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# Effects of Non-Integer Order Derivative over the Slippage of Fractionalized Second Order Fluid Flow

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## ABSTRACT

This article is dedicatedly analyzed for the investigation of exponential flow over the slippage of fractionalized second order fluid under the influence of exponential plate. Fractionalized second order fluid is analyzed for the expression of velocity and shear stress profiles under the existence and nonexistence of slip effects. The general solutions are perused with help of discrete Laplace transform along with its inverse and expressed in the form of newly defined generalized hyper geometric function. The contrast among the solutions has been declared for ordinary as well as fractional types of fluid. All the solutions satisfy usual conditions (natural, boundary and initial conditions) for verification as well. Finally, Rheology of slippage, viscosity, fractional parameters, material parameters and few others have been underlined in order to bring physical aspects through graphical depictions.
**KEY WORDS:** Non-integer derivative, Slippage, Second Order Fluid, Discrete Transforms, depictions of Graphs.

## 1. INTRODUCTION

Numerous constitutive models for partial differential equations have been suggested for non-Newtonians fluids due to their typical and diverse structures [20-21]. In general, models of non-Newtonians fluids are divided in three brands; they are (i) the integral brand (ii) the rate brand and (iii) the differential brand. The best brand amongst them is the differential brand so called $n^{th}$grade model. In brevity, the $n^{th}$grade model has gotten huge importance due its various industrial and technological applications. In this manuscript, the second grade fluid commonly known as a simplest subclass lies in the category of differential brand. For second grade fluid one can optimistically expect the prediction regarding differences among normal stress for steady exponential flow over a rigid boundary [1-3].In this manuscript, the assumptions of slippage are analyzed under the influence of exponential plate. In continuation, it is well known fact that the slip boundary assumptions are adequate for characteristics of Newtonian fluids but in comparison these assumptions are not sufficient for all characteristics of non-Newtonian fluids. In general, impacts of slippage on non-Newtonian fluids have not attained much interest. Despite slip effects occurs in many technological applications and experimental observations for instance, polymer melts, emulsions, fractional wave diffusions, non-linear creeping, micro and nano channels and several others. To best of our knowledge, the literature regarding no slip conditions for second grade fluid includes. Fetecau and Corina[4] has investigated solutions for unsteady unidirectional flow without considering slip effects for second grade fluid. Hayat et al. [5]has achieved analytical solution in cylindrical geometries for second grade fluid in the absence of slip assumptions. Investigation of first problem of stoke's without slip effects in presence of porous medium for second grade fluid is perused by Tan and Masuoka [6]. Kashif [7] has considered influences of magnetohydrodynamics flow for second grade fluid in nonexistence of slippage. Free convection unsteady flows on vertical oscillating plate over second grade fluid have been obtained by Farhad[8]. Athar et al.[9]has tracedout rotational flow through circular cylinder for second grade fluid using Caputo fractional derivatives. Generalized second grade fluid flow between two parallel plates with fractional calculus approach has been investigated by Tan and Mingyu [10]. Mohamad et al [11] analyzed heated generalized second grade fluid by implementing a new spectral collocation technique in which they acquired high accuracy via certain numerical tests. They focused the results obtained for multi-dimensional fractional stokes' first problem. Samiulhaq et al. [12] observed a porous flow of a second-grade fluid induced by an infinite plate between two side walls that exerts an accelerated shear stress. They investigated exact solutions by using Laplace transform, finite Fourier cosine and sine transform on governing partial differential equation to have solutions for velocity field and shear stress. Furthermore, the concept of fractional calculus has focused the attention of researchers in exploring the enormous applications for modeling of the fluid mechanics with non-local phenomenon. The fractional approach is widely used in the many engineering and scientific fields because of its remarkable expansion in providing the results, either it is used in numerical or differentials schemes. The most common non-integer order fractional derivatives with singular kernel, Riemann-Liouville and Caputo derivative both of them are better in dealing mathematical problems and they have effective results from applications point of view [2-21].In concision, we include here recent literature referenced in [13-17].Motivated by above research work, we

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are interested for the investigation of exponential flow over the slippage of fractionalized second order fluid under the influence of exponential plate. Fractionalized second order fluid is analyzed for the expression of velocity and shear stress profiles under the existence and nonexistence of slip effects. The general solutions are perused with help of discrete Laplace transform along with its inverse and expressed in the form of newly defined generalized hyper geometric function. The contrast among the solutions has been declared for ordinary as well as fractional types of fluid. All the solutions satisfy usual conditions (natural, boundary and initial conditions) for verification as well. Finally, Rheology of slippage, viscosity, fractional parameters, material parameters and few others have been underlined in order to bring physical aspects through graphical depictions.

**2. Formulation of Problem with Governing Equations**

Flow equations for incompressible fluid include in the nonexistence of body forces are[18]

$$\rho\frac{\partial \boldsymbol{V}}{\partial t}+\rho(\boldsymbol{V}.\nabla)\boldsymbol{V}-\nabla.\boldsymbol{T}=0, \qquad \nabla.\boldsymbol{V}=0, \tag{1}$$

Where, $t, \nabla, \boldsymbol{V}, \rho, \boldsymbol{T}$are time, gradient operator, velocity of fluid, density of fluid, Cauchy stress tensor respectively and the cauchy stress $\boldsymbol{T}$given by

$$\boldsymbol{T}=-\boldsymbol{pI}+\boldsymbol{S}, \qquad \boldsymbol{S}=\alpha_2\boldsymbol{A}_1^2+\alpha_1\boldsymbol{A_2}+\mu\boldsymbol{A_1}, \tag{2}$$

here, $\boldsymbol{A_1}, \boldsymbol{A_2}, \alpha_1, \alpha_2, \mu, \boldsymbol{S}, -\boldsymbol{pI}$are kinematic tensors, normal stress moduli, dynamic viscosity, extra tensor, hydrostatic pressure. The kinematic tensors are expresed as

$$\begin{gathered}\boldsymbol{A_1}=\nabla.\mathrm{V}+(\nabla.\mathrm{V})^T\\ \boldsymbol{A_2}=\boldsymbol{A_1}(\nabla.\mathrm{V})+\boldsymbol{A_1}(\nabla.\mathrm{V})^T+\frac{d\boldsymbol{A_1}}{dt},\end{gathered} \tag{3}$$

For the problem under consideration, it is assumed for velocity field $\boldsymbol{V}$ and extra-stress tensor $\boldsymbol{S}$ of the form

$$\boldsymbol{S}=\boldsymbol{S}(y,t), \quad \boldsymbol{V}=\boldsymbol{V}(y,t)=u(y,t)\boldsymbol{i}, \tag{4}$$

For these flows the constraint of incompressibility is automatically satisfied. If the fluid is at rest up to the moment $t = 0$, then

$$\boldsymbol{S}=(y,0)=0, \quad \boldsymbol{V}=(y,0)=0, \tag{5}$$

we obtained governing differential equations for second grade flow

$$\frac{\partial V(y,t)}{\partial t}-\frac{\partial^2 V(y,t)}{\partial y^2}\left(\alpha\frac{\partial}{\partial t}+\nu\right)=0, \tag{6}$$

$$\tau(y,t)-\frac{\partial V(y,t)}{\partial y}\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)=0. \tag{7}$$

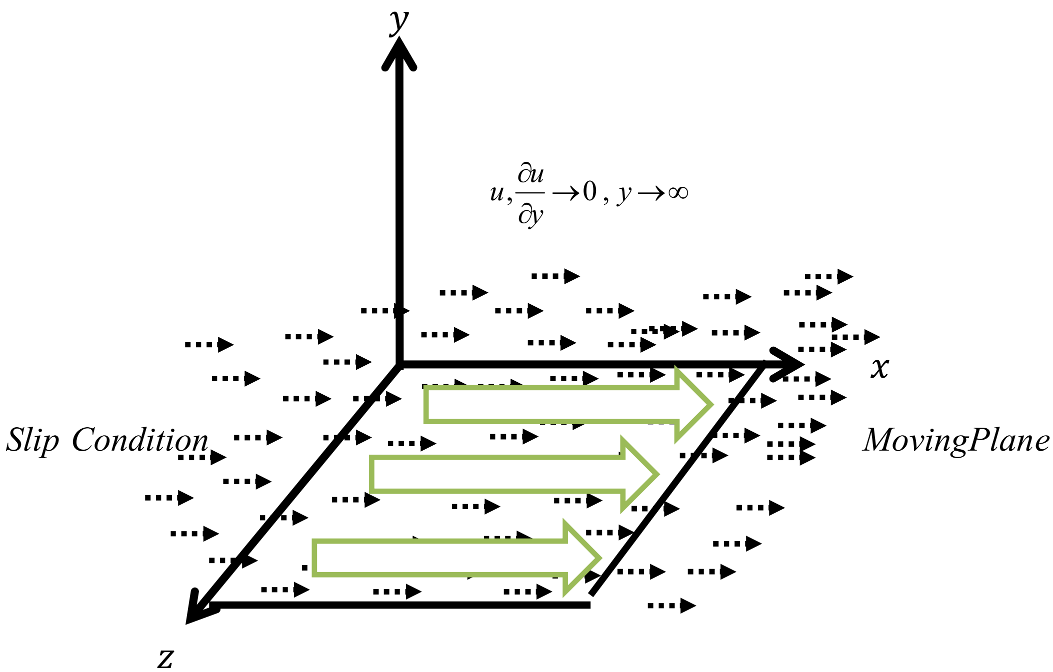
where, $\alpha=\alpha_1/\rho$ and $\nu=\mu/\rho$ are kinematic viscosity and $\alpha=\frac{\alpha_1}{\rho}$ the viscoelastic parameter of second grade fluid. using caputo operatorfor the fractional parameter is $0<\delta<1$ and the fractional differential operator $D_t^\delta$ is described as [19]

$$D_t^\delta W(t)=\begin{cases}\dfrac{1}{\Gamma(1-\delta)}\displaystyle\int_0^t\frac{W'(Y)}{(t-Y)^\delta}dY, & 0<\delta<1\\ \dfrac{dW(t)}{dt}, & \delta=1\end{cases}, \tag{8}$$

$$\frac{\partial V(y,t)}{\partial t}-\frac{\partial^2 V(y,t)}{\partial y^2}\left(\alpha D_t^\delta+\nu\right)=0, \tag{9}$$

$$\tau(y,t)-\frac{\partial V(y,t)}{\partial y}\left(\mu+\alpha_1 D_t^\delta\right)=0. \tag{10}$$

Assume that an unsteady fractionalized fluid of order second possessing the space lying over an interminably amplified plane having its location in $xz$ plane and vertical to $y$- axis. At the very initial, the fluid is at rest and the occasion $t=0^+$ the plane begins to waver in its own particular plane. Here we accept the presence of slip limit between the speed of the fluid at the plane. Because of shear, the liquid over the plane is steadily moved as described in goemetry of the problem:




**Depiction of Geometry of the problem**

The fitting initial and boundary conditions are

$$V(y,t), \frac{\partial V(y,t)}{\partial t} \to 0 \quad \text{as} \quad y \to \infty \quad and \quad t > 0, \tag{11}$$

$$V(0,t) = \Omega H(t) Exp(bt) + \chi V_t(y,t)|_{y=0} \quad t \geq 0. \tag{12}$$

$$V(y,0) = 0, \qquad \tau(y,0) = 0, \qquad y > 0, \tag{13}$$

are mollified as natural, boundary and initial conditions.

**3. Exploration of Velocity Field**

Applying Laplace trasform to equation (9) and keeing in consideration equations (11) and (13), we found

$$\frac{s}{(\alpha s^{\delta} + v)} V(y,s) = \frac{\partial^2 V(y,t)}{\partial y^2}, \tag{14}$$

Using bounary conditions in equation (14), we have

$$V(y,s) = \frac{\Omega e^{-y\sqrt{\frac{s}{(\alpha s^{\delta}+v)}}}}{(s-b)\left\{1 + \chi\sqrt{\frac{s}{(\alpha s^{\delta}+v)}}\right\}}, \tag{15}$$

Before applying discrete Laplace transform, firstly we rework on equation (15) for series form as

$$V(y,s) = \frac{\Omega}{(s-b)} + \Omega \sum_{\varepsilon=0}^{\infty} (b)^{\varepsilon} \sum_{\rho=1}^{\infty} \left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho} \sum_{\vartheta=0}^{\infty} \left(\frac{-v}{\alpha}\right)^{\vartheta} \frac{\Gamma\left(\varepsilon + \frac{\rho}{2}\right)}{\vartheta!\,\Gamma\left(\frac{\rho}{2}\right)} \frac{1}{s^{(\delta-1)\frac{\rho}{2}+\vartheta\delta+\varepsilon+1}} + \Lambda \sum_{\varepsilon=0}^{\infty} (b)^{\varepsilon} \sum_{\rho=0}^{\infty} \left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}$$
$$\times \sum_{\vartheta=0}^{\infty} \left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta} \frac{\Gamma\left(\varsigma + \frac{\vartheta+\rho}{2}\right)}{\varsigma!\,\Gamma\left(\frac{\vartheta+\rho}{2}\right)} \frac{1}{s^{(\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\delta\varsigma+\varepsilon+1}}, \tag{16}$$

Inverting equation (16) by Laplace Transfrom, we attain

$$V(y,t)=\Omega H(t)e^{bt}+\Omega H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=1}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\vartheta=0}^{\infty}\frac{\left(\frac{-\nu}{\alpha}t^{\delta}\right)^{\vartheta}\Gamma\left(\varepsilon+\frac{\rho}{2}\right)t^{(\delta-1)\frac{\rho}{2}+\varepsilon}}{\vartheta!\,\Gamma\left(\frac{\rho}{2}\right)\Gamma\left((\delta-1)\frac{\rho}{2}+\vartheta\delta+\varepsilon+1\right)}+\Omega\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}$$

$$\times\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\varsigma=0}^{\infty}\frac{\left(\frac{-\nu}{\alpha}t^{\delta}\right)^{\varsigma}\Gamma\left(\varsigma+\frac{\vartheta+\rho}{2}\right)t^{(\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\varepsilon}}{\varsigma!\,\Gamma\left(\frac{\vartheta+\rho}{2}\right)\Gamma\left((\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\delta\varsigma+\varepsilon+1\right)},\tag{17}$$

expressing equation (17) in terms of wright generalized Hyper-geometric function, we get simple expression for velocity as

$$V(y,t)=\Omega H(t)e^{bt}+\Omega H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=1}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\ {}_1\Psi_2\left[-\frac{\nu t^{\delta}}{\alpha}\middle|\begin{matrix}\left(\frac{\rho}{2},1\right)\\ \left(\frac{\rho}{2},0\right),\left((\delta-1)\frac{\rho}{2}+\varepsilon+1,\delta\right)\end{matrix}\right]t^{(\delta-1)\frac{\rho}{2}+\varepsilon}$$

$$+\Omega\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}\ {}_1\Psi_2\left[-\frac{\nu t^{\delta}}{\alpha}\middle|\begin{matrix}\left(\frac{\vartheta+\rho}{2},1\right)\\ \left(\frac{\vartheta+\rho}{2},0\right),\left((\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\varepsilon+1,\delta\right)\end{matrix}\right]t^{(\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\varepsilon}.\tag{18}$$

Where, the property of wright generalized Hyper-geometric function is

$$\sum_{i}^{\infty}\frac{(-B)^{i}\prod_{k=1}^{\alpha}\Gamma(c_k+C_k i)}{i!\,\prod_{k=1}^{\beta}\Gamma(d_k+D_k i)}={}_{\boldsymbol{\alpha}}\boldsymbol{\Psi}_{\boldsymbol{\beta}}\left[B\middle|\begin{matrix}(f_1,F_1),(f_2,F_2),\dots,(f_\alpha,F_\alpha)\\(g_1,G_1),(g_2,G_2),\dots,(g_\beta,G_\beta)\end{matrix}\right].$$

**4. Exploration of Shear Stress**

Applying Laplace trasform to equation (10) and keeing in consideration equations (11) and (13), we found

$$\bar{\tau}(y,s)-\frac{\partial\bar{V}(y,s)}{\partial y}\left(\alpha_1 s^{\delta}+\mu\right)=0,\tag{19}$$

Substituting the value of $\frac{\partial\bar{V}(y,s)}{\partial y}$ in equation (19), we get

$$\bar{\tau}(y,s)=\frac{-\Omega\,\rho\,e^{-y\sqrt{\frac{s}{(\alpha s^{\delta}+\nu)}}}\sqrt{(\alpha s^{\delta}+\nu)}}{(s-b)\left\{1+\chi\sqrt{\frac{r}{(\alpha s^{\delta}+\nu)}}\right\}},\tag{20}$$

Before applying Laplace transform, firstly we rework on equation (20) for series form as

$$\bar{\tau}(y,s)=-\Omega\sqrt{\alpha}\,\rho\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}\sum_{\varsigma=0}^{\infty}\frac{\left(\frac{-\nu}{\alpha}\right)^{\varsigma}\Gamma\left(\varsigma+\frac{\vartheta+\rho-1}{2}\right)}{\varsigma!\,\Gamma\left(\frac{\vartheta+\rho-1}{2}\right)s^{(\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\delta\varsigma+\varepsilon}},\tag{21}$$

applying Laplace transform to equation (21) and expressing it in the format of wright generalized Hyper-geometric function, we get simple expression for shear stress as

$$\tau(y,t)=-\Omega\sqrt{\alpha}\,\rho\,H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}$$

$$\times\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t^{\delta}\middle|\begin{matrix}\left(\frac{\vartheta+\rho-1}{2},1\right)\\ \left(\frac{\vartheta+\rho-1}{2},0\right),\left((\delta-1)\left(\frac{\vartheta+\rho-1}{2}\right)+\varepsilon,\delta\right)\end{matrix}\right]t^{(\delta-1)\left(\frac{\vartheta+\rho}{2}\right)+\varepsilon}.\tag{22}$$

Equations (18) and (22) are the solutions of velocity and shear stress respectively satisfying initial and boundary conditions as well.

**5. Special Solutions**

**Solutions of second grade fluid in the absence of slippage if $\chi \to 0$**

Permitting $\chi \to 0$ in equations (18) and (22), we acquire

$$V(y,t) = \Omega H(t)e^{bt} + \Omega H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\vartheta=1}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}$$

$$\times\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t^{\delta}\left|\begin{matrix}\left(\frac{\vartheta}{2},1\right)\\ \left(\frac{\vartheta}{2},0\right),\left((\delta-1)\left(\frac{\vartheta}{2}\right)+\varepsilon+1,\delta\right)\end{matrix}\right.\right]t^{(\delta-1)\left(\frac{\vartheta}{2}\right)+\varepsilon}. \tag{23}$$

$$\tau(y,t) = -\Omega\sqrt{\alpha}\,\rho\,H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}$$

$$\times\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t^{\delta}\left|\begin{matrix}\left(\frac{\vartheta-1}{2},1\right)\\ \left(\frac{\vartheta-1}{2},0\right),\left((\delta-1)\left(\frac{\vartheta-1}{2}\right)+\varepsilon,\delta\right)\end{matrix}\right.\right]. \tag{24}$$

Ordinary solutions of second grade fluid in the presence of slippage if $\delta \to 1$ and $\chi \neq 0$

Letting $\delta \to 1$ and $\chi \neq 0$ in equations (18) and (22), we acquire

$$V(y,t) = \Omega H(t)e^{bt} + \Omega H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=1}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t\left|\begin{matrix}\left(\frac{\rho}{2},1\right)\\ \left(\frac{\rho}{2},0\right),(\varepsilon+1,1)\end{matrix}\right.\right]t^{\varepsilon} + \Lambda\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}$$

$$\times\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t\left|\begin{matrix}\left(\frac{\vartheta+\rho}{2},1\right)\\ \left(\frac{\vartheta+\rho}{2},0\right),(\varepsilon+1,1)\end{matrix}\right.\right]t^{\varepsilon}. \tag{25}$$

$$\tau(y,t) = -\Omega\sqrt{\alpha}\,\rho\,H(t)\sum_{\varepsilon=0}^{\infty}(b)^{\varepsilon}\sum_{\rho=0}^{\infty}\left(\frac{\chi}{\sqrt{\alpha}}\right)^{\rho}\sum_{\vartheta=0}^{\infty}\left(\frac{-y}{\sqrt{\alpha}}\right)^{\vartheta}\ {}_1\Psi_2\left[-\frac{\nu}{\alpha}t\left|\begin{matrix}\left(\frac{\vartheta+\rho-1}{2},1\right)\\ \left(\frac{\vartheta+\rho-1}{2},0\right),(\varepsilon,1)\end{matrix}\right.\right]t^{\varepsilon}. \tag{26}$$

Furthermore, one can investigate the few limiting solutions for instance, when $\delta \to 1$ and $\alpha \to 0$ solutions are termed into ordinary fluid and Newtonian fluid from general solutions as well.

## 6. RESULTS AND DISCUSSIONS

In this portion, numerical discussion regarding results and their effects are highlighted for the investigation of exponential flow over the slippage of fractionalized second order fluid under the influence of exponential plate. Fractionalized second order fluid is analyzed with help of graphical depiction under the existence and nonexistence of slip effects. The general solutions are plotted using distinct rheology of slippage, viscosity, fractional parameters, and material parameters. It is worth pointed out that while depiction of graphical illustrations, we have considered couple of graphs for velocity field and shear stress respectively. However, the major outcomes are:

- In Fig.1, by fixing all rheology except time parameter in presence and absence of slip assumption, the effects display the velocity is decreasing while shear stress is increasing at variation of time for the whole domain.
- The sequestrating and scattering behavior of fluid flow has been identified on the plate by increasing the fractional parameter at domain in Fig. 2 in presence and absence of slip assumption. This phenomenon happens when we consider the order of fractionalization as $0.2 \leq \delta \leq 0.8$.

- Fig. 3 is prepared to display effects of viscosity in presence and absence of slip assumption in which velocity field and shear stress has contradictory behavior of fluid as expected on the exponential plate with the no slip assumptions.
- Due to increment in exponential flow shown in Fig. 4, we observed that the range of fluid flow for coincident without slip effect and maximum with slip effect at the free surface.
- Fig. 5 indicates the comparisons made on fractionalized and ordinary fluid flows in which it is observed that both the velocity field as well as shear stress have reciprocal behavior in four models of fluid namely (i) fractionalized second grade fluid, (ii) ordinary second grade fluid, (iii) fractionalized Newtonian fluid and (iv) ordinary Newtonian fluid.

## 7. CONCLUSION

The conclusion is based on the rheological and pertinent parameters, the effects of such parameters have key notes similarities and differences which are:

- The velocity is decreasing and shear stress is increasing at variation of time either slip strength is considered or not.
- Fractional parameter is considered for the order of fractionalization as $0.2 \le \delta \le 0.8$. which represent sequestrating and scattering behavior of fluid flows.
- Effects of viscosity in presence and absence of slip assumption has reversal behavior of fluid flows.
- The analysis for the comparisons on fractionalized and ordinary fluid flows suggested opposite effects on different models.

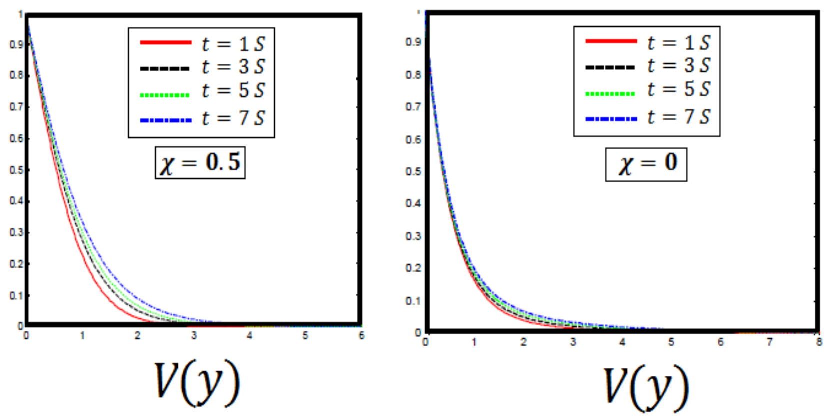



**Fig. 1: Effects of time parameter on the velocity field with and without slip assumptions.**

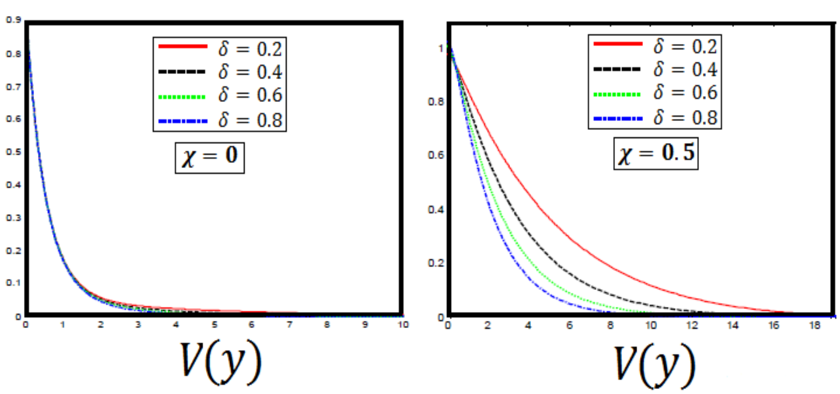



**Fig. 2: Effects of fractional parameter on the velocity field with and without slip assumptions.**

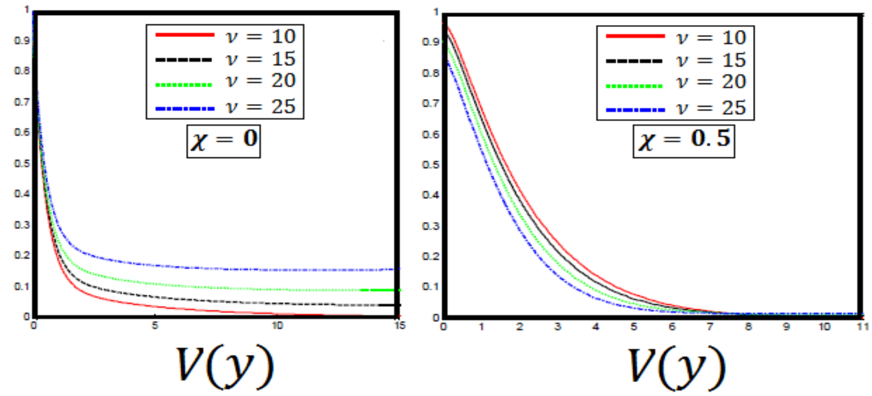



**Fig. 3: Effects of viscosity on the velocity field with and without slip assumptions.**

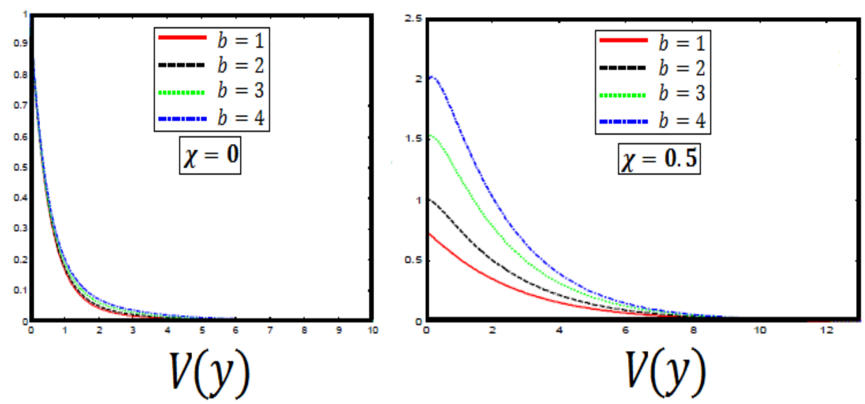



**Fig. 4: Effects of exponential flow on the velocity field with and without slip assumptions.**

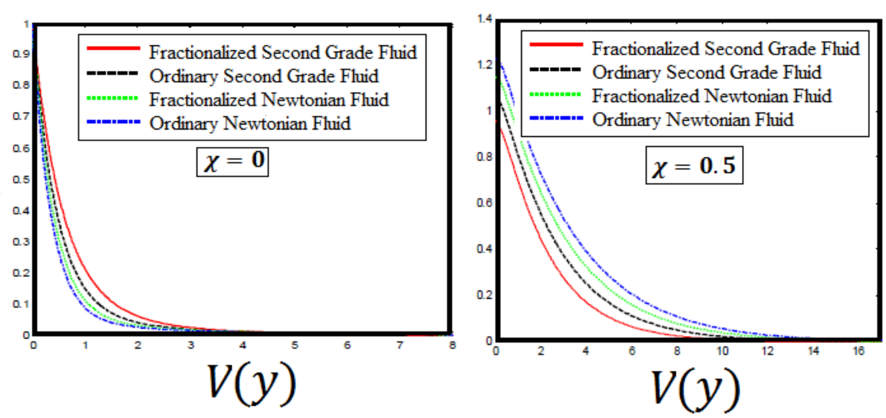



**Fig. 5: Comparison of four rheological models on the velocity field with and without slip assumptions.**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