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Some Contributions in Soft MTL-Algebras

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ABSTRACT

Characterizations of soft filters in MTL algebras are given. The concept of Boolean soft filters and MV-soft filters are presented & related possessions are explored. A situation for a softfilter to be Boolean is delivered. A characterization of a Boolean soft filter is given.
KEYWORDS:MTL-algebras, Soft sets, Soft filters, Boolean soft filters, MV-soft filter.

1. INTRODUCTION

In [2], Esteva and Godo introduced the logic known as, Monoidal t-norm grounded logic. [2] Presented a different algebra called an MTL-algebra and deliberate numerous rudimentary possessions of MTL-algebra. Haveshki et al [3] introduced the concept of filter in MTL-algebras. The concept of Fuzzy set was initiated by Zadeh [8]. Kim et al. [5] and jun et al. [3] studied the fuzzy structure of filters in MTL-algebras. The concept of soft set was initiated by Molodtsov [7]. In [6] Maji et defined some operations on soft set. These operations were corrected by Ali et al [1]. In this paper we introduced soft filters in MTL-algebras and investigate possessions of soft filters in MTL-algebras. In section 2, we recall some basic definitions of MTL-algebras. In section 4, we have given the definition of Boolean soft filter and some characterizations of Boolean soft filters of an MTL-algebra are investigated. In section 5, we have given the definition of an MV-soft filter and provided a situation for a Boolean soft filter to be an MV-soft filter.

2. PREMİLİNRİES

By a resituated framework we mean a lattice $E=(E,\le,\wedge,\vee,\otimes,\to,0,1)$ comprising the smallest component 0 and the biggest component 1, and capable through double binary procedures $\otimes$ (Product) & $\to$ (Residuum) such that

$\otimes$ is an isotone, commutative & associative.

$(x\otimes 1=x)\ \forall x\in E$

The Galois communicationgrips, which is,

$$x\otimes y\le z \Leftrightarrow x\le y\to z$$

for all $x,y,z\in E$.

2.1 Definition [4]

An MTL-algebra is a resituated framework $E=(E,\le,\wedge,\vee,\otimes,\to,0,1)$ sustaining the prelinearity equation:

$$(x\to y)\vee(y\to x)=1$$

for all $x,y\in E$.

2.2 Proposition [4]

The following are true for an MTL-algebra E. For all $x,y,z\in E$

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(u_1) $x \le y \Leftrightarrow x \to y = 1.$

(u_2) $0 \to x = 1, 1 \to x = x, x \to (y \to x) = 1.$

(u_3) $y \le (y \to x) \to x.$

(u_4) $x \to (y \to z) = (x ? y) \to z = y \to (x \to z).$

(u_5) $x \to y \le (z \to x) \to (z \to y), (x \to y) \le (y \to z) \to (x \to z).$

(u_6) $y \le x \Rightarrow x \to z \le y \to z, z \to y \le z \to x.$

(u_7) $(\vee_{i\in\Gamma} y_i) \to x = \wedge_{i\in\Gamma} (y_i \to x).$

We describe $x^* = \vee\{y \in E \mid x \otimes y = 0\}$, consistently, $x^* = x \to 0$. then

(u_8) $0^* = 1, 1^* = 0, x \le x^{**}, x^* = x^{***}.$

In a MTL-algebra, the following are true

(u_9) $x \to (y \vee z) = (x \to y) \vee (x \to z).$

(u_{10}) $x \otimes y \le x \wedge y.$

2.3 Definition **[4, 5]**

Let " E " is MTL-algebra. A non-empty sub-set F of E is said to be a filter of E if it fulfills

(a_1) $x \otimes y \in F$ for all $x, y \in F$.

(a_2) If $x \in F$, $x \le y$ then $y \in F$ for all $y \in E$.

2.4 Proposition [4, 5]

A non-empty sub-set F of MTL-algebra E is a filter of E only if it fulfills the following

(F_1) $1 \in F$

(F_2) $x \to y \in F \;\Rightarrow\; y \in F$. for all $x, y \in E$.

2.5 Definition [3]

Let " F " be a non-empty subset of MTL-algebra E. Then F is called a Boolean filter of E if, F is a filter of E such that $x^* \vee x \in F$, for all $x \in E$ where $x^* = x \to 0$.

2.6 Definition [3]

Let F be a non-empty subset of an MTL-algebra E. Then F is called an MV-filter if

$$x \to y \in F \text{ implies } (((y \to x) \to x) \to y) \in F.$$

2.7 Definition [1, 6, 7]

Suppose that U is a preliminary universe & E is the set of all parameters under consideration with respect to U. The power set of U $(i.e \text{ the set of all subsets of } U)$ is denoted by $P(U)$ and A is a subset of E.

Usually, parameters are attributes, characteristics, or properties objects in U.

A couple of (F, A) is said to be a soft set above U, where the mapping F is a given by

$$F : A \to P(U).$$

Additionally, a softset above U is a family which is parameterized of universal U subsets.

3. SOFT FILTERS

In this section, we define soft filter of an MTL-algebra. Some characterizations of a soft filter are investigated. Throughout this paper E is an MTL-algebra and U is a non-empty set.

3.1 Definition

A softset (F, E) above U is called a soft-filter of an MTL-algebra E if F satisfies

(1) $F(x \otimes y) \supseteq F(x) \cap F(y)$ for all $x, y \in E$.

(2) F is an order-preserving, that is, $x \leq y \Rightarrow F(x) \subseteq F(y)$, for all $x, y \in E$.

3.2 Example

Let $E = [0,1]$. Define $\otimes$ and $\rightarrow$ as follows

$$x \otimes y = \begin{cases} x \wedge y & \text{if } x + y > 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y, \\ (1-x) \vee y & \text{otherwise,} \end{cases}$$

For all $x, y \in E$. Then E is an MTL-algebra. Let us define a soft set (F, E) over E by

$$F : E \rightarrow P(E)$$

$$F(x) = \begin{cases} H \text{ if } x \in [0, 0.5] \\ E \text{ if } x \in (0.5, 1] \end{cases}$$

Where $H \subset E$. Then F is a soft filter of E.

3.3 Theorem

A softset (F, E) above U is a softfilter in an MTL-algebra E only if it fulfills the following conditions

(1) $F(1) \supseteq F(x)$

(2) $F(y) \supseteq F(x) \cap F(x \rightarrow y)$

For all $x, y \in E$.

Proof: Suppose that the soft set (F, E) over U satisfies conditions (1) and (2). Let x & y $\in E$ be like $x \leq y$ which implies that $x \rightarrow y = 1$. Now by (2)

$$\begin{aligned} F(y) &\supseteq F(x) \cap F(x \rightarrow y) \\ &= F(x) \cap F(1) \quad \text{because } (x \rightarrow y = 1) \\ &= F(x) \quad \text{by } (1) \\ &\Rightarrow F(x) \subseteq F(y). \end{aligned}$$

Since,

$x \rightarrow (y \rightarrow (x \otimes y)) = (x \otimes y) \rightarrow (x \otimes y) = 1$ because $x \rightarrow (y \rightarrow z) = (x \otimes y) \rightarrow z$.

We have from (2)

$$\begin{aligned} F(x \otimes y) &\supseteq F(y) \cap F(y \rightarrow (x \otimes y)) \\ &\supseteq F(y) \cap \{F(x) \cap F(x \rightarrow (y \rightarrow (x \otimes y)))\} \quad \text{by } (2) \\ &= F(y) \cap \{F(x) \cap F((x \otimes y) \rightarrow (x \otimes y))\} \quad \text{because } x \rightarrow (y \rightarrow z) = (x \otimes y) \rightarrow z \\ &= F(y) \cap \{F(x) \cap F(1)\} \quad \text{because } (x \rightarrow x = 1) \\ &= F(y) \cap F(x) \quad \text{because } F(x) \subseteq F(1) \end{aligned}$$

$$F(x \otimes y) \supseteq F(x) \cap F(y).$$

Thus F is a soft filter of E.

Conversely, Let F be a softfilter of E. Since $x \leq 1 \, \forall \, x \in E$. we have $F(x) \subseteq F(1) \, \forall \, x \in E$.

Now, let $x, y \in E$, $x \leq (x \rightarrow y) \rightarrow y$ which implies that

$x \otimes (x \rightarrow y) \leq y$ by Galois correspondence $(x \otimes y) \leq z \Rightarrow x \leq y \rightarrow z$

Hence,

$$F(y) \supseteq F(x \otimes (x \to y))$$
$$\supseteq F(x) \cap F(x \to y) \text{ by Definition 3.1}$$
$$\Rightarrow F(y) \supseteq F(x) \cap F(x \to y).$$

3.4 Theorem

A softset (F,E) above U is a soft filter of an MTL-algebra E only if it fulfills the following condition
$a \le b \to c \;\Rightarrow\; F(c) \supseteq F(a) \cap F(b)\ \forall\ a,b,c \in E.$

Proof. Suppose that F is a soft filter of E. Let a,b & c $a,b,c \in E$ be like that $a \le b \to c$. Then $F(a) \subseteq F(b \to c)$.

Also,

$$F(c) \supseteq F(b) \cap F(b \to c) \text{ by Theorem 3.3}$$
$$\supseteq F(b) \cap F(a) \quad \text{because } F(a) \subseteq F(b \to c)$$
$$\Rightarrow F(c) \supseteq F(a) \cap F(b).$$

Conversely, suppose that the soft set (F,E) over U satisfies, for all $a,b,c \in E$
$a \le b \to c$ implies $F(c) \supseteq F(a) \cap F(b)$.

We show that F is a soft filter in E.

Since, $x \le x \to 1$ for all $x \in E$, we have

$$F(1) \supseteq F(x) \cap F(x) = F(x)$$
$$\Rightarrow F(1) \supseteq F(x).$$

Since, $x \to y \le x \to y$ for all $x, y \in E$, we have

$$F(y) \supseteq F(x) \cap F(x \to y).$$

Hence by Theorem 3.3, F is a soft filter of E.

3.5 Definition

Let if (F,E) remain a soft set above U, where E is an MTL-algebra and $T \subseteq U$. Then the set

$$F_T = F[T] = \{x \in E \mid F(x) \supseteq T\}$$

is called a soft level subset of F.

3.6 Theorem

The soft set (F,E) above U which is soft filter in an MTL-algebra E if and only if the level soft set $F_T = F[T] = \{x \in E \mid F(x) \supseteq T\}$ is either empty or a filter of E.

Proof. Suppose F_T is a filter of E if non-empty. Let $x \in E$ exist like $F(x) \supset F(1)$.
Take $T = F(x)$. At that time $x \in F_T$ but $1 \notin F_T$, contradiction. Thus $F(1) \supseteq F(x)$.
Let $x, y \in E$ be like take

$$T_1 = F(x) \cap F(x \to y).$$

Then $x, x \to y \in F_{T_1}$ but $y \notin F_{T_1}$, contradiction.

Therefore

$$F(y) \supseteq F(x) \cap F(x \to y)$$

This implies that F is a soft filter of E.

Conversely, let F remain a soft filter of E and $T \in P(U)$, such that $F_T \ne \phi$. Then there exist $x_o \in F_T$ such that $F(x_o) \supseteq T$. As $F(1) \supseteq F(x_o)$. we have $F(1) \supseteq T$, that is $1 \in F_T$. If $x, x \to y \in F_T$ then

$$F(x) \supseteq T \text{ and } F(x \to y) \supseteq T$$
$$\Rightarrow F(x) \cap F(x \to y) \supseteq T.$$

Since

$$F(y) \supseteq F(x) \cap F(x \to y),$$

we have $F(y) \supseteq T$, that is $y \in F_T$. Thus F_T is a filter.

3.7 Theorem

If F is a soft filter of an MTL-algebra E, then the set

$$\Omega_a = \{x \in E \mid F(x) \supseteq F(a)\}$$

is a filter of E for all $a \in E$.

Proof. If F is a softfilter of E, so for all $x \in E$, $F(1) \supseteq F(x) \Rightarrow 1 \in \Omega_a$. Lets take $x, y \in E$ be like $x \in \Omega_a$ and $x \to y \in \Omega_a$. Then $F(x) \supseteq F(a)$ and $F(x \to y) \supseteq F(a)$. Since F is a soft filter of E, it follows that

$$F(y) \supseteq F(x) \cap F(x \to y) \supseteq F(a) \cap F(a)$$

Thus $F(y) \supseteq F(a)$. Hence $y \in \Omega_a$. This implies that Ω_a is the filter of E.

3.8 Theorem

Let $a \in E \,\&\, (F, E)$ is a soft-set over U. then

1. If Ω_a is the filter of an MTL-algebra E, then F fulfills the subsequent condition:

$F(a) \subseteq F(x \to y) \cap F(x) \Rightarrow F(a) \subseteq F(y)$ For all $x, y \in E$.

2. If F satisfies

$F(x) \subseteq F(1)$ and $F(a) \subseteq F(x) \cap F(x \to y) \Rightarrow F(a) \subseteq F(y)$.

For all $x, y \in E$, then Ω_a is a filter of E.

Proof. (1) Supposing that, Ω_a is the filter of E. Lets the $x, y \in E$ be like that $F(a) \subseteq F(x \to y) \cap F(x)$.

Then $x \in \Omega_a$ and $x \to y \in \Omega_a$. This implies that $y \in \Omega_a$ and so $F(y) \supseteq F(a)$.

(2) Assume that F fulfills given conditions. It monitors that $1 \in \Omega_a$. Let $x, y \in E$ be such that $x \in \Omega_a$ and $x \to y \in \Omega_a$ Then

$F(x) \supseteq F(a)$ and $F(x \to y) \supseteq F(a)$.

Whichsuggests that

$$F(a) \subseteq F(x) \cap F(x \to y).$$

Thus

$$F(a) \subseteq F(y),$$

so $y \in \Omega_a$. Therefore Ω_a is a filter of E.

3.9 Proposition

Let F be a soft filter of an MTL-algebra E. Formerly the subsequent conditions are equal:

(1) $F(x \to z) \supseteq F(x \to (y \to z)) \cap F(x \to y)$

(2) $F(x \to y) \supseteq F(x \to (x \to y))$

(3) $F((x \to y) \to (x \to z)) \supseteq F(x \to (y \to z))$

for all $x, y, z \in E$.

Proof:- $(1) \Rightarrow (2)$ Assume that F fulfills the situation

$F(x \to z) \supseteq F(x \to (y \to z)) \cap F(x \to y)$ for all $x, y, z \in E$.

Taking $y = z$ and $x = y$ we get

$$\begin{aligned} F(x \to y) &\supseteq F(x \to (x \to y)) \cap F(x \to x) \\ &= F(x \to (x \to y)) \cap F(1) \quad \text{because } (x \to x = 1). \\ &= F(x \to (x \to y)) \quad \text{by Theorem 3.3 } (1) \\ &\Rightarrow F(x \to y) \supseteq F(x \to (x \to y)) \end{aligned}$$

$\forall x, y \in E$.

$(2) \Rightarrow (3)$ Assume that F fulfills the situation

$F(x \to y) \supseteq F(x \to (x \to y))$ for all $x, y \in E$.

Let $x, y, z \in E$. Since

$$y \to z \leq (x \to y) \to (x \to z) \text{ because } (x \to y) \leq (z \to x) \to (z \to y)$$

$$\Rightarrow x \to (y \to z) \leq x \to ((x \to y) \to (x \to z)) \text{ because } x \leq y \Rightarrow (z \to x) \leq (z \to y)$$

$$\Rightarrow x \to (y \to z) \leq x \to ((x \to y) \to (x \to z)) ......(A)$$

It follows that

$$F(x \to y) \to (x \to z) = F(x \to ((x \to y) \to z)) \text{ because } (x \to (y \to z) = y \to (x \to z))$$

$$\supseteq F(x \to (x \to ((x \to y) \to z))) \text{ by } (2)$$

$$= F(x \to ((x \to y) \to (x \to z))) \text{ because } (x \to (y \to z) = y \to (x \to z)).$$

$$\supseteq F(x \to (y \to z)) \text{ by } (A)$$

$$\Rightarrow F(x \to y) \to (x \to z) \supseteq F(x \to (y \to z)).$$

for all $x, y, z \in E.$

$(3) \Rightarrow (1)$ Suppose that F satisfies

$$F(x \to y) \to (x \to z) \supseteq F(x \to (y \to z)).$$

Then

$$F(x \to z) \supseteq F((x \to y) \to (x \to z)) \cap F(x \to y) \text{ by Theorem 3.3}$$

$$\supseteq F(x \to (y \to z)) \cap F(x \to y) \quad \text{by } (3)$$

$$\Rightarrow F(x \to y) \supseteq F(x \to (y \to z)) \cap F(x \to y).$$

for all $x, y, z \in E.$

3.10 Theorem

The intersection of any family of soft filters of MTL algebra E is also a soft filter of $E.$

Proof Let $\{F_j \mid j \in \Lambda\}$ be a family of soft filters of E and $x, y \in E.$ Then

$$\left(\cap_{j\in\Lambda} F_j\right)(x \otimes y) = \cap_{j\in\Lambda}\left(F_j(x \otimes y)\right) \supseteq \cap_{j\in\Lambda}\left(F_j(x) \cap F_j(y)\right)$$

$$\supseteq \left(\cap_{j\in\Lambda} F_j\right)(x) \cap \left(\cap_{j\in\Lambda} F_j\right)(y).$$

If $x \leq y$ then

$$\left(\cap_{j\in\Lambda} F_j\right)(x) = \cap_{j\in\Lambda}\left(F_j(x)\right) \subseteq \cap_{j\in\Lambda}\left(F_j(y)\right) = \left(\cap_{j\in\Lambda} F_j\right)(y).$$

Hence $\cap_{j\in\Lambda} F_j$ is a soft filter of $E.$

4. BOOLEAN SOFT FILTERS

In this section, we give the definition of Boolean soft filter of an MTL-algebra and present some results on Boolean soft filters.

4.1 Definition

A soft filter F of an MTL-algebra E is understood to be Boolean if it fulfills the subsequent condition

$$F(x \vee x^*) = F(1)$$

$\forall\, x \in E,$ where $x^* = \vee\{y \in E \ : \ x \otimes y = 0\}$ or $x^* = x \to 0.$

4.2 Example

Let $E = \{0, l, m, 1\},$ where $0 < m < l < 1.$ Define $\otimes$ and $\to$ as follows

$\otimes$	0	l	m	1
0	0	0	0	0
l	0	l	0	l
m	0	0	m	m
1	0	l	m	1

$\to$	0	l	m	1
0	1	1	1	1
l	m	1	m	1
m	l	1	1	1
1	0	l	m	1

Then $E = (E, \leq, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is an MTL-algebra. Let us define the soft sets (F, E) and (F_1, E) over U as

$$F : E \rightarrow P(U)$$

$$F(x) = \begin{cases} H \text{ if } x \in \{0, m\} \\ U \text{ if } x \in \{l, 1\}, \end{cases}$$

and

$$F_1 : E \rightarrow P(U)$$

$$F_1(x) = \begin{cases} H & \text{if } x = 0 \\ U & \text{if } x \in \{l, m, 1\}. \end{cases}$$

where $H = \{b, c\} \subset U = \{a, b, c\}$. Then F and F_1 are Boolean soft filters of E .

4.3 Remark

Every soft filter of an MTL-algebra E is not a Boolean soft filter.

4.4 Example

Let $E = \{0, u, v, 1\}$, where $0 < u < v < 1$. We define $\otimes$ and $\rightarrow$ as follows

$\otimes$	0	u	v	1
0	0	0	0	0
u	0	0	0	u
v	0	0	u	v
1	0	u	v	1

$\rightarrow$	0	u	v	1
0	1	1	1	1
u	v	1	1	1
v	u	v	1	1
1	0	u	v	1

Then $E = (E, \leq, \vee, \wedge, \otimes, \rightarrow 0, 1)$ is a MTL-algebra. Letus define a softset (F, E) over $U = \{e_1, e_2, e_3, e_4\}$, that is $F : E \rightarrow P(U)$ by

$$F(x) = \begin{cases} U & \text{if } x = 1 \\ \{e_1, e_2, 1\} & \text{otherwise.} \end{cases}$$

Then F is soft.filter of E, but is not a Boolean soft filter of E, since

$$F(v \vee v^*) = F(v) \neq F(1).$$

4.5 Proposition

Let F and G be soft filters of an MTL-algebra E such that $F \subseteq G$ and $F(1) = G(1)$. If F is Boolean, then so is G.

Proof. Suppose that F is Boolean. Then forall $x \in E$

$$F(1) = F(x \vee x^*)$$

Since

$$\begin{aligned} G(1) = F(1) &= F(x \vee x^*) \\ &\subseteq G(x \vee x^*) \text{ because } (F \subseteq G) \\ &\Rightarrow G(1) \subseteq G(x \vee x^*). \end{aligned}$$

But $G(x \vee x^*) \subseteq G(1)$, for all $x \in E$. So, $G(1) = G(x \vee x^*)$ for all $x \in E$. Thus G is Boolean.

4.6 Proposition

Every Boolean soft filter F of an MTL-algebra E satisfies the following condition

$$F(x \rightarrow z) \supseteq F(x \rightarrow (z^* \rightarrow y)) \cap F(y \rightarrow z)$$

for all $x, y, z \in E$.

Proof We know that for all $x, y, z \in E$

$$y \to z \le (z^* \to y) \to (z^* \to z)$$
$$\le (x \to (z^* \to y)) \to (x \to (z^* \to z))$$
$$\Rightarrow y \to z \le (x \to (z^* \to y)) \to (x \to (z^* \to z)).$$

It follows from Definition 3.1 that;

$$F(y \to z) \subseteq F(x \to (z^* \to y)) \to (x \to (z^* \to z)))$$

From Theorem 3.3 (2) we have

$$F(x \to (z^* \to z)) \supseteq F(x \to (z^* \to y)) \cap F((x \to (z^* \to y)) \to (x \to (z^* \to z)))$$
$$\supseteq F((x \to (z^* \to y)) \cap F(y \to z)) \text{ because } (x \to y \le (z \to x) \to (z \to y))$$
$$\Rightarrow F(x \to (z^* \to z)) \supseteq F((x \to (z^* \to y)) \cap F(y \to z)) \ldots\ldots (1)$$

Since

$$z^* \vee z = ((z^* \to z) \to z) \wedge ((z \to z^*) \to z^*)$$
$$\le (z^* \to z) \to z \text{ because } a \wedge b \le a,\ a \wedge b \le b$$

We have

$z^* \vee z \le (z^* \to z) \to z.$

$$F((z^* \to z) \to z) \supseteq F(z^* \vee z) = F(1) \text{ by Definition 3.1}$$
$$\Rightarrow F((z^* \to z) \to z) = F(1) \ldots\ldots (2)$$

Since

$$x \to (z^* \to z) \le ((z \to z^*) \to z) \to (x \to z) \text{ because} (x \to y \le (y \to z) \to (x \to z))$$

now it follows from Definition 3.1 (1) that

$$F(x \to (z^* \to z)) \subseteq F(((z^* \to z) \to z) \to (x \to z))$$

Thus

$$F(x \to z) \supseteq F((z^* \to z) \to z) \cap F(((z^* \to z) \to z) \to (x \to z)) \text{ by Theorem 3.3}$$
$$= F(1) \cap F(((z^* \to z) \to z) \to (x \to z)) \quad \text{by } (2)$$
$$\supseteq F(1) \cap F(x \to (z^* \to z)) \text{ because } (x \to (y \to z)) = y \to (x \to z)$$
$$= F(x \to (z^* \to z)) \qquad \text{by Theorem 3.3 } (1)$$
$$\supseteq F((x \to (z^* \to y)) \cap F(y \to z)) \text{ by } (1)$$
$$\Rightarrow F(x \to z) \supseteq F((x \to (z^* \to y)) \cap F(y \to z)).$$

4.7 Proposition

If a soft filter F of an MTL-algebra E satisfies the following condition

$$F(x) \supseteq F((x \to y) \to x)$$

for all $x, y \in E$ then it is Boolean.

Proof. As we know that for all $x \in E$

$$1 = x \to ((x^* \to x) \to x) \quad \text{because} (x \to (y \to x) = 1)$$
$$\le ((x^* \to x) \to x)^* \to x^* \quad \text{because} (x \le x^*)$$
$$\le (x^* \to x) \to (((x^* \to x) \to x)^* \to x) \quad \text{because} (x \to y \le (y \to z) \to (x \to z))$$
$$= ((x^* \to x) \to x)^* \to ((x^* \to x) \to x) \quad \text{because} (x \to (y \to z)) = y \to (x \to z)$$
$$= (((x^* \to x) \to x) \to 0) \to ((x^* \to x) \to x) \quad \text{because} (x^* = x \to 0)$$
$$\Rightarrow 1 = (((x^* \to x) \to x) \to 0) \to ((x^* \to x) \to x).$$

Now, by Definition 3.1 (2) and by hypothesis

$$F(((x^* \to x) \to x)) \supseteq F((((x^* \to x) \to x) \to 0) \to (((x^* \to x) \to x))$$
$$= F(1) \text{ because } (1 = ((((x^* \to x) \to x) \to 0) \to ((x^* \to x) \to x))$$
$$\Rightarrow F(((x^* \to x) \to x)) \supseteq F(1).$$

Also, we have

$$F(((x^* \to x) \to x)) \subseteq F(1) \quad \text{by Theorem 3.3}$$

Therefore,

$$F(((x^* \to x) \to x)) = F(1).$$

Since

$$(x^* \to x) \to x \le ((x^* \to x) \to x) \vee ((x^* \to x) \to x^*)$$
$$= (x^* \to x) \to (x \vee x^*) \text{ because } (x \to (y \vee z)) = (x \to y) \vee (x \to z))$$
$$= (1 \wedge (x^* \to x)) \to (x \vee x^*) \qquad \text{because } (1 \wedge x = x)$$
$$= ((x \to x) \wedge (x^* \to x)) \to (x \vee x^*) \text{ because } (x \to x = 1)$$
$$= ((x \vee x^*) \to x) \to (x \vee x^*) \text{ because } (\vee_{i \in \gamma} y_i) \to x = \wedge_{i \in \gamma} (y_i \to x)$$

we have

$$(x^* \to x) \to x \le ((x \vee x^*) \to x) \to (x \vee x^*).$$

We get,

$$F(1) = F((x^* \to x) \to x)$$
$$\subseteq F(((x \vee x^*) \to x) \to (x \vee x^*)$$
$$\subseteq F(x \vee x^*) \text{ because } (F(x) \supseteq F((x \to y) \to x))$$
$$\Rightarrow F(1) \subseteq F(x \vee x^*).$$

From Theorem 3.3 (1), i have

$$F(x \vee x^*) \subseteq F(1) \text{ for all } x \in E.$$

Therefore,

$$F(x \vee x^*) = F(1).$$

Hence F is a Boolean soft filter of E .

4.8 Proposition

Let F be a soft filter of an MTL-algebra E which satisfies the condition

$$F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z)$$

for all $x, y, z \in E$. Then F satisfies the condition

$$F(x) \supseteq F((x \to y) \to x)$$

for all $x, y \in E$.

Proof. Since

$$(x \to y) \to x \le x^* \to x$$

and from Definition 3.1, we have

$$F((x \to y) \to x) \subseteq F(x^* \to x)$$

also, $x = 1 \to x$

$$F(x) = F(1 \to x)$$

so,

$$F(x) = F(1 \to x)$$
$$\supseteq F(1 \to (x^* \to x^*) \cap F(x^* \to x) \quad \text{by given condition}$$
$$= F((1 \otimes x^*) \to x^*) \cap F(x^* \to x) \quad \text{because } (x \to (y \to z)) = (x ? y) \to z$$
$$= F(x^* \to x^*) \cap F(x^* \to x) \quad \text{because } (1 \otimes x = x)$$
$$\supseteq F(1) \cap F((x \to y) \to x) \quad \text{because } (x \to y) \to x \le x^* \to x$$
$$= F((x \to y) \to x) \quad \text{because } F(1) \supseteq F(x)$$

this implies that

$$F(x) \supseteq F((x \to y) \to x).$$

4.9 Theorem

Let F be a soft filter of an MTL-algebra E. thus the subsequent assertions are equal.

(1) F is a Boolean.

(2) $F(x \to z) \supseteq F(x \to (z^* \to y)) \cap F(y \to z)$, For all $x, y, z \in E$.

(3) $F(x) \supseteq F((x \to y) \to x)$, For all $x, y \in E$.

Proof Combining the Propositions 4.6, 4.7 and 4.8

4.10 Proposition

If a soft filter F of an MTL-algebra E satisfies,

$$F(x) \supseteq F((x \to y) \to x)$$

for all $x, y \in E$. Then it satisfies

$$F(x \to z) \supseteq F(x \to (y \to z)) \cap F(x \to y)$$

for all $x, y, z \in E$.

Proof. Since we know that for all $x, y, z \in E$,

$$x \to (y \to z) = y \to (x \to z)$$
$$\le (x \to y) \to (x \to (x \to z)) \quad \text{because } (x \to y) \le (z \to x) \to (z \to y)$$
$$\Rightarrow x \to (y \to z) \le (x \to y) \to (x \to (x \to z)).$$

It follows from Definition 3.1 (2) that

$$F(x \to (y \to z)) \subseteq F((x \to y) \to (x \to (x \to z)))$$

Also from Theorem 3.3, we have

$$F(x \to (x \to z)) \supseteq F(x \to y) \cap F((x \to y) \to (x \to (x \to z)))$$
$$\supseteq F(x \to y) \cap F(x \to (y \to z)) \quad \text{because } (x \to y) \le (z \to x) \to (z \to y)$$
$$F(x \to (x \to z)) \supseteq F(x \to y) \cap F(x \to (y \to z)).$$

Now,

$$x \to (x \to z) \le x \to (((x \to z) \to z) \to z) \quad \text{because } y \le (y \to x) \to x$$
$$= ((x \to z) \to z) \to (x \to z) \quad \text{because } (x \to (y \to z)) = y \to (x \to z)$$
$$\Rightarrow x \to (x \to z) = ((x \to z) \to z) \to (x \to z).$$

We have

$$F(x \to z) \supseteq F(((x \to z) \to z) \to (x \to z)) \quad \text{because } F(x) \supseteq F((x \to y) \to x)$$
$$\supseteq F(x \to (x \to z) \quad \text{because } (x \to y) \le (z \to x) \to (z \to y)$$
$$\supseteq F(x \to y) \cap F(x \to (y \to z)).$$

We have

$$F(x \to z) \supseteq F(x \to y) \cap F(x \to (y \to z)).$$

4.11 Theorem

Every Boolean soft filter F of an MTL-algebra E satisfies the following condition

$$F(x \to z) \supseteq F(x \to y) \cap F(x \to (y \to z))$$

for all $x, y, z \in E$.

Proof. Since we know that for all $x, y, z \in E$,

$$\begin{aligned} x \to (y \to z) &= y \to (x \to z) \\ &\leq (x \to y) \to (x \to (x \to z) \end{aligned}$$

and

$$\begin{aligned} x \to (x \to z) &\leq x \to (((x \to z) \to z) \to z) \quad \text{because } y \leq (y \to x) \to x \\ &= ((x \to z) \to z) \to (x \to y) \quad \text{because } x \to (y \to z) = y \to (x \to z) \\ \Rightarrow x \to (x \to z) &\leq ((x \to z) \to z) \to (x \to y). \end{aligned}$$

It follows from Definition 3.1 and Theorem 4.9 (3)

$$\begin{aligned} F(x \to z) &\supseteq F(((x \to z) \to z) \to (x \to y)) \\ &\supseteq F(x \to (x \to z)) \\ &\supseteq F(x \to y) \cap F((x \to y) \to (x \to (x \to z))) \quad \text{by Theorem 3.3 } (2) \\ &\supseteq F(x \to y) \cap F(x \to (y \to z)) \\ \Rightarrow F(x \to z) &\supseteq F(x \to y) \cap F(x \to (y \to z)). \end{aligned}$$

5. MV-SOFT FILTER

In this section, we introduce MV-soft filter of an MTL-algebra. Also we provide the relation between an MV-soft filter and a Boolean soft filter in an MTL-algebra.

5.1 Definition

A soft set (F, E) over U is called an MV-soft filter of an MTL-algebra E if it is a soft filter of E which satisfies the following condition

$$F(x \to y) \subseteq F(((y \to x) \to x) \to y)$$

for all $x, y \in E$.

5.2 Example

Let $E = \{0, p, q, r, s, 1\}$, where $0 < p < q < s < r < 1$. Define $\otimes$ and $\to$ as follows

$\otimes$	0	p	q	r	s	1
0	0	0	0	0	0	0
p	0	0	0	p	0	p
q	0	0	q	q	q	q
r	0	p	q	r	q	r
s	0	0	q	q	s	s
1	0	p	q	r	s	1

$\to$	0	p	q	r	s	1
0	1	1	1	1	1	1
p	s	1	1	1	1	1
q	p	p	1	1	1	1
r	0	p	s	1	s	1
s	p	q	r	1	1	1
1	0	p	q	r	s	1

Then $E = (E, \leq, \vee, \wedge, \otimes, \to, 0, 1)$ is a MTL-algebra and let us define a softset (F, E) over U, that is $F : E \to P(U)$ by

$$F(x) = \begin{cases} A \text{ if } x \in \{0, p\}, \\ B \text{ if } x \in \{q, s\}, \\ U \text{ if } x \in \{r, 1\}, \end{cases}$$

where $U = \{a, b, c\}$, $A = \{a\}$ and $B = \{a, b\}$. Then F is an MV-soft filter of E.

5.3 Theorem

Every Boolean soft filter is an MV-soft filter.

Proof. Let the F is a Boolean soft filter of E. Since

$$y \leq ((y \to x) \to x) \to y$$

For all $x, y \in E$.

We have

$$(((y \to x) \to x) \to y) \leq y \to x \quad \text{because } (y \leq x \Rightarrow x \to z \leq y \to z)$$

Also,

$$x \to y \leq ((y \to x) \to x) \to ((y \to x) \to y) \quad \text{because } (x \to y \leq (y \to z) \to (x \to z))$$

$$= (y \to x) \to (((y \to x) \to x) \to y) \quad \text{because } (x \to (y \to z) = y \to (x \to z))$$

$$\leq ((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to y) \quad \text{because } (y \leq x \Rightarrow x \to z \leq y \to z)$$

$$\Rightarrow x \to y \leq ((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x)$$

Now, from Proposition 4.7 we have

$$F(((y \to x) \to x) \to y) \supseteq F(((((y \to x) \to x) \to y) \to x) \to (((y \to x) \to x) \to y))$$

$$\supseteq F(x \to y).$$

Thus

$$F(x \to y) \subseteq F(((y \to x) \to x) \to y).$$

Hence F is an MV-soft filter of E.

5.4 Remark

The converse of the Theorem u is not true in general,

5.5 Example

Let $E = \{0, r, s, 1\}$ where $0 < r < s < 1$. Define $\otimes$ and $\to$ as follows

?	0	r	s	1
0	0	0	0	0
r	0	0	0	r
s	0	0	r	s
1	0	r	s	1

$\to$	0	r	s	1
0	1	1	1	1
r	s	1	1	1
s	r	s	1	1
1	0	r	s	1

Then $E = (E, \leq, \vee, \wedge, \otimes \to 0, 1)$ is a MTL-algebra. Let us define a softset (F, E) over E, that is $F : E \to P(E)$ by

$$F(x) = \begin{cases} E & \text{if } x = 1 \\ \{a, b, 1\} & \text{otherwise.} \end{cases}$$

Where $U = \{a, b, c\}$, then F is an MV-soft filter of E. But F is still not a Boolean soft filter of E, because

$$r \vee r^* = r \vee s = s$$

$$\Rightarrow F(r \vee r^*) = F(s) \neq F(1).$$

6. CONCLUSION

We gave characterizations of soft filters and investigated some properties of soft filters in MTL-algebras. In this paper we defined Boolean soft filter and MV-soft filter in MTL-algebras, and studied related properties. We provided a condition for a soft filter to be Boolean.Further research will focus on constructing a quotient MTL-algebra by using a soft filter, on studying prime soft filter.

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