

## Simulation of Industrial Rotating Mixing Flow in a Tub

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### ABSTRACT

This study is the part of our previous numerical predictions of rotating mixing flow in a cylindrical tub. A pair of rotating stirrers is considered eccentrically for the computational analysis. The two-dimensional complex industrial mixing flow of incompressible generalised Newtonian fluid is analysed. A semi-implicit time-stepping Taylor-Galerkin/pressure-correction finite element multi stepping scheme adopted as numerical method, posed in a cylindrical coordinate system. The flow reports good behaviour with the actual industrial dough mixing process.

**KEYWORDS:** Finite Element Method, Mixing Flow, generalised Newtonian, Rotating Stirrer.

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### 1. INTRODUCTION

This is the part study of our previous studies [1–4], optimizing the industrial mixer design. The theme of this simulation is to reduce the consumption of power and to enhance the rate of work done [1]. The motivation for simulating this type of mixer is due to energy problems of our country. The main part of mixing industrial process is stirring; considered problem is generally related with process industries and particularly related with mixing of dough in food processing industry [5, 6]. The homogenised mixing of dough of breads and biscuits has many complications during the mixing process and challenges for design engineers at industrial level which needs to be improved.

Finite Element Method (FEM) has attracted great researcher and design engineers in the field of computational fluid dynamics during last couple of decades. This method has much flexibility to control the complex computational domain with the best accurate results. The success of this method in solving nonlinear problems in every phase of science and engineering is notable. The primitive variables formulation is employed [5, 7-9] and the geometry considered in this study consist a cylindrical tub with a pair of stirrers attached with lid eccentrically. For industrial mixers, the dough partially fills the geometry, is driven around a bowl by stirring rods [6]. The mixer is considered in horizontal ( $r, \theta$ ) direction and to be in part filled during the process. Model fluid has been used, that is, syrup (generalised Newtonian fluid). The numerical algorithm is used has proved the stability, accuracy of a numerical method which are important concerns for the robustness of the numerical algorithm [10–13], when applied to a challenging system of fundamental governing system of equations.

This study considered with Navier–Stokes equations in Two–dimensional cylindrical coordinates. A semi implicit time marching Taylor–Galerkin/Pressure–Correction (TGPC) multi stages scheme adopted [the detail of scheme can be referred through reference 13, 14]. The field of interests such as the effects of inertia and impact of rotational velocity are clearly mentioned. The contour plots are used to show the predicted solutions and the oval shape is used to show the non–dimensional minimum value and the square shape for maximum value. The model fluids, model dough and actual dough represented by Reynolds numbers are  $Re = 8.0$ ,  $Re = 0.8$  and  $Re = 0.08$  corresponding to zero shear viscosities  $\mu = 1.05$  Pas,  $\mu = 10.5$  Pas and  $\mu = 105.0$  Pas is covered respectively for a range of material properties [11–12].

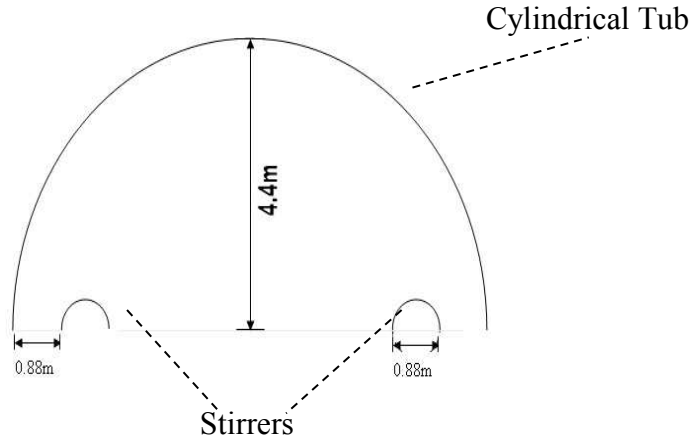
### 2 PROBLEM SPECIFICATION

A tub is considered with eccentrically fixed couple of stirrers due the importance in process technology [4]. In the industries, actually outer wall of tub is fixed but due to mathematical complexities and to preserve the originality of physical problem we considered it as rotating and three cases of rotating stirrers are analysed i.e., half, same and

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double with angular velocity  $\omega_0 = 0.5, 1.0$  and  $2.0$  respectively. The counter clockwise rotation is fixed of outer tub and the left stirrer rotates in same and the right in opposite direction (this is case of mix rotating stirrers).



**Figure 01: Two dimensional computational domain of eccentric rotating tub.**

For the prediction and the study of computational domain, the triangular finite elements discretisation adopts. The domain of interest involved eccentric rotating cylindrical flow shown in figure 01. For the simulation, a two-dimensional cylindrical coordinate frame of reference is taken over the domain.

### 3 GOVERNING SYSTEM OF EQUATIONS

In present study a generalised Newtonian Incompressible rotating flow is considered. The continuity equations as  $(\nabla \cdot \mathbf{u} = 0)$  and momentum transport equations  $(\rho \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \boldsymbol{\sigma} - \rho \mathbf{u} \cdot \nabla \mathbf{u})$  over the spatial domain  $\Omega$  are adopted in the absence body forces.

The adopted numerical scheme is fully detailed can be referred from previously published work [3, 10, and 11]. To define the initial and boundary conditions are essential for well-posed problem. Initially zero pressure is fixed on outer rotating wall of tub [6]. A time marching process is adopted with  $10^{-6}$  order of pre described level of tolerance achieved for the steady state convergence and Simulations starts from rest,  $\Delta t = 10^{-2}$  is fixed and the accuracy of mesh density is highly affected to the numerical predictions. 75 rpm is considered rotational velocity of outer boundary for dough mixing in experiment, which directly relate to inertial level (Reynolds number) equal to 0.08.

### 4 NUMERICAL RESULTS AND DISCUSSIONS

Predicted solutions for Generalised Newtonian Bird-Carreau model fluid are analysed through flow patterns and pressure isobars to demonstrate the effects of inertia ( $Re = 0.08, 0.8$  and  $8.0$ ). Impact of rotational velocities (half, same and double) of stirrers against the velocity of outer tub is also considered on various values of dimensionless parameter i.e., 0.6, 0.7 and 0.8. These numerical results illustrate typical solution of both fields of mix-rotating stirrers in figure-02 to 04 respectively. The graph of power and work-done with respect to time is shown in graph-01. The positions where positive maxima and negative minima occur are indicated by square and oval shapes respectively.

Computational results of mix-rotating stirrers for recirculation pattern and pressure isobars present and compared in three different views. Firstly, these are carried out at three inertial levels presented, (0.08, 0.8 and 8.0) displayed in figure-02. The results at all three inertial levels show maximum rate of recirculation outside the stirrers between the wide gap of centre and stirrer which rotates in opposite direction but when inertia reach at 8.0 centre of recirculation region twisted towards down and minimum rate observed near the outer wall and remained in same position in all three cases. The predicted results have close agreement between predicted results and experimental flow fields.

Symmetric pressure isobars come into view with equal magnitude in non-dimensional positive and negative extrema on both the sides (upper and lower) of the stirrers in the narrow gap when flow enters and releases the gap respectively at  $Re = 0.08$  and  $0.8$  and comparable symmetry influence relates across the geometry variants in pressure differential is noted. Asymmetric isobars are observed, with positive maximum and negative minimum on the top of the stirrer and the outer stirrer tip (near the narrow gap) respectively as inertia reaches up to  $O(10)$ .

Impact of rotational velocity of stirrers on flow patterns and pressure differentials of Bird–Carreau model fluid when inertial level is  $Re = 8.0$  and dimensionless parameter  $n = 0.8$  is displayed in figure–03. Notable phenomena observed in this case of study when velocity increased from half to double. The same fashion observed at half velocity as discussed in above paragraph. At same velocity, maxima shifted towards the opposite stirrer and twisted towards the in wide region but minima remained at same region i.e., outer wall of tub. As velocity reached at double, totally different scenario observed, maxima shifted in same region as half velocity but minima shifted from outer wall to near the right stirrer in wide region and a circulation region observed in narrow gap between right stirrer and outer wall.

The maximum pressure isobars appear inside the narrow gap between right stirrer and tub before the entry in the case at half speed. As stirrer rotates in counter direction of tub, the fluid gets compressed on entry to the constructed region between stirrer and tub and hence the maximum value of pressure is seemed on outer wall in this region. Minimum pressure is observed when the fluid releases from the gap and the flow expands at all inertial levels at same speed. At double speed asymmetric fashion is noticed and the maximum pressure is seemed near the tub wall before narrow gap between stirrer and outer tub. The figure–04 shows the influence of dimensionless parameter on flow patterns and pressure differentials of Bird–Carreau model fluid at half velocity and  $0.08$  inertial level. No any considerable change is noticed in all three cases of dimensionless parameter i.e.,  $0.6$ ,  $0.7$  and  $0.8$  on both field of interest.

The graph of power is high when time starts from  $10^0$ , it reduced approximately linearly as time passed, it matured steady state up to time reaches  $10^4$  and similarly graph of work done is increased with the same time interval. Same fashion is observed for all three inertial levels. The rate of work done also peaks in this region and this is an important quantity to state optimal mixer design. Numerical results tabulated in table–1 and table–2 for both variables on dimensionless parameters,  $0.6$  and  $0.8$  respectively.

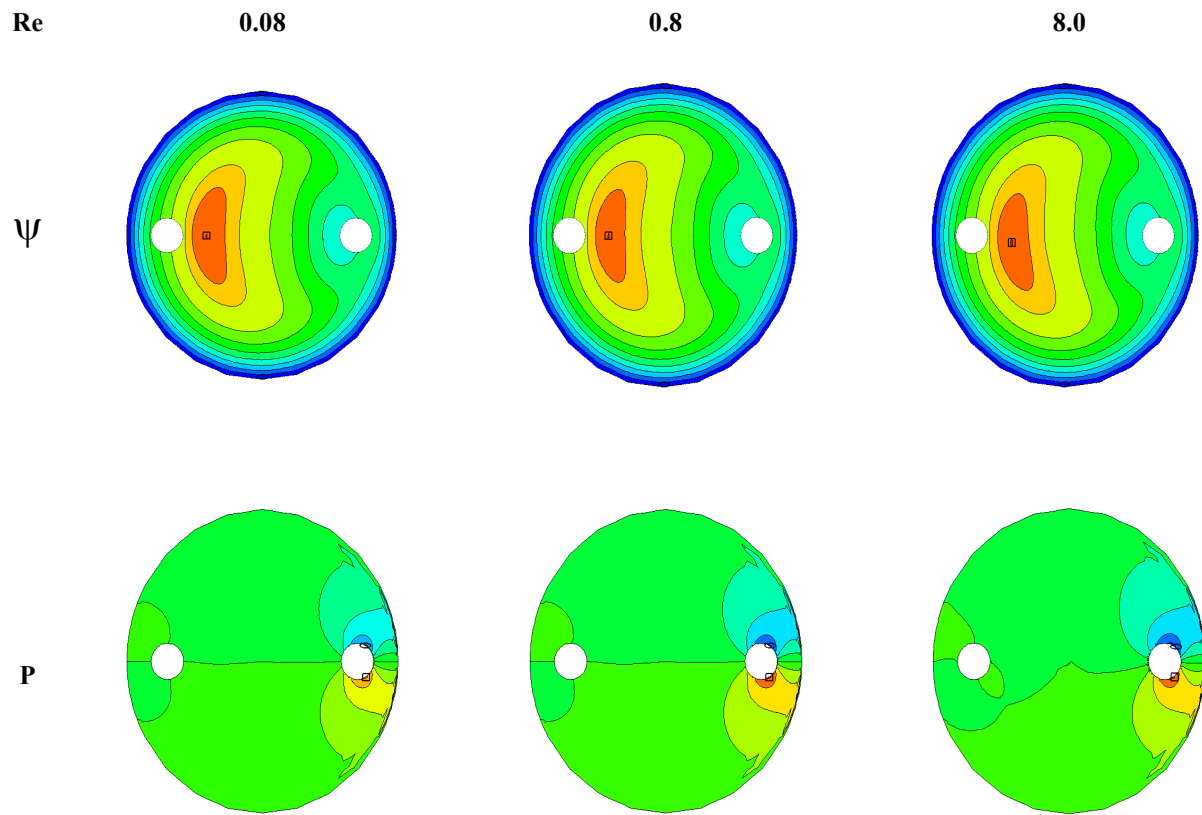
## 5. CONCLUSION

The case of demonstration successfully replicates the use of a numerical flow solver for generalised Newtonian fluid as a predictive tool for dough mixing process and to provide physically simulations of flows. As compared to previous cases of study (i.e., co-rotating and contra-rotating) this case (mix-rotating) of stirrers shows maximum homogenization and great form of required object. Maximum rate-of-work done noticed at the small gap between stirrers and tub. However, the power consumption is higher for this case of rotating stirrers. Thereverse scenario fully replication of the actual industrial tub rotating situation.

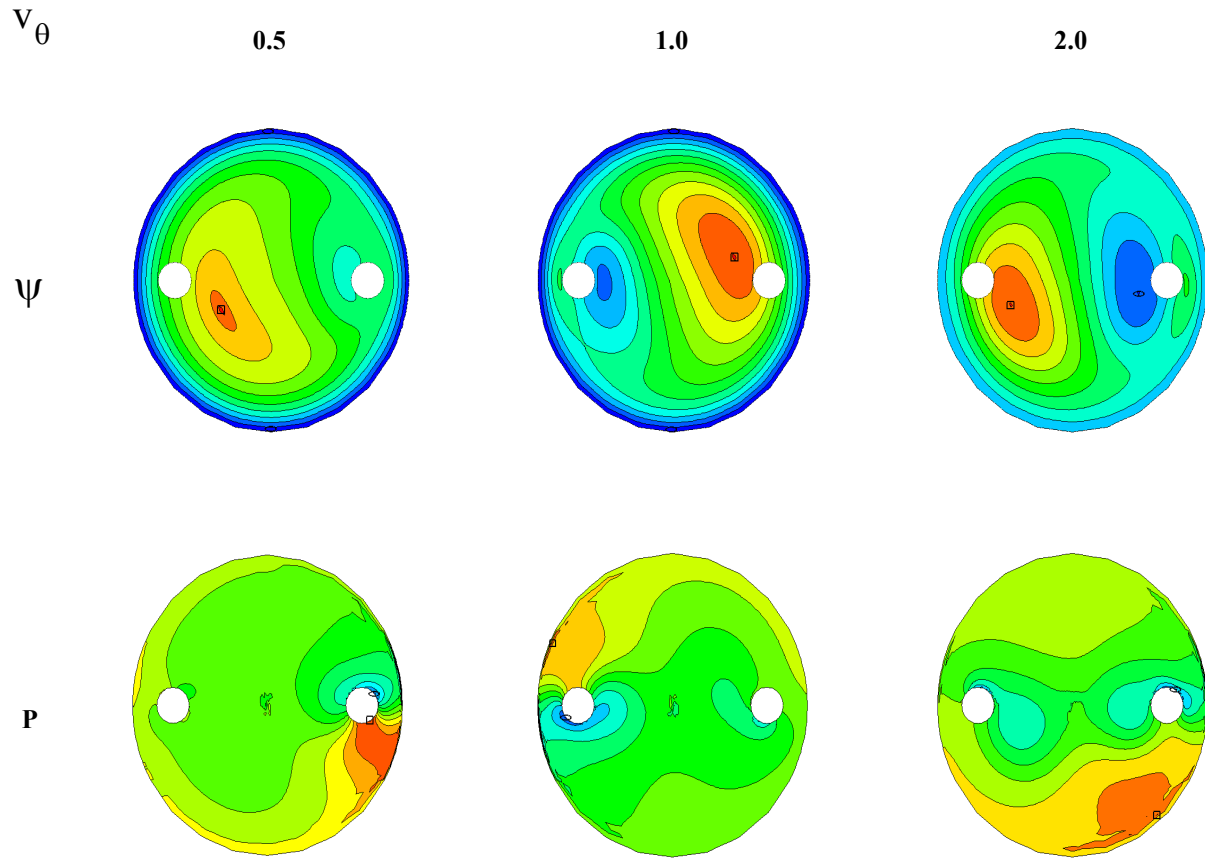
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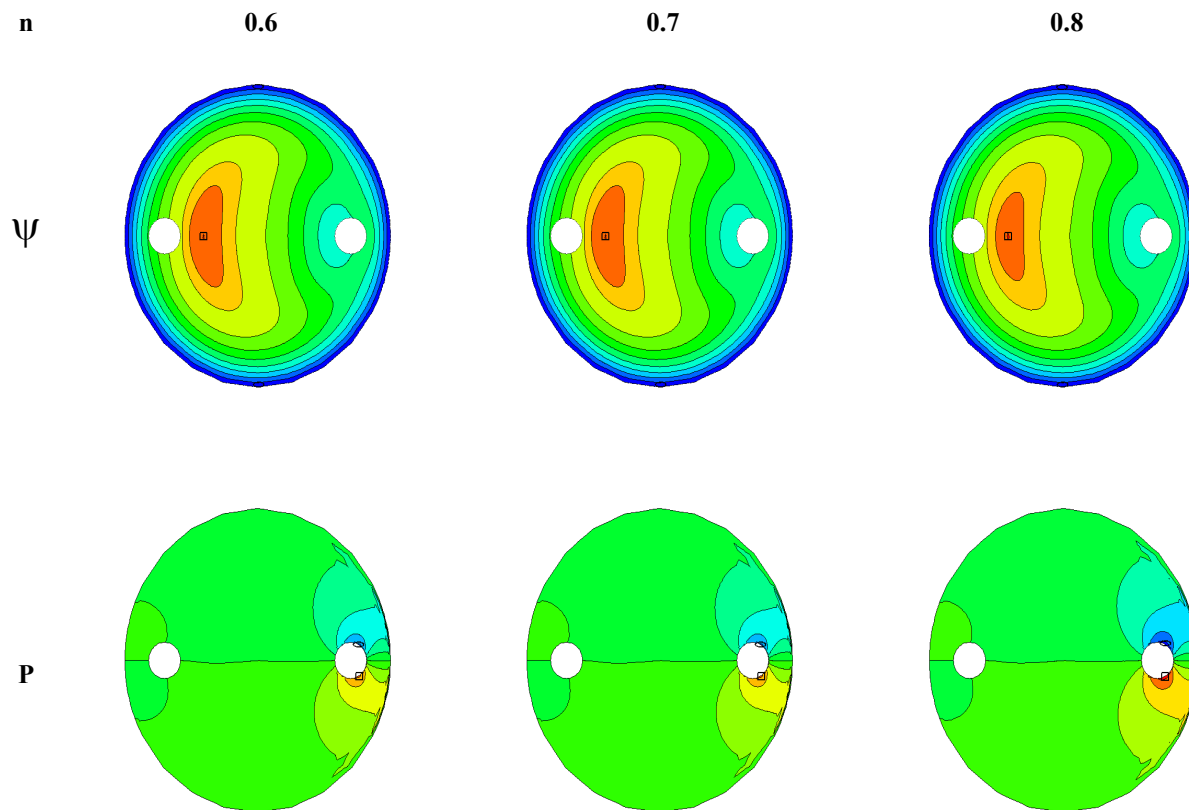
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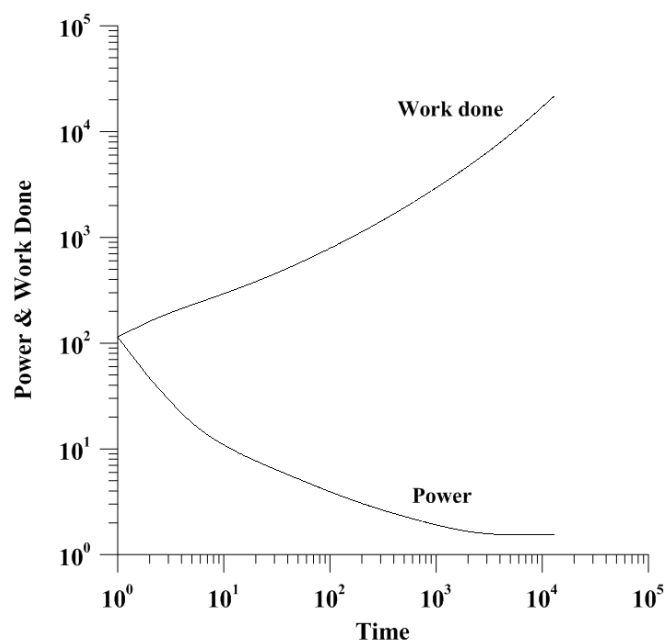
**Figure 2:** Effects of inertia on flow patterns, pressure differentials of when velocity of stirrer is half ( $v_0 = 0.5$ ) and dimensionless parameter  $n = 0.6$ .



**Figure 3:** Impact of rational velocity of stirrers on flow patterns, pressure differentials, when inertial level is  $Re = 8.0$  and dimensionless parameter  $n = 0.8$ .



**Figure 4:** Influence of dimensionless parameter on flow patterns, pressure differentials when velocity of stirrer is half ( $v_0 = 0.5$ ) and inertial level is  $Re = 0.08$ .



**Graph 1:** Power and work-done against time for mix-rotating stirrers.

Variables	Re	0.8		0.8		8.0	
	Speed of Stirrers	Min	Max	Min	Max	Min	Max
$\psi$	Half	0	4.294	0	4.294	0	4.304
	Same	0	5.494	0	5.479	0	5.480
	Double	-1.63	7.854	-1.61	7.870	-1.15	8.043
$P$	Half	-4.64	4.616	-4.64	4.616	-4.78	4.531
	Same	-6.58	6.509	-6.91	6.206	-13.2	4.729
	Double	-9.57	9.326	-31.8	7.472	-31.8	7.472

**Table 1:** Predicted results of flow patterns ( $\psi$ ), pressure differentials ( $P$ ) of Bird–Carreau model fluid when dimensionless parameter  $n = 0.6$ .

Variables	Re	0.08		0.8		8.0	
	Speed of Stirrers	Min	Max	Min	Max	Min	Max
$\psi$	Half	0	4.287	0	4.286	0	4.224
	Same	0	5.494	0	5.477	0	5.476
	Double	-1.60	7.850	-1.57	7.875	-1.10	8.035
$P$	Half	-4.95	4.929	-5.08	4.828	-7.99	4.178
	Same	-6.81	6.743	-7.14	6.438	-13.5	4.932
	Double	-10.2	9.975	-11.4	8.944	-33.2	7.564

**Table 2:** Predicted results of flow patterns ( $\psi$ ), pressure differentials ( $P$ ) of Bird–Carreau model fluid when dimensionless parameter  $n = 0.8$ .