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The Chemical Diffusion and Bouyancy Effects on MHD Flow of Casson Fluids Past a Stretching Inclined Plate with Non-Uniform Heat Source

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ABSTRACT

Theoretical and computational study for the magnetohydrodynamic flow of Casson fluids past a stretching inclined plate is considered to examine the effects of chemical diffusion and buoyancy. The governing equations of the problem are finally obtained in the ordinary differential form by employing similarity function which are than solved by a using straight forward and efficient coding in computational software Mathematica 11. An extensive computational work has been carried out to evaluate the effect of the pertinent parameters of the study to reveal the physical insight of the problem. Graphical patterns for velocity, chemical species concentration and heat function are presented for representative values of parameters of importance.

KEYWORDS: Computational Study, Chemical diffusion, Differential form , Casson fluids, magnetohydrodynamic flow.

1. INTRODUCTION

Non-Newtonian fluid flow arises in many diciplins of material processing and chemical engineering. There are different kinds of non-Newton fluids like power-law fluid and viscoelastic fluid etc. In addition, there is another non-Newtonian fluid model i.e. Casson fluid model., it is generally claimed that, the Casson model is better than the visco plastic models in fitting the rheological data. The influence of thermal radiation and chemical reaction on micro polar fluid flow in a rotating frame was discussed by Das [1] and concluded that an increase in the volume fraction of nano particles enhances the velocity profiles. Hayat et al. [2] discussed the cross diffusion effects on MHD flow of Casson fluid. Rashidi et al. [3] analytically discussed the flow due to a rotating disk in a porous medium and used homotopy analysis method (HAM). The effects of radiation on free convective flow of nanofluids, due to an infinite plate was considered by Sandeep et al. [4]. Further Sandeep and Sugunamma [5] discussed the effect of inclined magnetic force field on dusty viscous fluid. Nandy [6] studied the heat transfer for MHD flow of Casson fluid flow over a stretching sheet. Sandeep and Sugunamma [7] studied the radiation effects for inclined magnetic field for natural convection flow.The effects of chemical reaction on MHD flow near an expanding sheet with heat generation were investigated by Mohan krishna et al. [8]. Nandeppanavar [9-11] studied the heat transfer analysis of Casson fluids. Attia and Ahmed[12] examined the transient Coutte flow analysis of Casson fluid. Bhattacharyya et.al.[13] analyzed magnetohydrodynamic boundary layer flow of Casson fluid with wall mass transfer. Swati [14] observed the effect of radiation on the flow of Casson fluid over an unsteady stretching permeable sheet. Shehzad et.al.[15] considered chemical reaction magnetohydrodyanamic flow of Casson fluid. A comparative study has been done by Sandeep et al. [16] to analyze the heat and mass transfer for nanofluid past a permeable stretching surface. Raju et al. [17] discussed the effects of thermal diffusion on the flow over a stretching surface with inclined magnetic field. Very recently, the researchers [18-22] investigated the heat and mass transfer of non-Newtonian and Newtonian flows by considering various channels. Hassan et al [24] worked at chemical diffusion and radiative heat transfer heat transfer effects on magnetohydrodynamics stagnation oint flow of Casson fluid over a porous shrinking sheet. Recently, Hassan et al and Nadeem et al [25-26] investigated the unsteady magnetic hydrodynamic (MHD) stagnation point flow of Casson fluids with radiation heat transfer has been investigated. The fluid flows past porous shrinking sheet. Recently, Hassan et al [27-29] worked on the thermal effects in magnetohydrodynamics (MHD) stagnation point flow of Casson fluids over a flat stretching/shrinking sheet.

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2. MATHEMATICAL ANALYSIS

The steady incompressible and two dimensional flow of Casson fluid is considered in the presence of uniform magnetic field of strength B_0 .The flow of fluid is due to a vertical sheet inclined at an angle γ . The sheet is porous and stretches /shrinks. The fluid velocity components are u and v respectively in x and y directions.The temperature and velocity in the external flow are respectively T_∞ and *U*.The temperature at the surface of sheet is T_w and the fluid temperature is *T*. The fluid flows through a porous medium of permeability K_1 . The concentration of chemical diffusion is C,where concentration of species in the external flow in C_∞ .

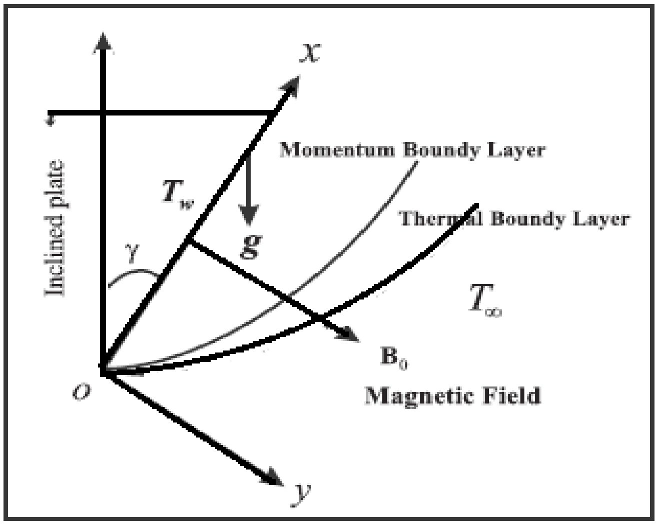

Figure 1: Flow Geometry

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\left(1+\frac{1}{\beta}\right)\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\upsilon}{K_1}\left(U-u\right)+\frac{\sigma_e B_0^2}{\rho}\left(U-u\right)+g\beta^*\left(T-T_\infty\right)\cos\alpha \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho C_p}q''' \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty) \quad (4)$$

Where β is the Casson parameter, $\upsilon=\frac{\mu}{\rho}$ is kinematic viscosity, μ is the coefficient of fluid viscosity, ρ is the fluid density σ_e is the electrical conductivity, K_1 is the permeability of the porous medium, β^* is the coefficient of thermal expansion, C_p is the specific heat capacity at constant pressure, α is thermal diffusivity. The non-uniform heat generated or absorbed per unit volume is taken as

$$q'''=\frac{kU_w}{xv}\left[A^*\left(T_w-T_\infty\right)e^{-\eta}+B^*\left(T-T_\infty\right)\right], \quad (5)$$

Where A^* and B^* are parameters of space-dependent and temperature-dependent heat generation/absorption. A^* and B^* are positive in case of internal heat source and negative in the case of internal heat sink. Thus $q^m > 0$ in the case of heat generation; it is negative in the case of heat absorption.

The boundary conditions are:

$$u = u_w(x) = bx, \quad T = T_w, C = C_w \quad \text{at } y=0$$
$$u = u_e(x) = ax, \quad T = T_\infty,\ C = C_\infty \quad \text{at } y \to \infty \tag{6}$$

The continuity equation (1) is identically satisfied by introducing a stream function

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$\psi(x,y) = \sqrt{a\alpha}\,xf(\eta), \eta = y\sqrt{\frac{a}{\alpha}}, T = T_\infty + (T_w - T_\infty)\theta(\eta), C = C_\infty + (C_w - C_\infty)\phi(\eta) \tag{7}$$

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$P_r\left(1+\frac{1}{\beta}\right)f''' + K(1-f') + M(1-f') + 1 = f'^2 - ff'' - \lambda\theta\cos\alpha \tag{8}$$

$$\theta'' + f\theta' + A^* e^{-\eta} + B^*\theta = 0 \tag{9}$$

$$\phi'' + S_c(f\phi' - \gamma\phi) = 0 \tag{10}$$

$$f = S, f' = c, \theta = 1, \phi = 1 \quad as \quad \eta = 0$$
$$f' \to 1, \theta \to 0, \phi \to 0 \quad as \quad \eta \to \infty \tag{11}$$

where $P_r = \frac{\upsilon}{\alpha}$ is the Prandtl number, $\lambda = g\beta^* \frac{T_0}{U_\infty}$ is the mixed convection parameter, $K = \frac{\upsilon}{aK_1}$ is the permeability parameter, $M = \frac{\sigma_e B_0^2 \upsilon \operatorname{Re}_x}{\rho U^2}$ is the magnetic parameter, $\gamma = \frac{R}{a}$ is non-dimensional rate of solutal. $Sc = \frac{\upsilon}{D}$ Schmidt number, S is the suction parameter and c is stretching/shrinking parameter.

3. RESULTS AND DISCUSSION

The resulting set of governing equations (8) to (11) is nonlinear in higher order. It is hard to find any analytical solution of this system of equations. Thus a numerical treatment of the situation has been employed to obtain a reliable solution of the problem. The higher order derivatives have been reduced to their first order form. We take $f' = u$, $u' = v$, $\theta' = w$, $\phi' = g$ Thus the resulting system of first order equations is as follows:

$$P_r\left(1+\frac{1}{\beta}\right)v' + K(1-u) + M(1-u) + 1 = u^2 - fv - \lambda\theta\cos\alpha$$

$$w' + fw + A^* e^{-\eta} + B^*\theta = 0$$

$$g' + S_c(fg - \gamma\phi) = 0$$

Fig.2 shows that velocity $f'(\eta)$ is enhanced in magnitude with increase in the value of parameter M. But the flow slows down as the magnitude of $f'(\eta)$ decreases with increase in P_r as depicted in fig. 3. Increase in the magnitude of $f'(\eta)$ is observed in increasing the values of K as shown in fig 4.

The mixed convection parameter λ implies small increasing effect on velocity $f'(\eta)$ as demonstrated in fig.5. The parameter c has significant increasing effect on $f'(\eta)$ as shown in fig.6. The Casson parameter β also has increasing effect on $f'(\eta)$ as presented in fig.7. Fig. 8 depicts the effect of inclination of the plate on $f'(\eta)$.
Fig.9 maps the temperature function under the effect of *Pr*. Fig.10 and fig.11 shows that the increase in the value of magnetic parameter M and porosity parameter K have small decreasing effects on temperature function $\theta(\eta)$.
Fig.12 shows that the temperature distribution increases with increase in values of shrinking parameter but decreases with increase in the values of stretching parameter.
The increase in the value of Casson parameter reduces heat distribution as shown in fig.13.

The heat distribution on $\theta(\eta)$ increases with increase in the values of parameter A^* and B^* as depicted in fig.14 and fig.15 respectively.

Fig.16 depicts the effect of Schmidt number S_c on $\phi(\eta)$. It is noted that $\phi(\eta)$ decreases with increase in the values of S_c. But fig.17. shows that $\phi(\eta)$ increases with increase in the values of parameter of *R*.

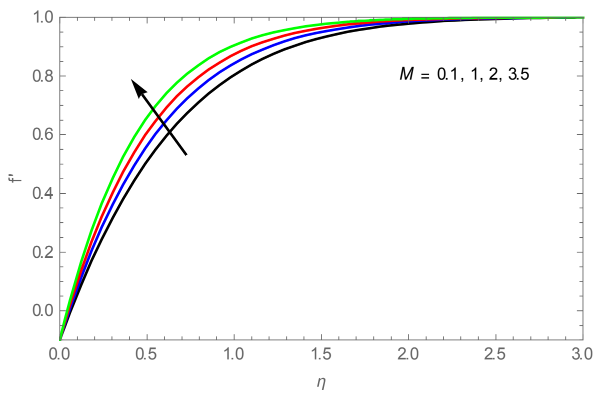

Fig.2: The plot for curves of f' under the effect of magnetic parameter M when S=1, c=–0.1, P_r=1, K=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

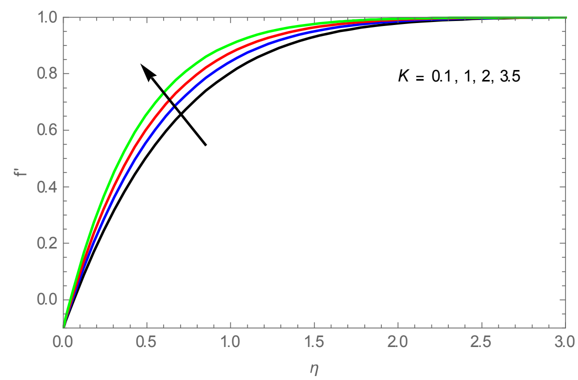

Fig.4: The plot for curves of f' under the effect of parameter K when S=1, c=–0.1, P_r =1, M=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

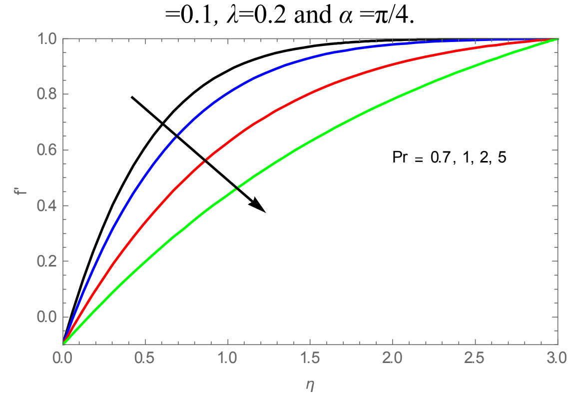

Fig.3: The plot for curves of f' under the effect of Prandtl number P_r when S=1, c=–0.1, M=0.1, K=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

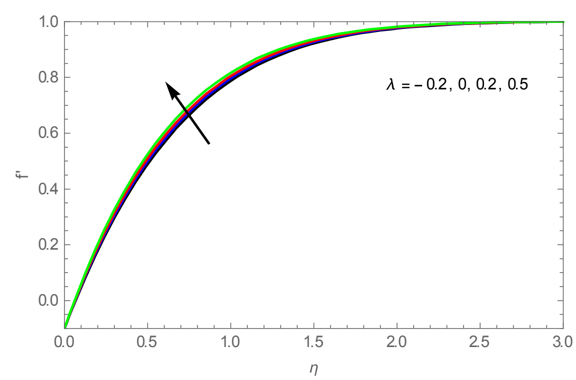

Fig.5: The plot for curves of f' under the effect of parameter λ when S=1, c=–0.1, P_r =1, M=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, K=0.1 and α =π/4.

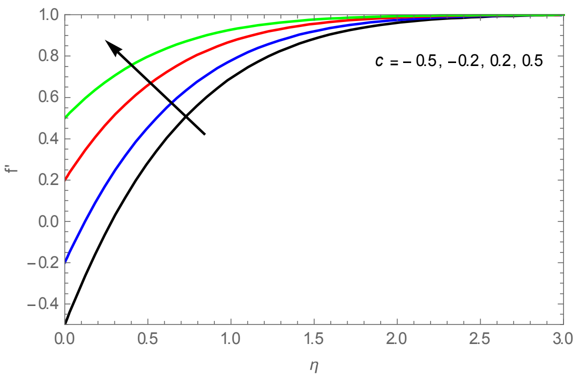

Fig.6: The plot for curves of f' under the effect of parameter c when S=1, K=0.1, P_r =1, M=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ =0.1 and α =π/4.

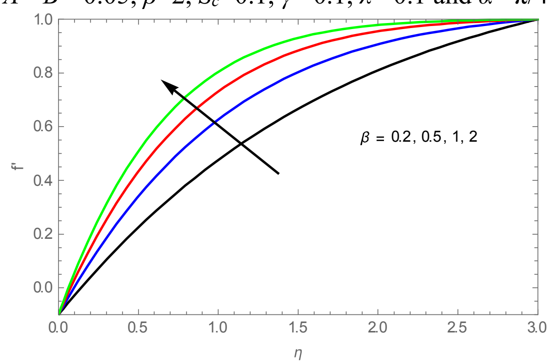

Fig.7: The plot for curves of f' under the effect of parameter β when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=B^*=0.05, S_c=0.1, γ =0.1, λ =0.1 and α =π/4.

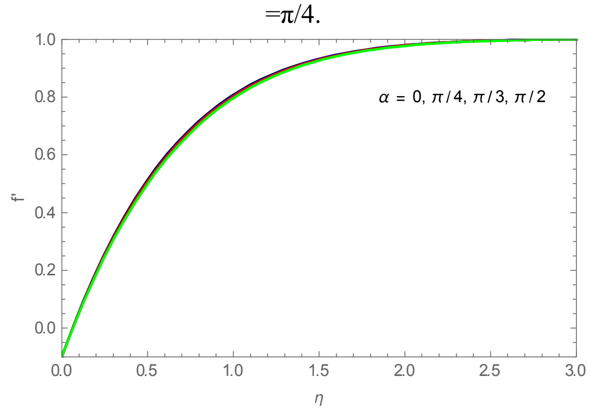

Fig.8: The plot for curves of f' under the effect of parameter α when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=B^*=0.05, S_c=0.1, γ =0.1, λ =0.1 and β =2

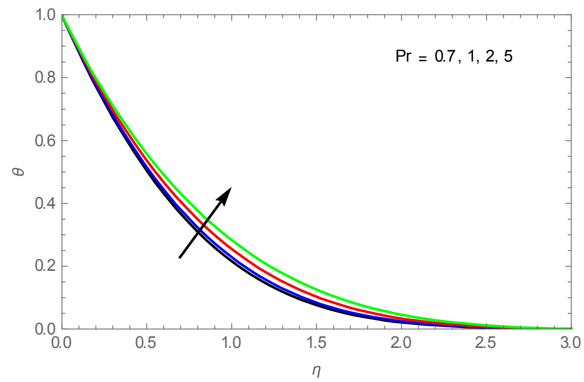

Fig.9: The plot for curves of θ under the effect of Prandtl number P_r when S=1, c=–0.1, M=0.1, K=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

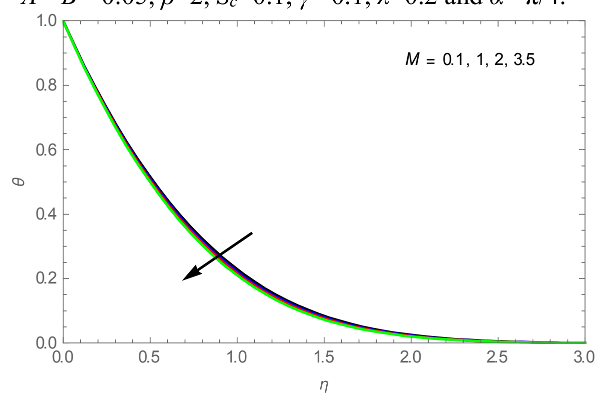

Fig.10: The plot for curves of θ under the effect of magnetic parameter M when S=1, c=–0.1, P_r=1, K=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

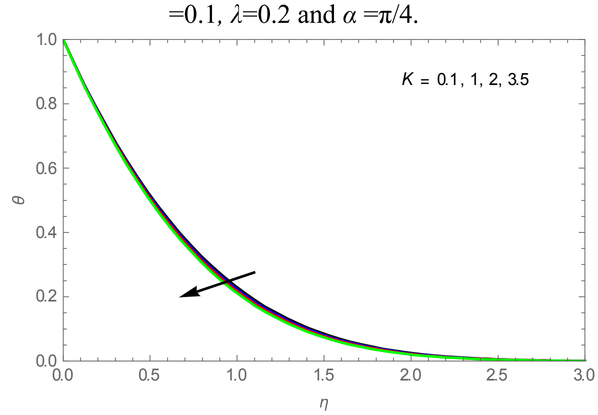

Fig.11: The plot for curves of θ under the effect of parameter K when S=1, c=–0.1, P_r =1, M=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ=0.2 and α =π/4.

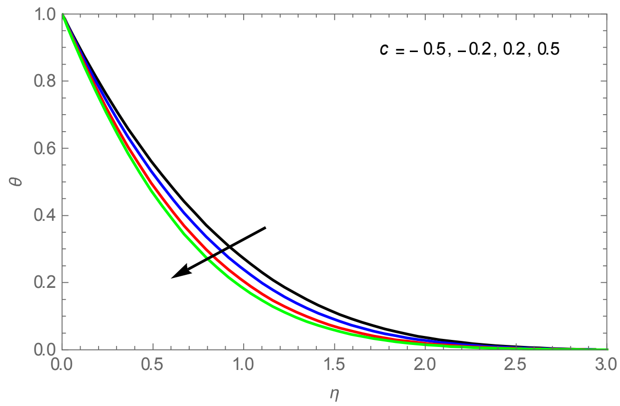

Fig.12: The plot for curves of θ under the effect of parameter c when S=1, K=0.1, P_r =1, M=0.1, A^*=B^*=0.05, β=2, S_c=0.1, γ =0.1, λ =0.1 and α =π/4.

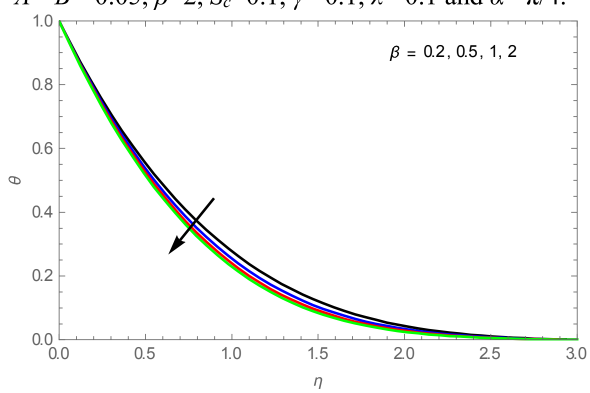

Fig.13: The plot for curves of θ under the effect of parameter β when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=B^*=0.05, S_c=0.1, γ =0.1, λ =0.1 and α =π/4.

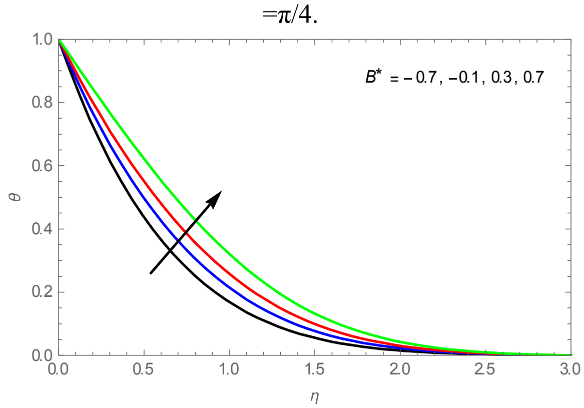

Fig.14: The plot for curves of θ under the effect of parameter B^* when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=0.05, S_c=0.1, γ =0.1, β=2, λ =0.1 and α =π/4.

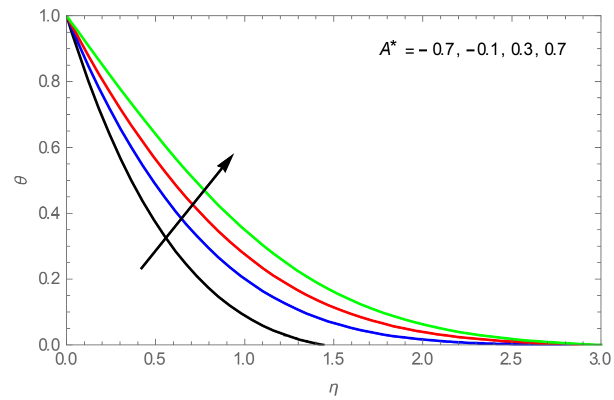

Fig.15: The plot for curves of θ under the effect of parameter A^* when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, B^*=0.05, S_c=0.1, γ =0.1, β=2, λ =0.1 and α =π/4.

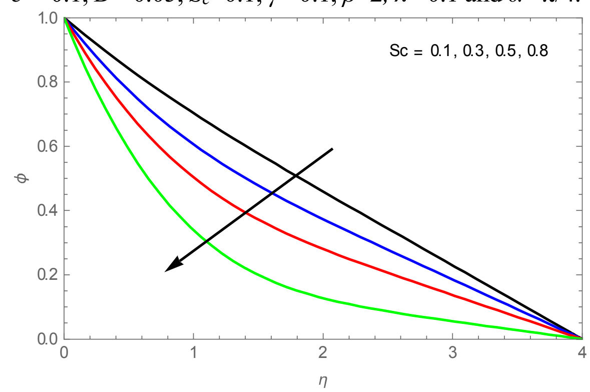

Fig.16: The plot for curves of ϕ under the effect of Schmidt number S_c when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=B^*=0.05, γ =0.1, β=2, λ =0.1 and α =π/4.

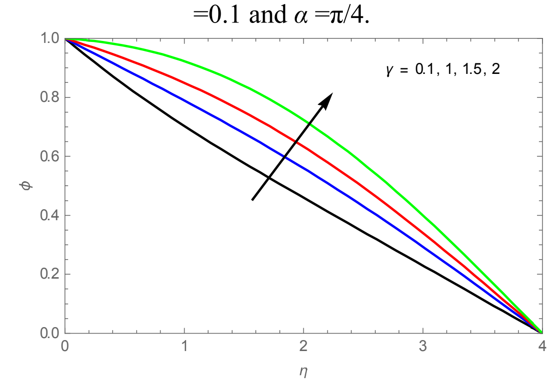

Fig.17: The plot for curves of ϕ under the effect of of Solutal number γ when S=1, K=0.1, P_r =1, M=0.1, c=–0.1, A^*=B^*=0.05, S_c=0.1, β=2, λ =0.1 and α =π/4.

REFERENCES

[1] K. Das, "Effect of Chemical Reaction and Thermal Radiation on Heat and Mass Transfer Flow of MHD Micropolar Fluid in a Rotating Frame of Reference", International Journal of Heat and Mass Transfer., vol. 54, no. 15-16, (2011), pp. 3505-3513.

[2] T. Hayat, S. A. Shehzad and A. Alsaedi, "Soret and Dufour Effects on Magneto Hydrodynamic (MHD) Flow of Casson Fluid", Applieed Mathematics and Mechanics, vol. 33, no. 10, (2012), pp. 1301-1312.

[3] M. M. Rashidi, S. A. M. Pour, T. Hayat and S. Obaidat, "Analytic Approximate Solutions for Steady Flow over a Rotating Disk in Porous Medium with Heat Transfer by Homotopy Analysis Method", Computers & Fluids., vol. 54, (2012), pp. 1-9.

[4] N. Sandeep, V. Sugunamma and P. Mohankrishna, "Effects of Radiation on an Unsteady Natural Convective Flow of a EG-Nimonic 80a Nanofluid Past an Infinite Vertical Plate", Advances in Physics Theories and Applications, vol. 23, no. 7, (2014), pp. 36-43.

[5] N. Sandeep and V. Sugunamma, "Effect of Inclined Magnetic Field on Unsteady Free Convection Flow of a Dusty Viscous Fluid between Two Infinite Flat Plates Filled by Porous Medium", International Journal of Applied Mathematics and Modeling, vol. 1, no. 1, (2013), pp. 16-33.

[6] S. K. Nandy, "Analytical Solution of MHD Stagnation-point Flow and Heat Transfer of Casson Fluid over a Stretching Sheet with Partial Slip", ISNR Thermodynamics., vol. 2013, (2013), Article Id:108264.

[7] N. Sandeep and V. Sugunamma, "Radiation and Inclined Magnetic Field Effects on Unsteady Hydrodynamic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium", Journal of Applied Fluid Mechanics., vol. 7, no. 2, (2014), pp. 275-286. International Journal of Advanced Science and Technology Vol.91 (2016) 36 Copyright Ⓒ 2016 SERSC .

[8] P. Mohan Krishna, N. Sandeep and V. Sugunamma, "Effects of Radiation and Chemical Reaction on MHD Convective Flow over a Permeable Stretching Surface with Suction and Heat Generation", Walailak Journal of Science and Technology, vol. 12, no. 9, (2014), pp. 831-847.

[9]. Mahantesh M. Nandeppanavar, Flow and Heat transfer analysis of Casson fluid due to a stretching sheet. Advances in Physics Theories and Applications, (2015) 50:27-34

[10]. MahanteshM.Nandeppanavar, Convective Heat Transfer Analysis of Non-Newtonian Fluid due to a Linear Stretching Sheet, Chemical and Process Engineering Research 41 (2016) 1-8.

[11]. MahanteshM.Nandeppanavar, Flow and Heat transfer of Casson fluid due to stretching sheet with convective boundary condition: an analytical solution, Chemical and Process Engineering Research, 41 (2016) 10-22.

[12]. H.A.Attia, M.E.S.Ahmed, Transient MHD Coutte flow of a Casson fluid between parallel flow with heat transfer., Italian J. Pure Appl. Math. 27(2010)19-38.

[13]. K.Bhattacharyya, T.Hayat, A.Alsaedi, Analytical solution for magnetohydrodynamic boundary layer flow Casson fluid over stretching sheet with wall mass transfer. Chin.Phys.B 22(2013)024702.

[14]. S. S.Mukhopadhyay, Effect of thermal radiation on Casson fluid flow and heat transfer over unsteady stretching surface subject to suction/blowing. Chin.Phys.B 22 (2013)114702(pp 1-7).

[15]. S.A.Shehzad, T.Hayat, M.Qasim and S.Aaghar, Effect of mass transfer on MHD flow of Casson Fluid with chemical reaction and suction , BrazillianJ.Chemical Engineering 30(2013)187-195.

[16] N. Sandeep, B. Rushi Kumar and M. S. Jagadeesh Kumar, "A Comparative Study of Convective Heat and Mass transfer in Non-Newtonian Nanofluid flow Past a Permeable Stretching Sheet", Journal of Molecular Liquids, vol. 212, (2015), pp. 585-591.

[17] C. S. K.Raju, N. Sandeep, C. Sulochana, V. Sugunamma and M. Jayachandrababu, "Radiation, Inclined Magnetic Field and Cross-Diffusion Effects on Flow over a Stretching Surface", Journal of the Nigerian Mathematical Society, vol. 34, no. 2, (2015), pp. 169-180.

[18] C. Sulochana and N. Sandeep, "Dual Solutions for Radiative MHD Forced Convective Flow of a Nanofluid over a Slandering Stretching Sheet in Porous Medium", Journal of Naval Architecture and Marine Engineering, vol. 12, (2015), pp. 115-124.

[19] C. S. K. Raju and N. Sandeep, "Heat and Mass Transfer in MHD Non-Newtonian Bio-convection Flow over a Rotating Cone/Plate with Cross Diffusion", Journal of Molecular Liquids, vol. 215, (2016), pp. 115-126.

[20] M. Satish Kumar, N. Sandeep and B. Rushi Kumar, "Dual Solutions for Heat and Mass Transfer in MHD Bio-convective Flow over a Stretching/Shrinking Surface with Suction/Injection", International Journal of Engineering Research in Africa, vol. 21, (2015), pp. 84-101.

[21] N. Sandeep, C. Sulochana and I. L. Animasaun, "Stagnation-point Flow of a Jeffrey Nanofluid over a Stretching Surface with Induced Magnetic Field and Chemical Reaction", International Journal of Engineering Research in Africa, vol. 20, (2016), pp. 93-111.

[22] N. Sandeep, A. VijayaBhaskar Reddy and V. Sugunamma, "Effect of Radiation and Chemical Reaction on Transient MHD Free Convective flow over a Vertical Plate through Porous Media", Chemical and Process Engineering Research, vol. 2, (2012), pp. 1-9.

[23] H .waqas, S. Hussain, S.Khalid. Chemaical diffusion and thermal effects on MHD stagnation oint flow of Casson fluids past a porous surface International Journal of Scientific and Engineering Research. 8(3), march, (2017) 1092-1103.

[24] H. Waqas, S. Rafique, , S. Khalid3, F. Ahamad, S. Hussain. Thermal radiation effects on unsteady MHD flow of Casson fluids through porous medium over a permeable shrinking sheet. *J. Appl. Environ. Biol. Sci.*, 7(4)201-209, 2017.

[25] N. Tariq, S.Hussain Magnetohydrodynamic flow of Casson fluids over a moving boundary surface. J. Appl. Environ. Biol. Sci.,7(4)192-200, 2017

[26] H. Waqas, S. Rafique, , S. Khalid3, F. Ahmad, S. Hussain. The influence of radiation on MHD slip flow and heat transfer of Casson fluids due to shrinking sheet immersed in porous medium. Sci. Int. (Lahore),29(2),453-457,2017.

[27] H Waqas, U Rasheed, S. Farooq, S. Hussain, S Khalid. An analysis of thermal effects on electrically conducting Casson fluid flow past a permeable shrinking surface in a porous medium. Sci. Int. (Lahore), 29(2), 459-464,2017.

[28] H. Waqas, A. Maryam, S. Khalid3, F. Ahmad, S. Hussain. Radiation effects and chemical reaction on boundary layer flow of electrically conducting Casson fluids owing to porous shrinking sheet. J. Appl. Environ. Biol. Sci., 7(5)117-125, 2017