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Numerical Solution of Hydromagnetic Boundary Layer Flow Due to Shrinking Surface with Prescribed Heat Flux

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ABSTRACT

The hydromagnetic boundary layer flow due to shrinking surface with prescribed heat flux has been considered for its numerical solution to observe the effects of physical parameters namely suction parameter *S,* the magnetic parameter M^2, Prandtl number *Pr*, and temperature index parameter *n,* on fluid flow and temperature distribution. For the purpose of numerical solution of the problem, suitable similarity transformations have been used to obtain the ordinary differential form of the governing equations of motion and energy. The results have been obtained and presented graphically in the form of non-dimensional velocity and temperature function.

KEYWORDS: hydromagnetic, boundary layer, shrinking sheet,suction parameter, Prandtl number

1. INTRODUCTION

The researchers are consistently interested in the study of hyderomagnetic boundary layer flow due to moving boundaries. The hyderomagnetic flows have applications in astrophysics, geophysics and many industrial processes such as accelerators, generators, bearing, coils, pumps etc. Arthur and Seini [1] investigated the hydromagnetic stagnation point flow of an incompressible viscous electrically conducting fluid towards a stretching sheet in the presence of radiation and viscous dissipation. Chakrabarti and Gupta [2] studied the hydromagnetic flow and heat transfer in a fluid, initially at rest and at uniform temperature, over a stretching sheet at a different uniform temperature. Banerjee [3] studied the effect of rotation on the hydromagnetic flow between two parallel plates where the upper plate is porous and solid, and the lower plate is a stretching sheet. Sheikholeslami et al. [4] analyzed hydromagnetic flow between two horizontal plates in a rotating system, where the lower plate is a stretching sheet and the upper is a porous solid plate and studied heat transfer in an electrically conducting fluid bonded by two parallel plates is in the presence of viscous dissipation using homotopy perturbation method. Ganji et al. [5] reported the analytical solution of the magnetohydrodynamic flow over a nonlinearly stretching sheet. Pandya [6] considered hydromagnetic flow between two horizontal plates in a rotating system where the lower plate is a stretching sheet and obtained numerical solution for the governing coupled ordinary differential equations by employing quintic spline collocation method.

Sakiadis [7, 8] was the pioneer to consider the surface stretching problem due to the boundary layer flow. Crane [9] discussed the flow caused by the stretching of a sheet. Many authors including Xu and Liao [10], Cortell [11, 12], Farooq [13] and Hayat et al. [14] studied various aspects of the fluid flow due to stretching boundaries. Ahmad et al. [15] obtained closed form solution for a viscous, incompressible, MHD flow over a porous stretching sheet. But the flow induced by a shrinking sheet is different from forward stretching flow, as first observed by Wang [16]. Miklavcˇicˇ and Wang [17] established the existence and uniqueness of the similarity solution of the equation for the steady flow due to a shrinking sheet and they also reported that an adequate suction is necessary to maintain the steady flow. Goldstein [18] opinioned the shrinking flow is essentially a backward flow. Also, Fang and Zhang [19] found a closed-form analytic solution for two dimensional MHD flow over a porous shrinking sheet subjected to wall mass transfer. Further, Fang and his co-authors [20–24] discussed some other important aspects of shrinking flow. Bhattacharyya [30] studied the flow over an exponentially shrinking sheet. Recently, an analytic solution of steady two dimensional MHD rotating flow of a second grade over a porous shrinking surface was reported by Faraz and Khan [25] using homotopy perturbation method. Yacob and Ishak [26] and Bhattacharyya et al. [27] discussed the micropolar fluid flow over a shrinking sheet with out and with thermal radiation, respectively.

This work examines the numerical solution for hydromagnetic boundary layer flow due to shrinking surface with prescribed heat flux. Siti et al [28] examined hydromagnetic boundary layer flow over a stretching surface with thermal radiation. We obtained the results for velocity and temperature distribution for flow due to shrinking surface. Some interesting results have been obtained and presented in graphical form.

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2. MATHEMATICAL ANALYSIS

Consider a viscous, incompressible and electrically conducting fluid. The fluid flow is assumed to be steady and two-dimensional and it is due to a vertical shrinking surface. The x-axis is along the sheet and y-axis is perpendicular to it, where the origin is fixed. A magnetic field of strength B_0 is applied normal to the surface. The magnetic Reynolds number is small and the induced magnetic field is negligible. Here $\underline{V} = V(u,v)$ is the velocity vector and $T_w = T_\infty + bx^n$ is surface temperature where T_∞ is free stream temperature and b is a constant. The viscous dissipation is neglected.

Under the above assumptions, the governing equations of motion become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \pm g\beta(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(\frac{\partial^2 T}{\partial y^2}) \tag{3}$$

where ρ, υ, α and T are fluid density, kinematic viscosity, thermal diffusivity and fluid temperature respectively.

The boundary conditions are :

$$\begin{aligned} &y = 0, u = -ax^m, v = -V, T = T_w(x) \\ &y \to \infty, u \to 0, T \to T_\infty \end{aligned} \tag{4}$$

where a is positive constant.

Introducing the stream function $\psi(x, y)$, we have

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = \frac{-\partial \psi}{\partial x} \tag{5}$$

The continuity equation (1) is satisfied identically. By using the similarity transformation:

$$\psi = (a\upsilon x^{m+1})^{1/2} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \sqrt{\frac{ax^{m-1}}{\upsilon}}y, \tag{6}$$

The equations (2) and (3) in dimensionless form lead to:

$$f''' + \frac{m+1}{2}ff'' - mf'^2 - M^2 f' + \lambda\theta = 0 \tag{7}$$

$$\theta'' + P_r f\theta' = 0 \tag{8}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &\eta = 0, f = f_w, f' = -1, \theta = 1 \\ &\eta \to \infty, f' = 0, \theta = 0 \end{aligned}\right\} \tag{9}$$

where prime denotes differentiation with respect to η, $S = \frac{V}{\sqrt{a\upsilon}}$ is suction parameter, $P_r = \frac{\upsilon}{\alpha}$ is the Prandtl number, $M^2 = \frac{\sigma B_0^{\ 2}}{\rho a}$ is the magnetic parameter

3. RESULTS AND DISCUSSION

The numerical solution for the highly non- linear ODEs (7) and (8) together with boundary conditions (9) has been obtained by using Mathematica software. The graphical pattern of the curves for the results shows that the fluid flow develops a different shape and behavior from the flow due to stretching phenomenon as presented by [28]. More over new results have been established for temperature distribution with prescribed heat flux condition.

The curves for velocity f' as presented respectively in fig.1 and fig.2 show that the velocity increases with increase in the values of magnetic parameter M^2 as well as the suction parameter *S*. But the temperature function decreases with increase in the values of M^2 and *S* as shown respectively in fig.3 and fig.4. The fig.5 and fig.6 depict that the temperature distribution increases with increase in the values of Prandtl number P_r and temperature index parameter *n*.

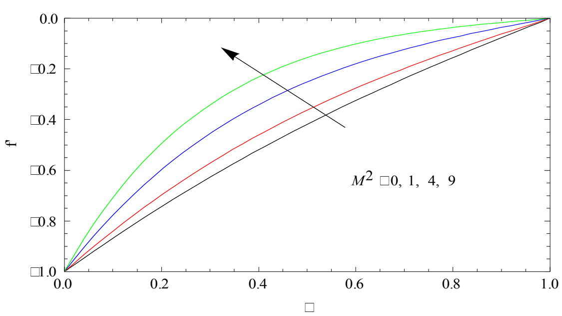

Fig.1: Graph of f' for different values of M^2.

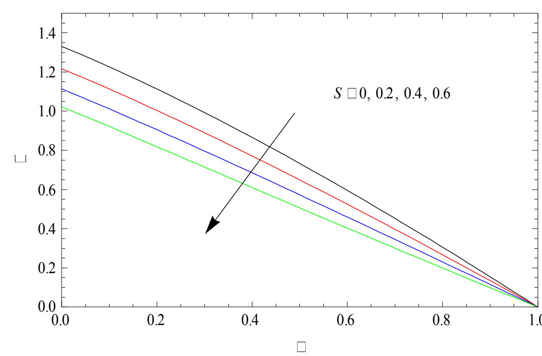

Fig.4: Graph of θ for different values of S.

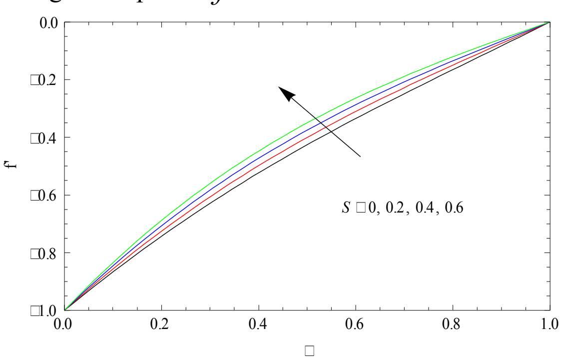

Fig.2: Graph of f' for different values of *S*.

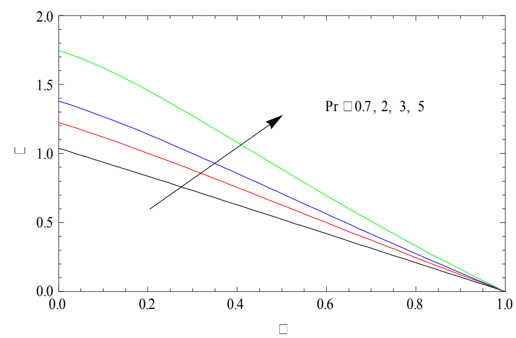

Fig.5: Graph of θ for different values of P_r.

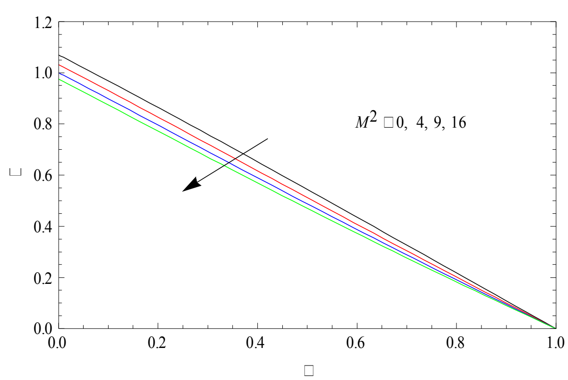

Fig.3: Graph of θ for different values of M^2

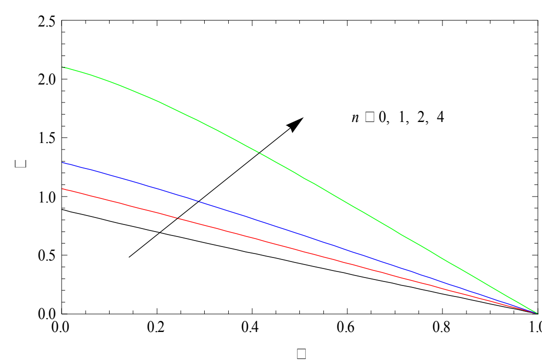

Fig.6: Graph of θ for different values of

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