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Heat Transfer and Unsteady MHD Flow of Third Grade Fluid Past on Vertical Oscillating Belt

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ABSTRACT

In this paper an investigation is made for the thin film unsteady MHD flow of a third grade fluid with heat transfer analysis on vertical oscillating belt. The resulting problems of velocity and temperature distribution are solved by means of OHAM. It is noted that the graphical results for several interesting physical parameters are also presented and analyzed.

KEYWORDS: Unsteady thin film flows, third order fluid, MHD, Heat and OHAM.

1 INTRODUCTION

In the study of fluid dynamics many engineering and industrial problems are usually described by non-Newtonian fluids. Several industrial and biological fluids show the non-Newtonian behavior such as molten plastics, polymers, blood, tomato ketchup, honey, mud and slurries. For these types of fluids the fundamental governing equations of motion are highly nonlinear and complex partial differential equations. Certain important studies dealing with the flow of non-Newtonian fluids are [1-3]. In various types of flow situations the subclass of differential type fluids such as second and third grade fluid have received special attention. So the present analysis deals with the problem of non-Newtonian third grade fluid due to extensive uses. Third grade fluid captures the non-Newtonian effects as shear thinning or shear thinking and also normal stress. Ariel [4] studied the steady flow of third grade fluid through porous channel. He obtained the result of the corresponding BVP by applying two numerical techniques. Keimanesh et al. [5] examined the three types of flows of third grade fluid. They used multi-step differential transform method (MDTM) for the solutions and compare the result with fourth order Runge-Kutta method. Fakhar and Chen [6] analyzed the approximate solution of the steady flow of third grade fluid on porous plate. Therefore a number of researchers have discussed the flow of third grade fluid in different configurations, see [7-10].In the literature physical structures of thin film have been examined and discussed by various scholars [11-15].

In several areas of technologies the flow of an electrically conducting fluid in the presence of a magnetic field is of great significance such as MHD power generation and MHD pumps. Hameed and Nadeem [16] investigated the flow of MHD second grade fluid on porous plate. They find the exact solutions of problems and discussed the effect of the model variable on velocity field. Kumari and Nath [17] studied the MHD unsteady flow of fluid on rotating disc. Das et al. [18] studied the exact solution of unsteady MHD couette flow in a rotating system by using Laplace transforms. Jha and Apere [19] also investigated the MHD couette flow and studied the effect of various parameters on velocity field. The heat transfer problem of third grade fluids has been studied by several authors. Nasir et al. [20] analyzed heat transfer and MHD flow of third grade fluid on inclined belt. They have been used OHAM and HPM techniques to obtain the solution of heat and velocity profile.

Different asymptotic techniques are used for the approximate solution of nonlinear problems because there is a paucity of the exact solution of these problems. Here we successfully applied a new powerful and straightforward technique OHAM. Marinca and Herisanu [21] studied the approximate analytical solution of the oscillating particle using OHAM. Some other important investigation dealing with OHAM solutions are [22- 24].

The central goal of the present work is to determine analytical solutions for heat transfer and unsteady MHD flow of a third-grade fluid over a moving and oscillating vertical belt. OHAM technique has been applied to the relevant problem of heat and velocity profile. This paper is planned in the following manner. Section 2 contains the basic theory of the OHAM. Section 3 deals with applications of the OHAM to the Cauchy reaction-diffusion problem. Section 4 is reserved for conclusions.

2 Basic idea of OHAM

In this section we establish the basic concept of OHAM, consider the boundary value problem [21, 22]

$$\mathfrak{L}(v(x,t)) + \mathcal{K}(v(x,t)) + \mathcal{F}(v(x,t)) = 0, \qquad \mathfrak{B}(v(x,t)) = 0, \tag{1}$$

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Here$\mathfrak{T}$, is the linear operator in differential equation, $v(x, t)$ is unknown function, $\mathcal{K}$ is non linear term, $\mathcal{F}$ is source term, x is independent variable and $\mathfrak{B}$ is boundary operator. According to OHAM first we construct a family of equation as

$$[1-p][\mathfrak{T}\psi(x,p)+\mathcal{F}(x)] = H(p)[\mathfrak{T}\psi(x,p)+\mathcal{F}\psi(x,p)+\mathcal{K}\psi(x,p)], \tag{2}$$

Where p is embedding parameter and$p \in [0,1]$. We choose the auxiliary function $H(p)$ as $H(p) = pc_1 + p^2c_2 + p^3c_3 \dots\dots$ (3)

Here$c_1, c_2, c_3, \dots\dots$ are called auxiliary constants and the values of these constant determined latter.
Here in Eq. (2) $\psi(x,p)$ is unknown function. Obviously when $p = 0$ and $p = 1$ it holds that

$$\psi(x,0) = v_0(x)\ \ ,\ \psi(x,1) = v(x), \tag{4}$$

To get the approximate solution, we expand $\psi(x,p,c_j)$ in the following form

$$\psi(x,p,c_i) = v_0(x) + \sum_{k\geq 1} v_k(x,c_i)p^k\ \ ,\ i = 1,2,3,\dots m, \tag{5}$$

Now substituting Eq. (5) into Eq. (2) collecting the same power of p and equating each coefficient of p equal to zero. We get the component of unknown function $v(x)$ which is zero, first, second components are given by$v_0(x)\ , v_1(x,c_1)\ \ and\ \ v_2(x,c_2)$. We obtained the general solution of Eq. (2) is

$$v^n = v_0(x) + \sum_{k-1}^{n} v_k(x,c_1,c_2,c_3,\dots\dots,c_m), \tag{6}$$

In OHAM method, we note that the last coefficient of c_i is a function of x.
Substituting Eq. (5) in Eq. (1), we obtained the following expression for residual

$$\mathrm{R}(x,c_1,c_2,c_3,\dots\dots,c_m) = \mathfrak{T}(v(x,c_1,c_2,c_3,\dots\dots,c_m)) + \mathcal{F}(x) + \mathcal{K}(v(x,c_1,c_2,c_3,\dots\dots,c_m)), \tag{7}$$

There are different methods used to find the optimal value of auxiliary constants $c_i\ , i = 1,2,3\ \dots.$ like Galerkin's Method, Ritz Method, Least Squares Method and Collocation Method. Here we apply the Method of Least Squares in the current problem

$$\mathcal{J}(c_1.c_2,c_3,\dots,c_m) = \int_a^b \mathrm{R}^2(x,c_1.c_2,c_3,\dots,c_m)dx, \tag{8}$$

Where a, b are two values depending on the problem.
The auxiliary constants $c_i\ , i = 1,2,3,\dots.m$ can be identifiedfrom the given conditions

$$\frac{\partial \mathcal{J}}{\partial c_1} = \frac{\partial \mathcal{J}}{\partial c_2} = \frac{\partial \mathcal{J}}{\partial c_3} = \cdots = \frac{\partial \mathcal{J}}{\partial c_m} = 0, \tag{10}$$

With these auxiliary constants $(c_1.c_2,c_3,\dots,c_m)$the approximate solution is well determined.

3 Statement of lifting problem

We consider heat transfer and unsteady flow of third grade fluid on a flat oscillating and vertically moving upward belt with velocity V. During the motion the belt take itself a thin layer of liquid with thickness δ. A uniform magnetic field $B = (0, B_0, 0)$ transversely applied to the belt. Chosen the coordinate axis in such a way that x-axis is perpendicular and y-axes parallel to the belt as shown (Figs.1).

Thus, for the problem under consideration we seek velocity and temperature fields of the form [11]

$$\mathbf{V} = (0, v(x,t), 0), \tag{11}$$

$$\Theta = \Theta(x,t). \tag{12}$$

The prescribed boundary conditions are

$$\mathbf{v}(0,t) = V + V\cos\omega t \quad \text{and} \quad \frac{\partial \mathbf{v}(\delta,t)}{\partial x} = 0, \tag{13}$$

$$\Theta(0,t) = \Theta_0 \text{and} \quad \Theta(\delta,t) = \Theta_1, \tag{14}$$

here$v(x,t)$ is velocity component in x-direction, $\Theta(x,t)$is the temperature and $\omega$is the frequency of the oscillating belt.

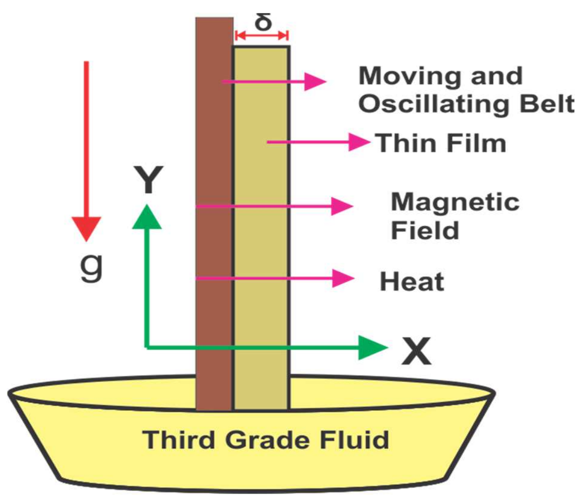


Figure 1: Geometry of lift problem

3.1 Mathematical analysis of Flow and Heat

The continuity and momentum equations for the MHD unsteady flow of incompressible third grade fluid [11, 12] are given

$$\mathrm{div}\,\mathbf{v} = 0, \tag{15}$$

$$\rho \frac{D\mathbf{v}}{Dt} = \mathrm{div}\mathbf{T} + \rho g + \mathbf{J}\times\mathbf{B}, \tag{16}$$

$$\rho c_p \frac{D\Theta}{Dt} = k\nabla^2\Theta + tr(\boldsymbol{T}.\boldsymbol{L}), \tag{17}$$

$$\mathbf{J}\times\mathbf{B} = \left[0, \sigma {B_0}^2 v(x), 0\right]. \tag{18}$$

where ρ is the fluid density, $\mathbf{J}$ is the current density, $\mathbf{B} = (0, B_0, 0)$ is the uniform magnatic filed, σ is the electrical conductivity, μ_0 is the magnetic permeability, $\mathbf{E}$ is electric field, $\frac{D}{Dt}$ is the material time derivative, g is the external body force, k is the thermal conductivity and c_p is specific heat.

T is the Cauchy stress tensor for third grade fluid model [1, 11] is defined as

$$\mathbf{T} = -pI + \mu\mathbf{A}_1 + \alpha_1\mathbf{A_2} + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A_2}\mathbf{A}_1) + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1, \tag{19}$$

where **I** is identity tensor, p is fluid pressure, μ is coefficient of viscosity, $\alpha_i (i = 1,2)$ are the material constants second grade fluid and $\beta_j (j = 1,2,3)$ are the material constants third grade

The first three Rivlin-Ericksen tensors $\mathbf{A}_n (n = 1,2,3)$ are defined by the following relations [11]

$$\mathbf{A}_0 = \mathrm{I},\ \mathbf{A}_1 = \boldsymbol{L} + \boldsymbol{L}^T,\ \boldsymbol{L} = grad(\mathbf{v}), \tag{20}$$

$$\mathbf{A}_n = \frac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}(\mathrm{L}) + (\mathrm{L})^T\mathbf{A}_{n-1},\ \ n = 2,3. \tag{21}$$

With above form of velocity field the continuity Eq. (15) is identically satisfied while the momentum and heat Eqs. (16), (17) takesthe form

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \frac{\partial}{\partial x} T_{yx} + \rho g - \sigma {B_0}^2\mathbf{v}, \tag{22}$$

$$\rho c_p \frac{\partial\Theta}{\partial t} = k\left(\frac{\partial^2\Theta}{\partial x^2}\right) + T_{yx}\left(\frac{\partial \mathrm{v}}{\partial x}\right). \tag{23}$$

Using Eq. (19) we obtain the T_{xy}

$$T_{xy} = \mu\frac{\partial \mathbf{v}}{\partial x} + \alpha_1\frac{\partial}{\partial t}\left(\frac{\partial \mathbf{v}}{\partial x}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial \mathbf{v}}{\partial x}\right) + 2(\beta_2+\beta_3)\left(\frac{\partial \mathbf{v}}{\partial x}\right)^3 = T_{yx}, \tag{24}$$

Substituting Eq. (24) into Eq. (22) and (23) we get

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \mu\frac{\partial^2 \mathbf{v}}{\partial x^2} + \rho\alpha_1\frac{\partial}{\partial t}\left(\frac{\partial^2 \mathbf{v}}{\partial x^2}\right) + 6\beta_3\left(\frac{\partial \mathbf{v}}{\partial x}\right)^2\left(\frac{\partial^2 \mathbf{v}}{\partial x^2}\right) - \rho g - \sigma B_0\mathbf{v}, \tag{25}$$

$$\rho c_p\frac{\partial\Theta}{\partial t} = k\left(\frac{\partial^2\Theta}{\partial x^2}\right) + \mu\left(\frac{\partial \mathbf{v}}{\partial x}\right)^2 + \alpha_1\left(\frac{\partial^2 \mathbf{v}}{\partial t\partial x}\right)\left(\frac{\partial \mathbf{v}}{\partial x}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial \mathbf{v}}{\partial x}\right) + 2(\beta_2+\beta_3)\left(\frac{\partial \mathbf{v}}{\partial x}\right)^4. \tag{26}$$

We introducing some non-dimensional quantities as fallows

$$\tilde{v} = \frac{\mathbf{v}}{V},\ \tilde{x} = \frac{x}{\delta},\ \tilde{t} = \frac{t\mu}{\delta^2\rho},\ \alpha = \frac{\alpha_1}{\rho\delta^2},\ \beta = \frac{\beta_3 V^2}{\mu\delta^2},\ S_t = \frac{\rho\delta^2 g}{\mu V},\ M = \frac{\delta^2\sigma B_0}{\mu V},\ \widetilde{\beta_1} = \frac{\beta_1\mu}{\delta^2}, P_r = \frac{c_p\mu}{k},\ E_c = \frac{V^2}{c_p(\Theta_1-\Theta_0)}, \overline{\Theta} = \frac{\Theta-\Theta_0}{\Theta_1-\Theta_0}. \tag{27}$$

In above expression P_r is Prandtl number, E_c is Eckert number, $\tilde{t}$ is a dimensionless time parameter and S_t is the stock number.

Substitute the non-dimensional variables Eq. (27) into Eqs. (25) and (26), and dropping bars we obtained

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) - S_t - Mv, \tag{28}$$

$$P_r\frac{\partial\Theta}{\partial t} = \frac{\partial^2\Theta}{\partial x^2} + P_r E_c\left\{\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\left(\frac{\partial^2 v}{\partial t\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + \beta_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial v}{\partial x}\right) + 2\beta\left(\frac{\partial v}{\partial x}\right)^4\right\}. \tag{29}$$

And the boundary conditions are

$$v(o,t) = 1 + cos\omega t\ ,\quad \frac{\partial v(1,t)}{\partial x} = 0\ , \tag{30}$$

$$\Theta(0,t) = 0\ ,\quad \Theta(1,t) = 1. \tag{31}$$

3.3 OHAM Solution of lifting problem

In this section we applying the basic idea of the optimal homotopy asymptotic method (OHAM) presented in [20, 30] also in [30, 31] on Eqs. (28) and (39). We obtained the zeroth, first and second order problems with boundary conditions Eq. (30), (31) are as given below

Zeroth order velocity and heat problems:

$$q^0:\ \frac{\partial^2 v_0(x,t)}{\partial x^2} = S_t, \tag{32}$$

$$q^0:\ \frac{\partial^2 \Theta_0(x,t)}{\partial x^2} = 0, \tag{33}$$

First order velocity and heat problems:

$q^1: \ \frac{\partial^2 v_1(x,t)}{\partial x^2} = -S_t - c_1\left(S_t + \mathrm{M}v_0 - \left(\frac{\partial v_0}{\partial \mathrm{t}}\right) - \left(\frac{\partial^2 v_0}{\partial x^2}\right)\right) + \frac{\partial^2 v_0}{\partial x^2}\left(1 + 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\right) + \ \alpha\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_0}{\partial x^2}\right),$
(34)

$q^1: \ \frac{\partial^2 \Theta_1(x,t)}{\partial x^2} = -P_r c_3 \frac{\partial \Theta_0}{\partial \mathrm{t}} + P_r E_c c_3\left(\left(\frac{\partial v_0}{\partial x}\right)^2 + 2\beta\left(\frac{\partial v_0}{\partial x}\right)^4 + \alpha\left(\frac{\partial v_0}{\partial x}\right)\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial v_0}{\partial x}\right) + \beta_1\frac{\partial^2}{\partial \mathrm{t}^2}\left(\frac{\partial v_0}{\partial x}\right)\right) + \frac{\partial^2 \Theta_0}{\partial x^2}(1 + c_3),$
(35)

Solution of Eq. (32)- (35) by using boundary condition in Eq. (30) and (31) is

$$v_0(x,t) = (1 + \mathrm{Cos}[t\omega]) - \left(1 + \mathrm{Cos}[t\omega] + \frac{S_t}{2}\right)x + \left(\frac{S_t}{2}\right)x^2, \quad (36)$$

$$\Theta_0(x,t) = x, \quad (37)$$

$v_1(x,t) = c_1\Big[\Big[\frac{2}{3}M\mathrm{Cos}[\frac{t\omega}{2}]^2 - \frac{2}{3}\omega\mathrm{Cos}[\frac{t\omega}{2}]\mathrm{Sin}[\frac{t\omega}{2}] - \frac{1}{24}MS_t + \frac{3}{4}\beta\omega^2 S_t - \frac{3}{4}\beta\omega^2\mathrm{Cos}[2t\omega]S_t\Big]x +$
$\Big[\omega\mathrm{Cos}[\frac{t\omega}{2}]\mathrm{Sin}[\frac{t\omega}{2}] - M\mathrm{Cos}[\frac{t\omega}{2}]^2 - \frac{3}{2}\beta\omega^2 S_t + \frac{3}{2}\beta\omega^2\mathrm{Cos}[2t\omega]S_t\Big]x^2 + \Big[\frac{1}{3}M\mathrm{Cos}[\frac{t\omega}{2}]^2 - \frac{1}{3}\omega\mathrm{Cos}[\frac{t\omega}{2}]\mathrm{Sin}[\frac{t\omega}{2}] +$
$\frac{1}{12}MS_t + \beta\omega^2 S_t - \beta\omega^2\mathrm{Cos}[2t\omega]S_t\Big]x^3 - \frac{S_t}{4}\Big[\frac{1}{6}M - \beta\omega^2 + \beta\omega^2\mathrm{Cos}[2t\omega]\Big]x^4\Big],$
(38)

$\Theta_1(x,t) = P_r E_c c_3\Big[-2\mathrm{Cos}[\frac{t\omega}{2}]^4(1 + 5\beta) - \mathrm{Cos}[\frac{t\omega}{2}]^3\left(5\beta\mathrm{Cos}\left[\frac{3t\omega}{2}\right] + \beta\mathrm{Cos}\left[\frac{5t\omega}{2}\right] - 2\alpha\omega\mathrm{Sin}[\frac{t\omega}{2}]\right) -$
$\frac{1}{3}\mathrm{Cos}[\frac{t\omega}{2}]^2 c_3 S_t(1 + 10\beta) - \frac{S_t}{3}\mathrm{Cos}[\frac{t\omega}{2}]\left(5\beta\mathrm{Cos}\left[\frac{3t\omega}{2}\right] + \beta\mathrm{Cos}\left[\frac{5t\omega}{2}\right] - \frac{1}{2}\alpha\omega\mathrm{Sin}[\frac{t\omega}{2}]\right) - \frac{\beta}{4}S_t^2\Big(\frac{1}{4\beta} - 3 - \mathrm{Cos}[t\omega] -$
$\mathrm{Cos}[2t\omega] - \frac{4S_t}{5}\mathrm{Cos}[\frac{t\omega}{2}]^2 - \frac{1}{20}S_n^2\Big)\Big]x + \Big[2\mathrm{Cos}[\frac{t\omega}{2}]^4(1 + 5\beta) + \mathrm{Cos}[\frac{t\omega}{2}]^3\Big(5\beta\mathrm{Cos}[\frac{3t\omega}{2}] + \beta\mathrm{Cos}[\frac{5t\omega}{2}] -$
$2\alpha\omega\mathrm{Sin}[\frac{t\omega}{2}]\Big) + \mathrm{Cos}[\frac{t\omega}{2}]^2 S_t(1 + 10\beta) + 5\beta S_t\mathrm{Cos}[\frac{t\omega}{2}]\left(\mathrm{Cos}[\frac{3t\omega}{2}] + \mathrm{Cos}[\frac{5t\omega}{2}] - \frac{1}{2\beta}\alpha\omega\mathrm{Sin}[\frac{t\omega}{2}]\right) + \frac{1}{2}\beta S_t^2\Big(\frac{1}{2\beta} + \frac{9}{4} +$
$6\mathrm{Cos}[t\omega] + \frac{3}{2}\mathrm{Cos}[2t\omega] + 2S_t\mathrm{Cos}[\frac{t\omega}{2}]^2 + \frac{1}{8}S_n^2\Big)\Big]x^2 - S_t\Big[\frac{2}{3}\mathrm{Cos}\left[\frac{t\omega}{2}\right]^2\left(1 + 10\beta\mathrm{Cos}\left[\frac{t\omega}{2}\right]^2\right) +$
$\frac{2}{3}\beta\mathrm{Cos}\left[\frac{t\omega}{2}\right]\left(5\mathrm{Cos}\left[\frac{3t\omega}{2}\right] + \mathrm{Cos}\left[\frac{5t\omega}{2}\right] - \frac{1}{2}\alpha\omega\mathrm{Sin}\left[\frac{t\omega}{2}\right]\right) + S_t\Big(\frac{1}{6} - 3\beta - 4\beta\mathrm{Cos}[t\omega] - \beta\mathrm{Cos}[2t\omega] -$
$2\beta\mathrm{Cos}\left[\frac{t\omega}{2}\right]^2 S_t - \frac{1}{6}\beta S_t^2\Big)\Big]x^3 + \beta S_t^2\Big[\frac{1}{12\beta} + \frac{3}{2} + 2\mathrm{Cos}[t\omega] + \frac{1}{2}\mathrm{Cos}[2t\omega] + 2\mathrm{Cos}[\frac{t\omega}{2}]^2 + \frac{1}{4}S_n^2\Big]x^4 -$
$\frac{\beta S_t^3}{5}\left[4\mathrm{Cos}\left[\frac{t\omega}{2}\right]^2 - S_t\right]x^5 + \left[\frac{1}{15}\beta S_t^4\right]x^6.$ (39)

The solution of the second component of velocity distribution is too large. So derivation is given up to first order while, graphical solutions are given up to second order.
Using the residual optimal value of c_j,for velocity distribution $v_0(x,t), v_1(x,t,c_1)$are
$c_1 = -0.34866293270937915, c_2 = -0.27149642971648824.$
The value of $c_j j = 1,2,3,4$ for temperature distribution is
$c_1 = 0.09746278469603249, c_2 = 0.010772390418597814, c_3 = -2.421544014021652, c_4 =$
$-2.010885173749646.$

4 Statement of drainage problem

In this section we consider the belt is only oscillating and not moving upward show in fig. 2. The fluid layer is draining down the belt due to gravity. The remaining assumptions are same as in the previous problem for heat and velocity, but the stock number into the equation (28) positively taken.
Boundary condition for the electrically conducting drainage problem is

$$v(0,t) = V\cos(\omega t) \quad , \quad \frac{\partial v(\delta,t)}{\partial x} = 0 \quad , \quad (40)$$

Due to downward flow of the fluid film equation (28) reduced as.

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \alpha\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) + 6\beta\left(\frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2}\right) + S_t - Mv, \quad (41)$$

Boundary conditions are

$$v(0,t) = \mathrm{Cos}(\omega t)\ , \quad \frac{\partial v(1,t)}{\partial x} = 0 \quad at \ \ x = 0, \quad (42)$$

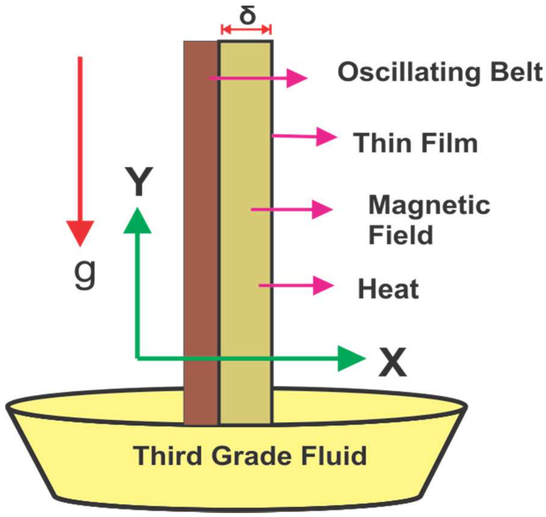


Figure 2: Geometry of drainage problem

4.1OHAM solution of drainage problem

In this section we apply an OHAM technique on Eq. (41) and obtained zero, first and second component velocity and temperature problems

Zero component problem:

$$q^0: \quad \frac{\partial^2 v_0(x,t)}{\partial x^2} = -S_t \tag{42}$$

$$q^0: \quad \frac{\partial^2 \Theta_0(x,t)}{\partial x^2} = 0, \tag{43}$$

$$q^1: \quad \frac{\partial^2 v_1(x,t)}{\partial x^2} = S_t + c_1\left(S_t - \mathrm{M}v_0 - \left(\frac{\partial v_0}{\partial \mathrm{t}}\right) + \left(\frac{\partial^2 v_0}{\partial x^2}\right) + \alpha \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_0}{\partial x^2}\right)\right) + \frac{\partial^2 v_0}{\partial x^2}\left(1 + 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\right), \tag{44}$$

$$q^1: \quad \frac{\partial^2 \Theta_1(x,t)}{\partial x^2} = -P_r c_3 \frac{\partial \Theta_0}{\partial \mathrm{t}} + P_r E_c c_3\left(\left(\frac{\partial v_0}{\partial x}\right)^2 + 2\beta\left(\frac{\partial v_0}{\partial x}\right)^4 + \alpha\left(\frac{\partial v_0}{\partial x}\right)\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial v_0}{\partial x}\right) + \beta_1 \frac{\partial^2}{\partial \mathrm{t}^2}\left(\frac{\partial v_0}{\partial x}\right)\right) + \frac{\partial^2 \Theta_0}{\partial x^2}(1 + c_3), \tag{45}$$

$$q^2: \frac{\partial^2 v_2(x,t)}{\partial x^2} = c_2\left(S_t - M v_0 - \frac{\partial v_0}{\partial \mathrm{t}} + \alpha \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_0}{\partial x^2}\right) + \frac{\partial^2 v_0}{\partial x^2}\left(1 + 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\right)\right) + c_1\left(12\beta\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_1}{\partial x}\right)\left(\frac{\partial^2 v_0}{\partial x^2}\right) - M v_1 - \frac{\partial v_1}{\partial \mathrm{t}} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial x^2}\left(1 + 6\beta\left(\frac{\partial v_0}{\partial x}\right)^2\right) + \alpha \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial^2 v_1}{\partial x^2}\right)\right). \tag{46}$$

Solution of zero, first and second component problems using boundary condition in Eq. (42) is

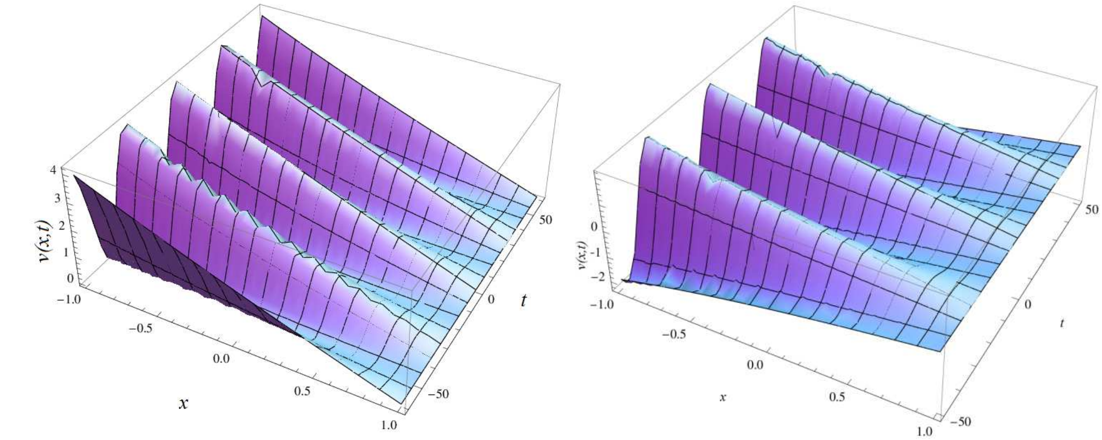
$$v_0(x,t) = \mathrm{Cos}[t\omega] + \left[\frac{S_t}{2} - \mathrm{Cos}[t\omega]\right]x - \left[\frac{S_t}{2}\right]x^2, \tag{47}$$

$$\Theta_0(x,t) = x, \tag{48}$$

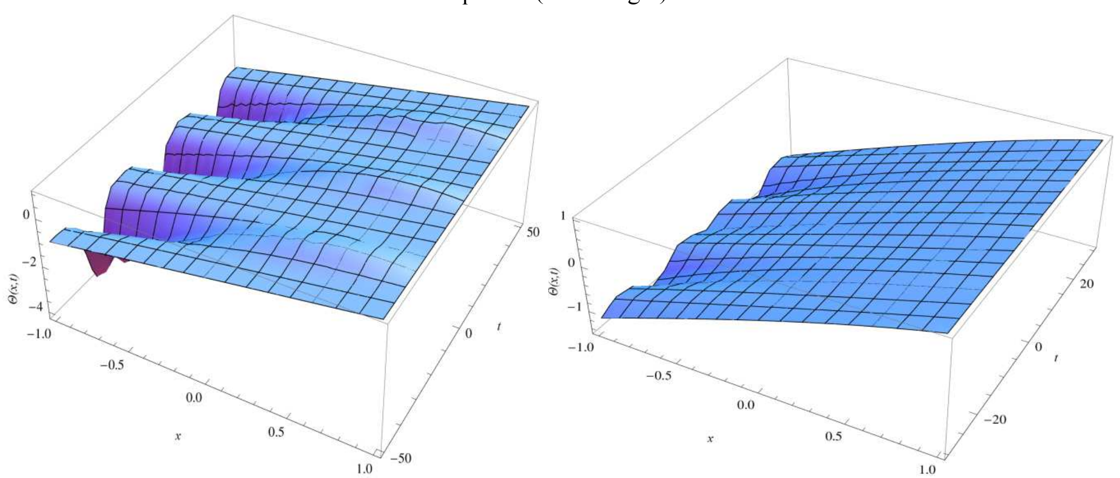
$$v_1(x,t) = c_1\Bigg[\left[\frac{1}{3}M\mathrm{Cos}[t\omega] - \frac{1}{3}\omega\mathrm{Sin}[t\omega] + \frac{1}{24}MS_t - \frac{3}{4}\beta\omega^2 S_t + \frac{3}{4}\beta\omega^2\mathrm{Cos}[2t\omega]S_t\right]x + \left[-\frac{1}{2}M\mathrm{Cos}[t\omega] + \frac{1}{2}\omega\mathrm{Sin}[t\omega] + \frac{3}{2}\beta\omega^2 S_t - \frac{3}{2}\beta\omega^2\mathrm{Cos}[2t\omega]S_t\right]x^2 + \left[\frac{1}{6}M\mathrm{Cos}[t\omega] - \frac{1}{6}\omega\mathrm{Sin}[t\omega] - \frac{1}{12}MS_t - \beta\omega^2 S_t + \beta\omega^2\mathrm{Cos}[2t\omega]S_t\right]x^3 + \frac{S_t}{4}\left[\frac{M}{6} + \beta\omega^2 - \beta\omega^2\mathrm{Cos}[2t\omega]\right]x^4\Bigg], \tag{49}$$

$$\Theta_1(x,t) = P_r E_c c_3\Bigg[\frac{1}{6}\beta\mathrm{Cos}[3t\omega]S_t - \frac{1}{2}\mathrm{Cos}[t\omega]^2(1+3\beta) - \mathrm{Cos}[t\omega]\left(\frac{1}{4}\beta\mathrm{Cos}[3t\omega] + \frac{1}{2}\alpha\omega\mathrm{Sin}[t\omega] + \frac{1}{6}S_t + \frac{1}{2}\beta S_t + \frac{1}{10}\beta S_t^3\right) - \frac{1}{12}\alpha\omega\mathrm{Sin}[t\omega]S_t - \frac{\beta S_t^2}{4}\left(\frac{1}{6\beta} - 1 - \mathrm{Cos}[2t\omega] - \frac{1}{20}S_t^2\right)\Bigg]x + \Bigg[\frac{1}{2}\mathrm{Cos}[t\omega]^2(1+3\beta) + \frac{1}{2}\mathrm{Cos}[t\omega]\left(\frac{1}{2}\beta\mathrm{Cos}[3t\omega] - \alpha\omega\mathrm{Sin}[t\omega] - S_t - 3\beta S_t - \beta S_t^3\right) - \frac{1}{2}\beta\mathrm{Cos}[3t\omega]S_t + \frac{1}{4}\alpha\omega\mathrm{Sin}[t\omega]S_t + \frac{\beta}{4}S_t^2\left(\frac{1}{2\beta} + 3 + 3\mathrm{Cos}[2t\omega] + \frac{1}{4}S_t^2\right)\Bigg]x^2 + \Bigg[\mathrm{Cos}[t\omega]S_t\left(\frac{1}{3} + \beta + \beta S_t^2\right) + \frac{1}{3}\beta\mathrm{Cos}[3t\omega]S_t - \frac{1}{6}\alpha\omega\mathrm{Sin}[t\omega]S_t - \frac{1}{6}S_t^2(1 + 6\beta + 6\beta\mathrm{Cos}[2t\omega] - \beta S_t^2)\Bigg]x^3 + \frac{\beta S_t^2}{2}\left[\frac{1}{6\beta} + 1 + \mathrm{Cos}[2t\omega] - 2\mathrm{Cos}[t\omega]S_t + \frac{1}{2}S_t^2\right]x^4 + \frac{\beta S_t^3}{5}[2\mathrm{Cos}[t\omega] - S_t]x^5 + \left[\frac{1}{15}\beta S_t^4\right]x^6. \tag{50}$$

The solution of the second component of velocity distribution is too large. So derivation is given up to first order while, graphical solutions are given up to second order
The values of c_j,for drainage velocity distribution are
$c_1 = -0.000237, c_2 = -0.452582388.$
The values of $c_{j,}\ j = 1,2,3,4$ for drainage temperature distribution are
$c_1 = 0.0003093, c_2 = 0.00342825, c_3 = -0.87601824, c_4 = -0.0270931$.



Figures 3, 4: In 3D graphs, influence of time level in lift velocity profile (on the left) and drainage velocity profile (on the right).



Figures 5, 6: In 3D graphs, effect of time level on lift temperature profile (on the left) and drainage temperature profile (on the right).

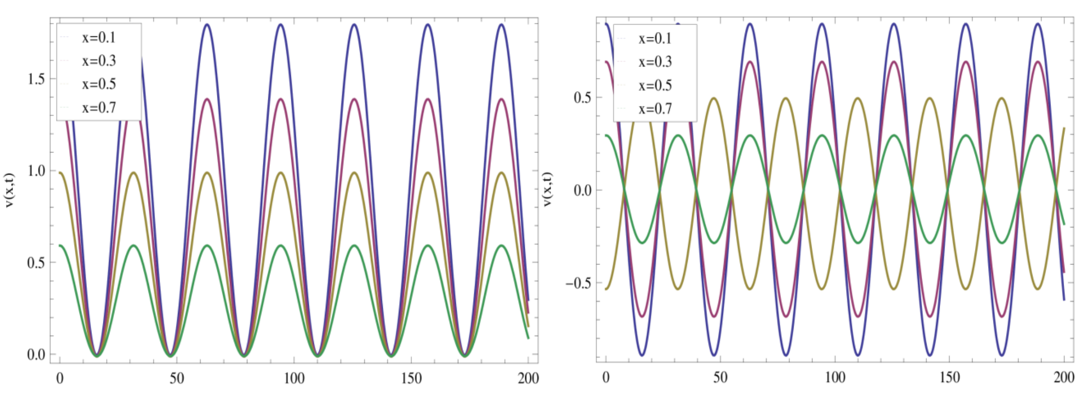


Figure 7, 8: Lift velocity distribution of fluid (on left) and drainage velocity distribution of fluid (on right) by taking $\omega = 0.2, \alpha = 0.2, S_t = 0.1, M = 0.5, \beta = 0.2, t = 0.7.$

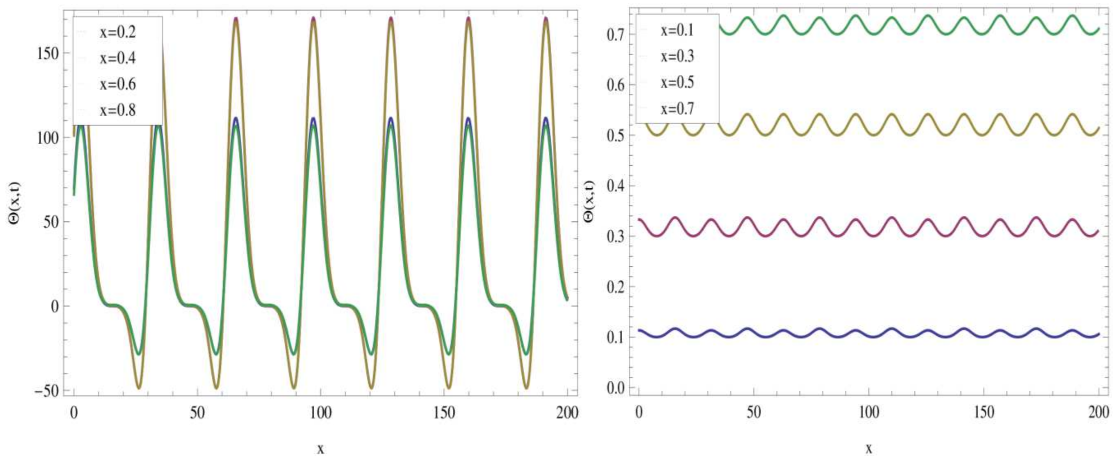


Figure 9, 10: Lift temperature distribution of fluid (on left) and drainage temperature distribution of fluid (on right) by taking $\omega = 0.2, \alpha = 0.2, S_t = 0.3, M = 0.5, P_r = 10, E_c = 5, \beta = 0.4, \beta_1 = 0.5, t = 0.6$.

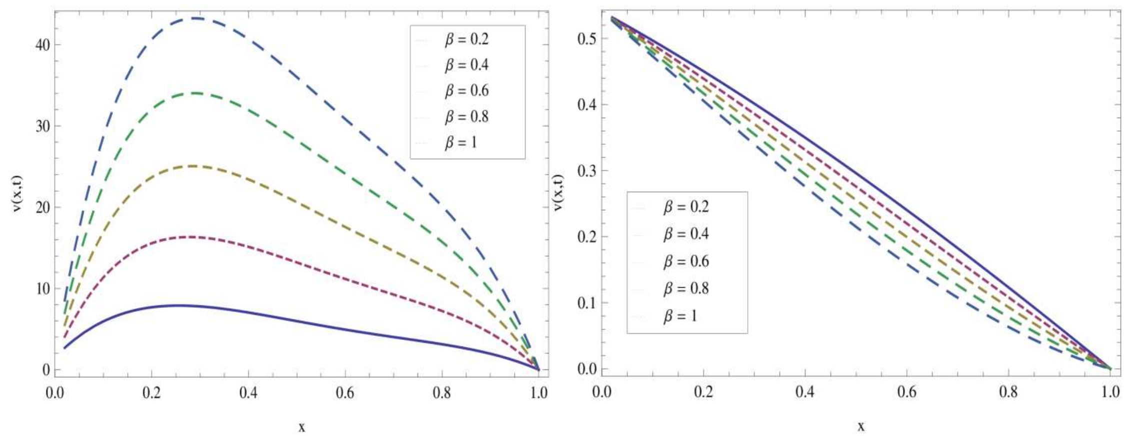


Fig 11, 12: Effect of the Non-Newtonian parameter βon theLift velocity (on left)and drainage velocity profile (on right) by taking $\omega = 0.2, \alpha = 0.2, S_t = 0.1, M = 0.5, t = 0.7$.

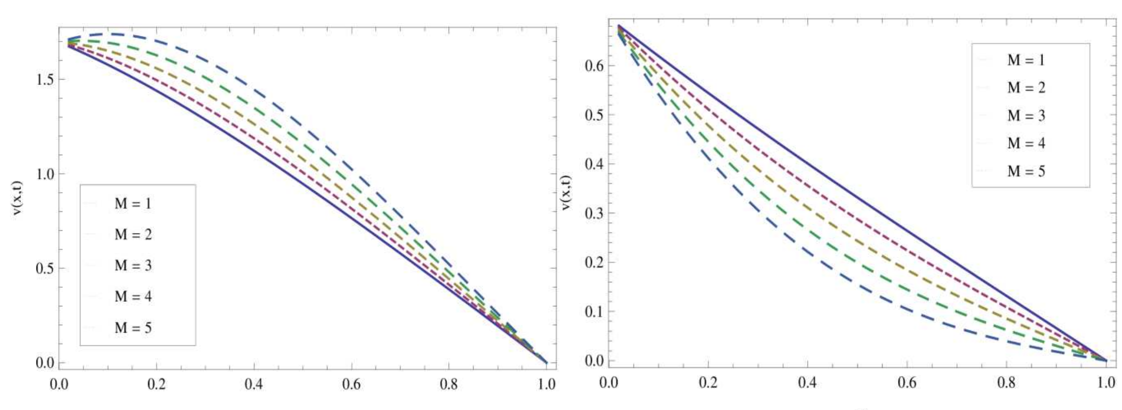


Fig 13, 14: Effect of the Magnetic parameter M on the Lift velocity profile (on left)and drainage velocity profile (on right) by taking $\omega = 0.2, \alpha = 0.2, S_t = 0.1, \beta = 0.2, t = 0.5$.

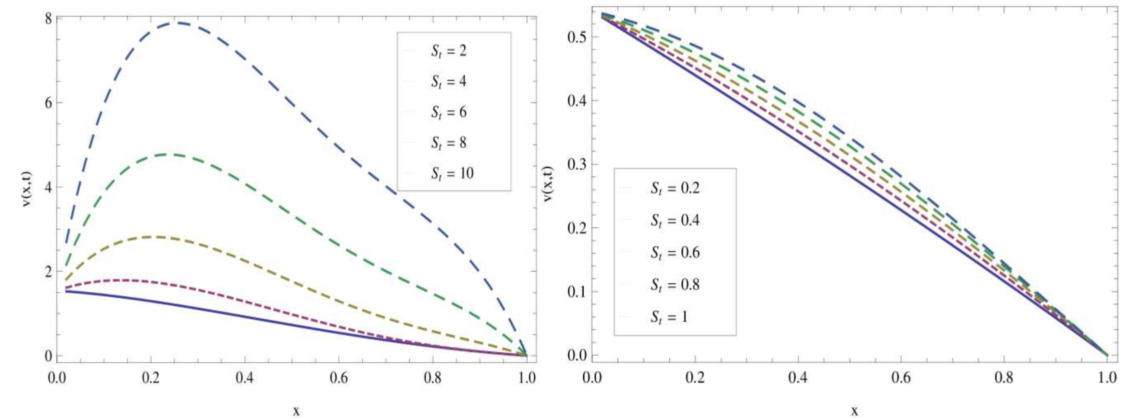


Fig 15, 16: Effect of the stock number on the Lift velocity (on left) and drainage velocity profile (on right) by taking $\omega = 0.2, \alpha = 0.2, M = 0.5, \beta = 0.2, t = 0.7$.

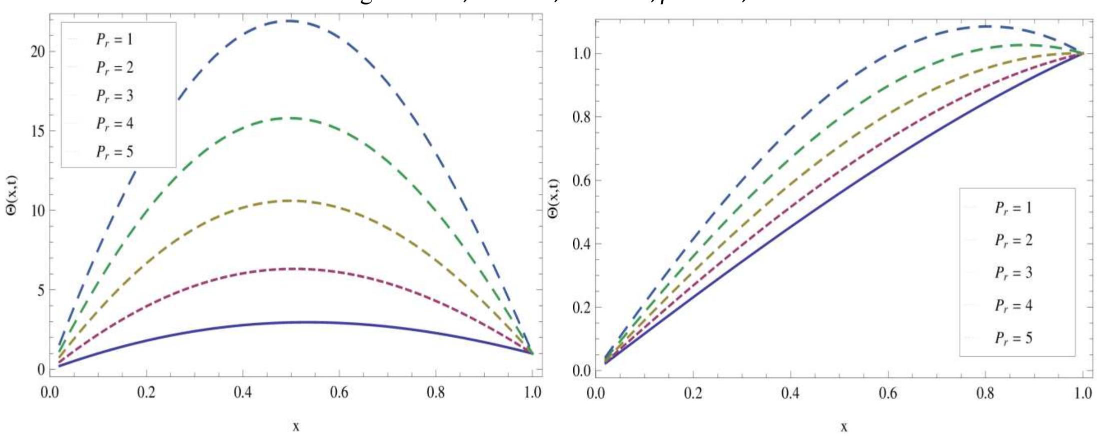


Fig 17, 18: Effect of the P_rPrandtl number on the Lift temperature (on left)and drainage temperature profile (on right) by taking$\omega = 0.2, \alpha = 0.2, S_t = 0.3, M = 0.5, E_c = 5, \beta = 0.4, \beta_1 = 0.5, t = 0.6$.

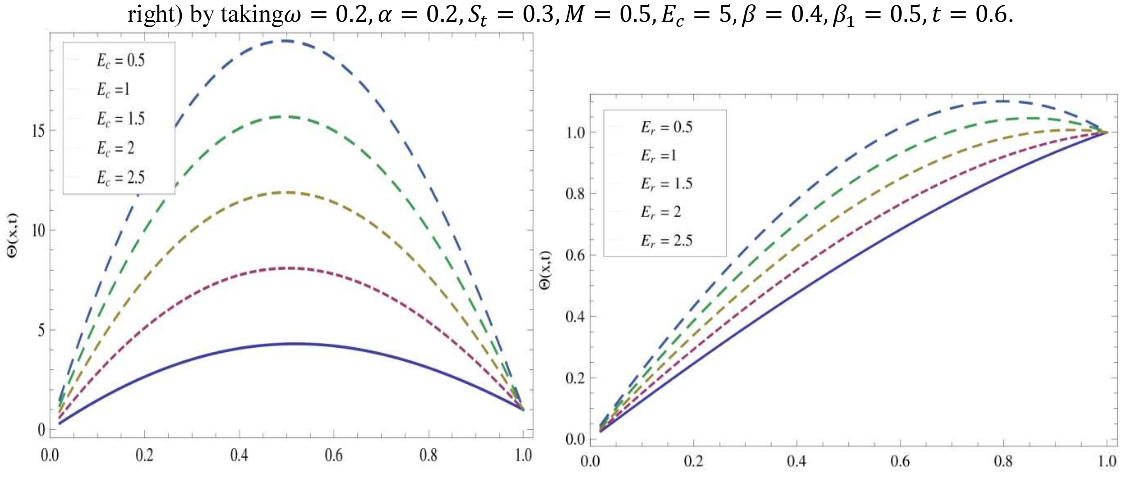


Fig 19, 20: Effect of the E_cEckert number on the Lift temperature (on left)and drainage temperature profile (on right) by taking $\omega = 0.2, \alpha = 0.2, S_t = 0.3, M = 0.5, P_r = 10, \beta = 0.4, \beta_1 = 0.5, t = 0.3$.

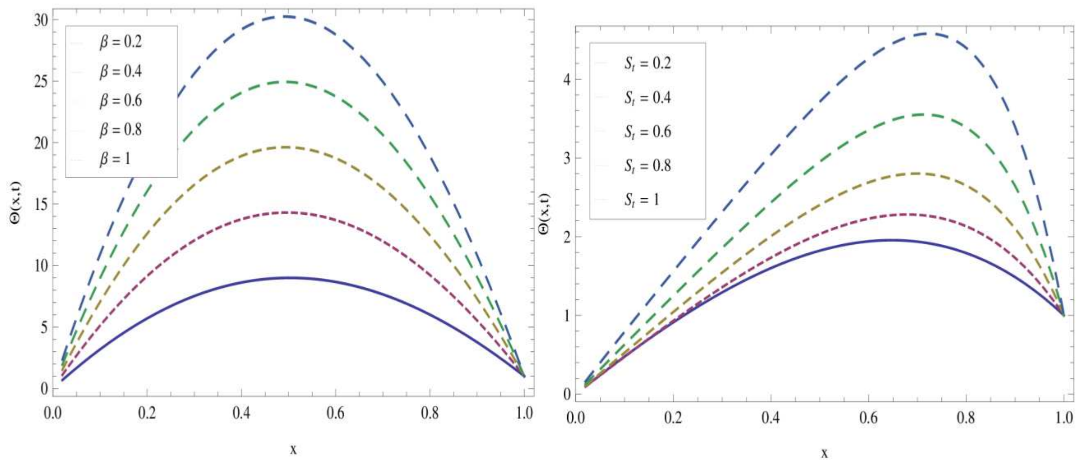


Fig 21, 22: Effect of the Non-Newtonian parameter βon theLift temperature (on left)and stock number on drainage temperature profile (on right) by taking $\omega = 0.2, \alpha = 0.2, M = 0.5, P_r = 10, E_c = 5, \beta_1 = 0.5, t = 0.6$.

5 RESULTS AND DISCUSSION

In the present study, we employed OHAM to obtain the heat and velocity fields for unsteady flow of third grade fluid on a vertical oscillating and moving belt under the transverse magnetic field. The result of velocity and heat are presented graphically because $v(x,t)$and $\Theta(x,t)$, were too long to be mentioned here. The schematic diagrams of lift and drainage problems are shown in Figs. 1, 2. The 3D influences of different time level of velocity and temperature have been presented in Figs. 3-6. Similarly Figs. 7- 10 are plotted to observe the effect of different time level on the velocity and temperature field. The fluid oscillates together with the belt in the same domain with the same time period and the amplitude of fluid motion increases gradually towards the free surface, because the friction force reduces away from the belt. Figs. 11- 16 have been plotted to understand the variation of different model variable on velocity. The effect of these parameters is similar as mentioned by Nasir and Gul et al. [20]. Figures 17- 20 show the effect of prandtl number and Eckert number. Increasing these parameters the fluid motion increases gradually towards the free surface. The reason is that the cohesive forces reduces with the increase of these parameters. Similarly, the effect of non-Newtonian parameter and stock number are shown in Figures 21 and 22. The increase in these parameters increases fluid motion gradually towards the free surface and this increase become more rapid with the increase of these parameters. The reason is that the density of fluid increases with the increase of these parameters.

6 CONCLUSION

The unsteady thin film flow of an MHD third grade fluid on a vertical oscillating belt has been discussed. The constitutive equation governing the flow of a third grade fluid for lifting and drainage problems has been solved analytically by using the Optimal Homotopy Asymptotic Method (OHAM). The effect of stock number S_t, non- Newtonian parameter β, magnetic parameter M and other parameters involved in the problem are discussed and result are displayed in graph to observe the effect of these parameters on lifting and drainage velocity profile. It is concluded that fluid motion increases with the increasing prandtl and Eckert numbers, because the cohesive forces become weaker with the increase of these parameters.

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