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MHD Forced Convective Boundary Layer Flow of Micropolar Fluids past a Shrinking Porous Sheet Prescribed with Variable Heat Flux and Heat Source

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ABSTRACT

This article intends to investigate MHD forced convective boundary layer flow of micropolar fluids past a shrinking porous sheet prescribed with variable heat flux and heat source. The highly nonlinear partial differential equations of motion have been transformed to ordinary differential form by using similarity functions. Numerical solution of the resulting equations is obtained using Runge Kutta fourth order method with shooting technique. The results have been computed for several values of the parameters namely magnetic parameter, velocity ratio parameter, suction parameter, Prandtl number, heat source parameter and the non-dimensional micropolar parameters D, C_1 and C_2. The effects of these parameters have been observed on fluid velocity, microrotation and heat function.

KEYWORDS: Forced convection, micropolar fluids, shrinking sheet, porous sheet, suction parameter

1. INTRODUCTION

Over the past few decades, fluid dynamics at micro and Nano scales has been expanding research field. The spinning of molecules affects significantly the flow field as observed in [1, 2]. Such motions can be described with Micropolar theory offered by Eringen [3]. Physically, micropolar fluids can be seen in Ferro fluids, blood flows, bubbly liquids, liquid crystals, and so on, all of them containing intrinsic polarities. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Khonsari and Brewe [4], Chamkha et al. [5], Bachok et al. [6], Kim and Lee [7] and Sajjad and Anwar [8].

The flow and heat transfer over a stretching surface bears important research interest due to its various applications in industries such as hot rolling, wire drawing, glass fiber production, manufacturing plastic films and extrusion of a polymer in a melt spinning process. Extensive work has been done in this research area by Sakiadis [9-10], Crane [11], Gupta and Gupta [12] and Dutta et al. [13]. The flow induced by a shrinking sheet is different from forward stretching flow, as first observed by Wang [14]. Shrinking sheet is a surface which decreases in size to a certain area due to an imposed suction or external heat. Shrinking problem can also be applied to study the capillary effects in smaller pores, the shrink-swell behavior and the hydraulic properties of agricultural clay soils. One of the most common applications of shrinking sheet problems in industries and engineering is shrinking film. In packaging of bulk products, shrink film is very useful as it can be unwrapped easily with adequate heat. Goldstein [15] opinioned the shrinking flow is essentially a backward flow. Fang [16] reported an analytic solution of the boundary layer flow over a shrinking sheet with a power law surface velocity and wall mass transfer. Also, Fang and Zhang [17] found a closed-form analytic solution for two dimensional MHD flow over a porous shrinking sheet subjected to wall mass transfer. Sajjad et al. [18] considered MHD boundary layer flow of micropolar fluids over a permeable shrinking sheet.

This article examines the numerical solution for MHD forced convective boundary layer flow of micropolar fluids past a shrinking porous sheet prescribed with variable heat flux and heat source. The results have been computed for several values of the parameters of the study in order to observe their effects on fluid flow and heat distribution.

2. MATHEMATICAL ANALYSIS

We consider a steady, two dimensional, laminar nonlinear hydromagnetic boundary layer flow of a viscous, incompressible, electrically conducting micropolar fluid, caused by a stretching/shrinking sheet subjected to suction

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in the presence of uniform transverse magnetic field. Cartesian coordinate system is chosen. A magnetic field of strength B_0 is applied normal to the boundary. The fluid flows with velocity $\underline{V}$ = V(u ,v).The microrotation vector is $\underline{\omega} = (0,0,\omega_3)$ All the fluid properties are assumed to be constant. The body couple is neglected.

Keeping in view, the governing equations of motion as given by Eringen [3] become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$(\mu + \kappa)(\frac{\partial^2 u}{\partial y^2}) + \kappa(\frac{\partial \omega_3}{\partial y}) - \rho\sigma B_0^2 u = \rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) \tag{2}$$

$$\gamma(\frac{\partial^2 \omega_3}{\partial x^2} + \frac{\partial^2 \omega_3}{\partial^2 y}) - \kappa\frac{\partial u}{\partial y} - 2\kappa\omega_3 = \rho j(u\frac{\partial \omega_3}{\partial x} + v\frac{\partial \omega_3}{\partial y}) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) \tag{4}$$

where κ and γ are material constants, ρ is the density, μ is the coefficient of viscosity, υ is kinematic viscosity coefficient, σ is the electrical conductivity of the fluid, T is the temperature, T_∞ is the free stream temperature, K is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure.

The boundary conditions are

The boundary conditions are:

$$u = -ax, \quad v = -v_0, \quad -K\frac{\partial T}{\partial y} = q_w = nx^{n-1}, \quad \omega_3 \to 0 \quad \text{at} \quad y=0$$

$$u = -bx, \quad T \to T_\infty, \quad w_3 \to 0 \quad \text{as} \quad y \to \infty \tag{5}$$

Where a is stretching velocity constant, b proportionality constant of free steam velocity and T_∞ is the free stream temperature.

The stream function ψ (x, y) is such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Using similarity transformations:

$$\psi = \sqrt{a\upsilon}xf(\eta), \quad \eta = y\sqrt{\frac{a}{\upsilon}} \tag{7}$$

$$u = axf'(\eta), \quad v = -\sqrt{a\upsilon}f(\eta), \quad \omega_3 = a\sqrt{\frac{a}{\upsilon}}xh(\eta) \tag{8}$$

$$T - T_\infty = \frac{nx^{n-1}}{K}\sqrt{\frac{\upsilon}{a}}\theta(\eta)$$

Equation of continuity (1) is identically satisfied.

Substituting (8) in to equations (2) to (4), we have

$$(1+D)f''' + ff'' - f'^2 + Dh' - M^2 f' = 0 \quad (9)$$

$$h'' - C_1 f'' - 2C_1 h = C_2(f'h - fh') \quad (10)$$

$$\theta'' + \Pr(f\theta' - nf'\theta + B\theta) = 0 \quad (11)$$

where D, C_1 and C_2 are dimensionless material constants, $M^2 = \frac{\sigma B_0^2}{\rho a}$ is magnetic parameter, $\Pr = \frac{\mu c_p}{K}$ is Prandtl number and $B = \frac{Q}{a\rho c_p}$ is heat source parameter.

$$f(0) = S, \quad f'(0) = -1, \quad h(0) = 0, \theta'(0) = -1$$
$$f'(\infty) = \varepsilon, \quad h(\infty) = 0, \quad \theta(\infty) = 0 \quad (12)$$

Here $S = \frac{v_0}{\sqrt{a\upsilon}}$ is suction parameter, $\varepsilon = \frac{b}{a}$ is velocity ratio parameter

3. RESULTS AND DISCUSSION

The numerical solution of the resulting ordinary differential equations (9) to (11) with boundary conditions (12) has been obtained by using Mathematica software. The results for non-dimensional velocity f', micro rotation h and temperature function θ have been obtained for representative values of the physical parameter namely, S, M^2, ε, Pr, B and n. It is to be mentioned that when D=0, the problem reduces to Newtonian fluid flow.
The graphical pattern of the results reveals the very nature of the flow and heat distribution.

It can be observed that the velocity f' increases with increase in the values of the parameters S, M^2 as depicted in fig.1 and fig.2 respectively. It is noticed in fig.3 that velocity f' increases with increase in the magnitude of ε. Fig.4 shows that f' decrease with increase in the values of D.

Fig.5 and Fig.6 respectively demonstrate the behavior of microrotation function, it is observed that the microrotation h decreases with increase in the values of S but decreases with increase in D.

The temperature function decreases with increase in Pr and B as shown respectively in fig.7 and fig.8. But it increases with increase in the magnitude of n as depicted in fig.9.

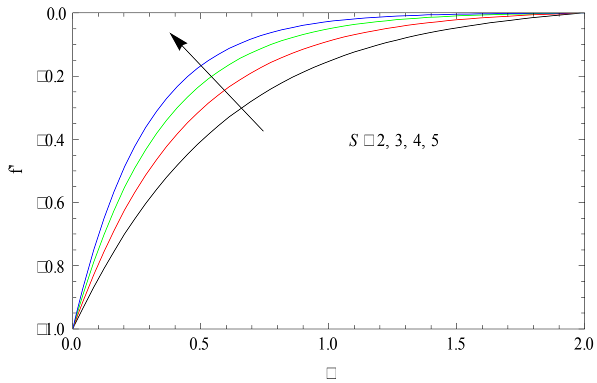

Fig.1: Graph of f' for different values of S

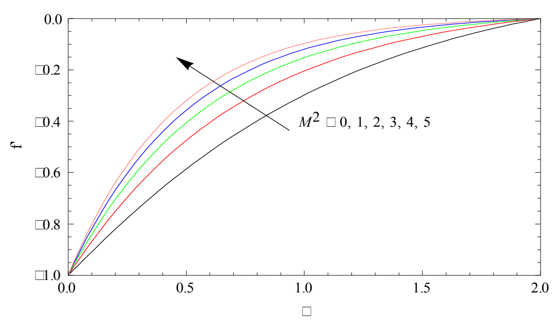

Fig.2: Graph of f' for different values of M^2

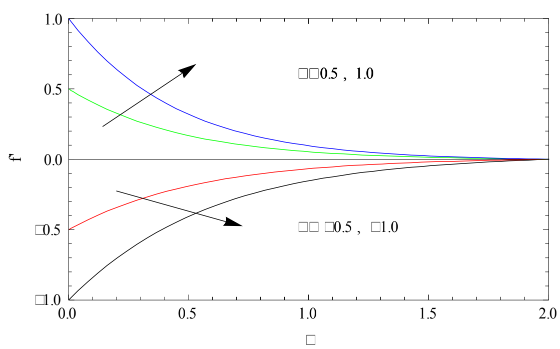

Fig.3: Graph of f' for different values of ε

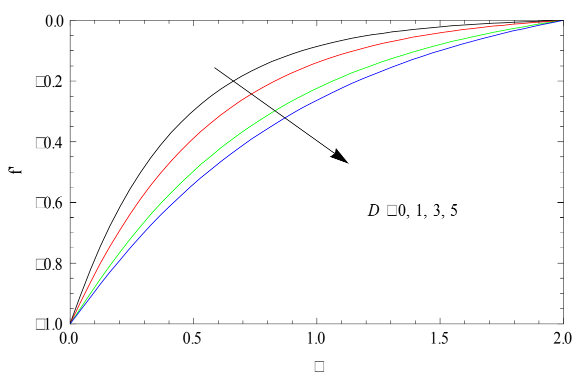

Fig.4: Graph of f' for different values of D

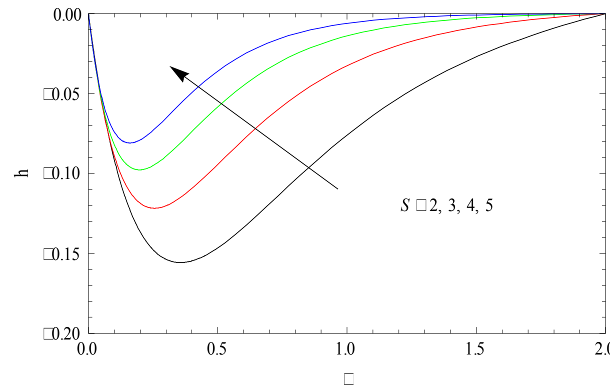

Fig.5: Graph of h for different values of S

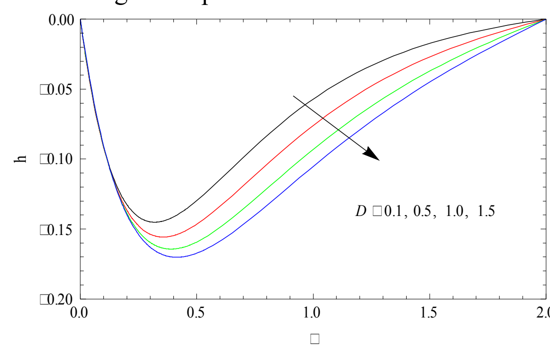

Fig.6: Graph of h for different values of D

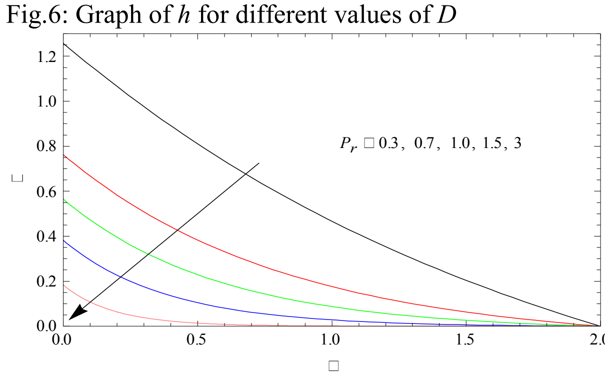

Fig.7: Graph of θ for different values of Pr

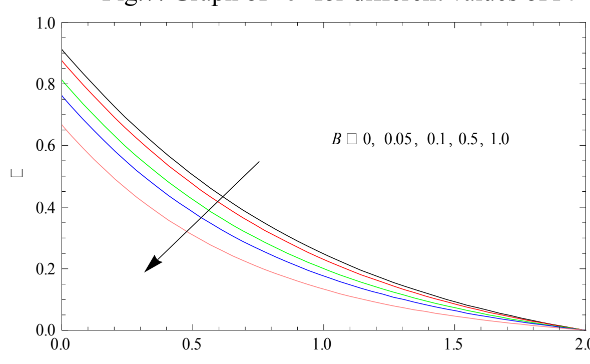

Fig.8: Graph of θ for different values of B

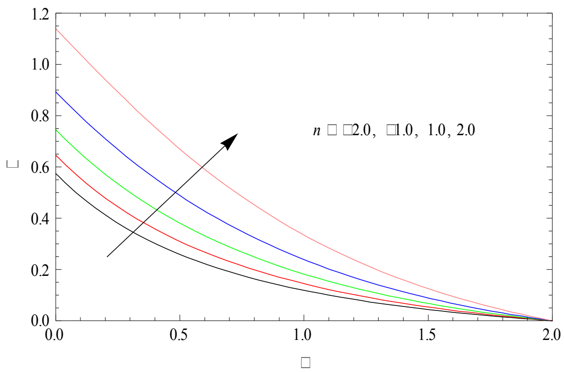

Fig.9: Graph of θ for different values of *n*

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