\}$  and  $\left\{ \varepsilon \right\}$  are, respectively, defined as

$$\left\{ \boldsymbol{\xi}^{T} = \left\{ \boldsymbol{\alpha}_{1}^{0} \boldsymbol{\varepsilon}_{2}^{0} \boldsymbol{\varepsilon}_{2}^{0} \boldsymbol{\kappa}_{1} \boldsymbol{k}_{2} \boldsymbol{\kappa}_{1} \boldsymbol{k}_{2} \boldsymbol{\kappa}_{1} \boldsymbol{k}_{2} \boldsymbol{\kappa}_{1} \boldsymbol{k}_{2} \boldsymbol{\kappa}_{2} \boldsymbol{\kappa}_{3}^{0} \boldsymbol{\gamma}_{3}^{0} \boldsymbol{\gamma}_{2}^{2} \boldsymbol{\gamma}_{3}^{2} \boldsymbol{\gamma}_{3}^{2} \boldsymbol{\gamma}_{3}^{3} \boldsymbol{\gamma}_{3}^{3} \right\}$$
(26)

and [S] is defined as

$$[S] = \begin{pmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \\ [E'] & [G] & [H'] \\ [A'] & [D'] & [E'] \\ [D'] & [F'] & [G] \end{pmatrix}$$
(27)

where A, B, E, D, F, H and G are the extensional, coupling and bending stiffness matrices and  $Q_{ij}$ are functions of z for functionally gradient materials. Here  $A_{ij}$  denote the extensional stiffness,  $D_{ij}$ the bending stiffness,  $B_{ij}$  the bending-extensional coupling stiffness and  $E_{ij}$ ,  $F_{ij}$ ,  $G_{ij}$ ,  $H_{ij}$  are the extensional, bending, coupling, and higher-order stiffness. Defining

$$\left[A_{j},B_{j},D_{j},E_{j},F_{j},G_{j},H_{j}\right] = \int_{W^{2}}^{W^{2}} Q_{j}\left[1,\alpha_{5},\alpha_{5}^{2},\alpha_{5}^{3},\alpha_{5}^{4},\alpha_{5}^{5},\alpha_{5}^{6}\right] d\alpha_{5}$$
(28)

The strain energy and kinetic energy of a cylindrical shell can be defined as

$$U = \frac{1}{2} \iiint \{\varepsilon\}^T \{\sigma\} \quad dV$$
<sup>(29)</sup>

$$T = \frac{1}{2} \iiint \left[ \left( \frac{\partial t_1}{\partial} \right)^2 + \left( \frac{\partial t_2}{\partial} \right)^2 + \left( \frac{\partial t_3}{\partial} \right)^2 \right] dV$$
(30)

where,  $\rho$  is the mass density,  $\{\varepsilon\}$  is the strain vector and  $\{\sigma\}$  is the stress vector. By substituting from Eq. (14), the strain and kinetic energies can be written as

$$U = \frac{1}{2} \int_{\circ}^{L} \int_{\circ}^{2\pi} \left\{ \varepsilon \right\}^{T} \left[ S \right] \left\{ \varepsilon \right\} R d\theta \, dx \tag{31}$$

$$T = \frac{1}{2} \int_{a}^{L} \int_{a}^{2\pi} \rho_{T} \left[ \left( \frac{\partial t_{1}}{\partial} \right)^{2} + \left( \frac{\partial t_{2}}{\partial} \right)^{2} + \left( \frac{\partial t_{3}}{\partial} \right)^{2} \right] R d d x$$
(32)

where  $\{\varepsilon\}$  is the strain vector defined in Eq. (26) and [S] is the stiffness matrix defined in relation (27). The parameter  $\rho_T$  is the density per unit length defined as

$$\rho_T = \int_{-h/2}^{h/2} \rho \, dz \tag{33}$$

The displacement fields for a cylindrical shell can be written as:

$$u_{1} = \overline{A} \frac{\partial \phi(x)}{\partial x} \cos(n\theta) \cos(\omega t)$$

$$u_{2} = \overline{B} \phi(x) \sin(n\theta) \cos(\omega t)$$

$$u_{3} = \overline{C} \phi(x) \cos(n\theta) \cos(\omega t)$$

$$\phi_{1} = \overline{D} \frac{\partial \phi(x)}{\partial x} \cos(n\theta) \cos(\omega t)$$

$$\phi_{2} = \overline{E} \phi(x) \sin(n\theta) \cos(\omega t)$$
(34)

where,  $\overline{A}$ ,  $\overline{B}$ ,  $\overline{C}$ ,  $\overline{D}$  and  $\overline{E}$  are the constants denoting the amplitudes of the vibrations in the  $x, \theta$  and z directions,  $\phi(x)$  is the axial function that satisfies the geometric boundary conditions, n denotes the number of circumferential waves in the mode shape and  $\omega$  is the natural angular frequency of the vibration. The axial function  $\phi(x)$  is chosen as the beam function as:

$$\phi(x) = \alpha_1 \cosh(\frac{\lambda_m x}{L}) + \alpha_2 \cos(\frac{\lambda_m x}{L}) - \zeta_m(\alpha_3 \sinh(\frac{\lambda_m x}{L}) + \alpha_4 \sin(\frac{\lambda_m x}{L}))$$
(35)

where  $\alpha_i$  (i = 1,...,4) are some constants with value 0 or 1 chosen according to the boundary conditions.  $\lambda_m$ , are the roots of some transcendental equations and  $\zeta_m$  are some parameters dependent on  $\lambda_m$ . The  $\alpha_i$  (i = 1,...,4), the transcendental equations and the parameters  $\zeta_m$  for the simply supported boundary condition considered. The geometric boundary condition for simply supported boundary condition can be expressed mathematically in terms of  $\phi(x)$  as:

$$\phi(x) = \phi''(x) = 0 \tag{36}$$

To determine the natural frequencies, the Rayleigh-Ritz method is used. The energy functional  $\Pi$  defined by the Lagrangian function as

$$\Pi = T_{\max} - U_{\max} \tag{37}$$

Substituting Eq. (34) into Eqs. (31) and (32) and minimizing the energy functional  $\Pi$  with respect to the unknown coefficients as follows,

$$\frac{\partial \Pi}{\partial \overline{A}} = \frac{\partial \Pi}{\partial \overline{B}} = \frac{\partial \Pi}{\partial \overline{C}} = \frac{\partial \Pi}{\partial \overline{D}} = \frac{\partial \Pi}{\partial \overline{E}} = 0$$
(38)

In Eq. (37),  $T_{max}$  and  $U_{max}$  are the maximum kinetic energy and strain energy, respectively. In Eq. (38), the five governing eigenvalue equations can be obtained. These five governing eigenvalue equation can be expressed in matrix from as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \begin{bmatrix} \overline{A} \\ \overline{B} \\ \overline{C} \\ \overline{D} \\ \overline{E} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
(39)

The eigenvalue equations are solved by imposing the non-trivial solutions condition and equating the determinant of the characteristic matrix  $[C_{ij}]$  to zero. Expanding this determinant, a polynomial in even powers of  $\omega$  is obtained

$$\beta_{\circ}\omega^{10} + \beta_{1}\omega^{8} + \beta_{2}\omega^{6} + \beta_{3}\omega^{4} + \beta_{4}\omega^{2} + \beta_{5} = 0$$

$$\tag{40}$$

where  $\beta_i$  (i = 0,1,2,3,4,5) are some constants. Eq. (40) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell in the x,  $\theta$  and z directions. The smallest of the five roots is the natural angular frequency studied in the present study.

## **3- RESULTS AND DISCUSSION**

In this paper studies are presented on the vibration of simply supported functionally graded (FG) cylindrical shell. The functionally gradient material (FGM) considered is composed of stainless steel and nickel and its properties are graded in the thickness direction according to the volume fraction power-law distribution. The influence of constituent volume fractions is studied by varying the volume fractions of the stainless steel and nickel. This is carried out by varying the value of the power law

exponent N. The effects of the FGM configuration are studied by studying the frequencies of two FG cylindrical shells. Type I FG cylindrical shell and Type II FG cylindrical shell. Type I FG cylindrical shell has Nickel on its inner surface and stainless steel on its outer surface and Type II FG cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. The material properties for stainless steel and nickel, calculated at T = 300K, are presented in table 1.

Coefficients	St	Nickel				
	Е	V	ρ	Е	V	ρ
<b>Р</b> <sub>0</sub>	$201.04 \times 10^{9}$	0.3262	8166	$223.95 \times 10^{9}$	0.3100	8900
Р_1	0	0	0	0	0	0
Р 1	$3.079 \times 10^{-4}$	$-2.002 \times 10^{-4}$	0	$-2.794 \times 10^{-4}$	0	0
P 2	-6.534 × 10 <sup>-7</sup>	$3.797 \times 10^{-7}$	0	-3.998 × 10 <sup>-9</sup>	0	0
Р 3	0	0	0	0	0	0
	$2.07788  imes 10^{11}$	0.317756	8166	2.05098×10 <sup>11</sup>	0.3100	8900

TABLE I: PROPERTIES OF MATERIALS

To validate the present analysis, results for cylindrical shells are compared with Chung [20]. The comparisons show that the present results agreed well with those in the literature.

TABLE II: COMPARISON OF FREQUENCY	$\left(\frac{rad}{s}\right)$	PARAMETER $\Omega = \omega R \sqrt{\frac{((1 - v^2)\rho)}{E}}$ FOR A CLAMPED-
CLAMPED IS	OTROF	PIC CYLINDRICAL SHELL

Ω							
Case	$\omega\left(\frac{rad}{s}\right)$	Chung [20]	Present				
(L / R) = 10 (R / h) = 500 n = 4	327.5406	0.01508	0.0154656				
(L / R) = 10 (R / h) = 20 n = 2	1254.2173	0.05787	0.0592211				
(L / R) = 2 (R / h) = 20 n = 3	1380.3668	0.3117	0.235887				

Tables 3 and 4 show the variations of the volume fractions  $V_f$  of Nickel and Stainless Steel, respectively, in the thickness direction z for a Type I FG cylindrical shell. the volume fraction for Nickel  $V_{fN}$  decreased from 1 at z = -0.5h to 0 at z = 0.5h and the volume fraction of Stainless Steel  $V_{fSS}$  increased from 0 at z = -0.5h to 1 at z = 0.5h.

TABLE III: VARIATION OF THE VOLUME FRACTION  $\,V_{f\!s\!s}$  in the thickness direction  $\,z$  for a type I FG cylindrical shell

$\overline{z}$	$V_{fss}$							
_	N=0.5	N=0.7	N=1	N=2	N=5	N=15		
-0.5h -0.4h -0.3h -0.2h -0.1h 0 0.1h 0.2h 0.3h 0.4h 0.5h	0 0.3162 0.4472 0.5477 0.6324 0.707 0.7745 0.8366 0.8944 0.9486 1	0 0.1995 0.3241 0.4305 0.5265 0.6155 0.6993 0.7790 0.8553 0.9289 1	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1	$\begin{array}{c} 0\\ 0.01\\ 0.04\\ 0.09\\ 0.16\\ 0.25\\ 0.36\\ 0.49\\ 0.64\\ 0.81\\ 1\end{array}$	0 0.00001 0.00032 0.00243 0.01024 0.03125 0.07776 0.1680 0.3276 0.3276 0.5904 1	$\begin{matrix} 0 \\ 10^{-15} \times 1 \\ 10^{-11} \times 3.27 \\ 10^{-8} \times 1.43 \\ 0.0000107 \\ 0.00003051 \\ 0.0004701 \\ 0.004747 \\ 0.03518 \end{matrix}$		
						0.20589		

	$V_{\ell N}$								
7	JIV								
4	N=0.5	N=0.7	N=1	N=2	N=5	N=15			
-0.5h	1	1	1	1	1	1			
-0.4h	0.6837	0.8004	0.9	0.99	0.9999	1			
-0.3h	0.5527	0.6758	0.8	0.96	0.9996	1			
-0.2h	0.4522	0.5694	0.7	0.91	0.9975	0.9999			
-0.1h	0.3675	0.4734	0.6	0.84	0.9897	0.9999			
0	0.2928	0.3844	0.5	0.75	0.9687	0.9999			
0.1h	0.2254	0.3006	0.4	0.64	0.9222	0.9995			
0.2h	0.1633	0.2209	0.3	0.51	0.8319	0.9952			
0.3h	0.1055	0.1449	0.2	0.36	0.6723	0.9648			
0.4h	0.0513	0.0710	0.1	0.19	0.4095	0.7941			
0.5h	0	0	0	0	0	0			

TABLE IV: VARIATION OF THE VOLUM FRACTION  $V_{fN}$  in the thickness direction z for a type I FG Cylindrical shell

In this section variations of natural frequencies with the circumferential wave number n for two type functional graded cylindrical shells with different volume fractions are presented. Tables 5 and 6 show variations of natural frequencies for type I FG cylindrical shell and type II FG cylindrical shell. The influence of the constituent volume fraction on the frequencies for Type I and II FG cylindrical shells has been found to be different. For the Type I FG cylindrical shells, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N increased. In Types I and II FG cylindrical shells, the natural frequencies for a stainless steel and Nickel cylindrical shells. For N < 1, the natural frequencies for Type I FG cylindrical shells are higher than for Type II FG cylindrical shells and for N > 1, the natural frequencies for Type II FG cylindrical shells are higher than Type I FG cylindrical shells.

## TABLE V: VARIATION OF NATURAL FREQUENCIES WITH THE CIRCUMFERENTIAL WAVE NUMBER n FOR A TYPE I FG CYLINDRICAL SHELL. m=1, h/R=0.002, L/R=20

	f(Hz)								
n	N=0.5	N=0.7	N=1	N=2	N=5	N=15			
1	13.319	13.267	13.209	13.101	12.996	12.930			
2	4.514	4.496	4.476	4.440	4.4046	4.382			
3	4.190	4.173	4.156	4.123	4.0914	4.070			
4	7.101	7.074	7.044	6.989	6.9357	6.899			
5	11.345	11.301	11.254	11.166	11.080	11.022			
6	16.609	16.545	16.475	16.348	16.222	16.1374			
7	22.848	22.760	22.664	22.489	22.315	22.199			
8	29.052	29.937	29.811	29.580	29.351	29.198			
9	38.219	38.072	37.912	37.618	37.328	37.133			
10	47.347	47.166	46.967	46.604	46.244	46.002			

TABLE VI: VARIATION OF NATURAL FREQUENCIES WITH THE CIRCUMFERENTIAL WAVE NUMBER n FOR A TYPE II FG CYLINDRICAL SHELL. m=1, h/R=0.002, L/R=20

	<i>f(Hz)</i>									
n	N=0.5	N=0.7	N=1	N=2	N=5	N=15				
1	13.321	13.269	13.211	13.103	12.997	12.932				
2	32.679	32.553	32.416	32.165	31.917	31.751				
3	91.388	91.037	90.652	89.951	89.258	88.795				
4	174.998	174.326	173.590	172.246	170.920	170.035				
5	282.855	281.769	280.578	278.407	276.263	274.832				
6	414.818	413.225	411.480	408.295	405.151	403.053				
7	570.840	568.648	566.246	561.863	557.537	554.650				
8	750.900	748.016	750.853	739.091	733.400	729.603				
9	954.987	951.319	947.300	939.968	932.731	927.902				
10	1183.096	1178.552	1173.573	1164.489	1155.52	1149.542				

#### **4-** Conclusions

A study on the vibration of functionally graded (FG) Cylindrical shell with clamped-clamped boundary condition composed of stainless steel and nickel has been presented. The study was carried

out for two types of functionally graded cylindrical shells where the configurations of the constituent materials in the functionally graded cylindrical shells are different. One is termed as a Type I FG cylindrical shell and has properties that vary continuously from nickel on its inner surface to stainless steel on its outer surface. The other is termed as a Type II FG cylindrical shell and has properties that vary continuously from stainless on its inner surface to nickel on its outer surface. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. Studied were made on study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies for two kind of FG cylindrical shell and the influence of boundary conditions simply support on the frequencies. The study showed that the constituent volume fractions and the configurations of the constituent materials affect the natural frequencies. However, because of the functionally graded cylindrical shells exhibit interesting frequency characteristics when the constituent volume fractions are varied. This is done by varying the power law exponent N. The influence of the constituent volume fraction on the frequencies for Type I and II FG cylindrical shells has been found to be different. For the Type I FG cylindrical shells, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N decreased. In Types I and II FG cylindrical shells, the natural frequencies for all values of N lie between those for a stainless steel and Nickel cylindrical shells. For N < 1, the natural frequencies for Type I FG cylindrical shells are higher than for Type II FG cylindrical shells and for N > 1, the natural frequencies for Type II FG cylindrical shells are higher than Type I FG cylindrical shells. Thus the constituent volume fractions and the configurations of the constituent materials affect the natural frequencies.

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