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Ideal μ- Weak Structure Space with Some Applications

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ABSTRACT

A. Császár , [13] introduced generalized structures on a nonempty set X called a generalized topology. Also, A. Császár , [2] introduced and studied more generalized topology and generalized continuity between generalized spaces. The purpose of this paper is to define $i^*_{\tau_\mu}$ and $c^*_{\tau_\mu}$ under more general conditions and to show that the important properties of these operations remain valid under these conditions, Also we define and study weaker form of τ_μ-open sets , τ_μ- continuity and an ideal $*$- μ- weak structure space and μ -weak local function with some applications on a set X are defined and their properties are discussed.

KEYWORDS: ideal $*$- μ- weak structure space, μ -weak local function,

1. INTRODUCTION

A. Császár, [3] has introduced a new notion of structures called weak structure. Every generalized topology is a weak structure. In [3], A. Császár, defined some structures and operators under more general conditions. Let X be a nonempty set and $\tau \in P(X)$where $P(X)$ is the power set of X. Then τ is called a weak structure [3] on X if $\phi \in \tau$. A nonempty set X with a weak structure τ is denoted by the pair (X,τ) and is called simply a space (X,τ). The elements of τ are called τ-open sets and the complements of τ-open sets are called τ-closed sets [14]. For a weak structure τ on X, the intersection of all τ-closed sets containing a subset λ of X is denoted by $c_\tau(\lambda)$ and the union of all τ-open sets contained in λ is denoted λ by $i_\tau(\lambda)$. A subfamily $\tau \subset P(X)$ is called a generalized topology [2] if $\phi \in \tau$ and τ is closed under arbitrary union. A generalized topology on X is said to be a quasi-topology [14] if τ is closed under finite intersection. The concept of ideals in topological spaces is treated in the classic text by Kuratowski [5] and Vaidyanathaswamy [11]. Jankovic and Ha- Mlett [4] investigated further properties of ideal spaces. An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X which Satisfies the following properties: (1) $A \in I$ and $B \subseteq A$ implies $B \in I$; (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space (or an ideal space) is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I). For a subset $A \subseteq X$, $A^*(I,\tau) = \{x \in X : A \cap U \notin I$ for every $U \in \tau(X,x)\}$ is called the local function of A with respect to I and τ (see [5]). We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $Cl^*(.)$ or a topology $\tau^*(I,\tau)$ called the $*$- topology , finer than τ , is defined by $Cl^*(A) = A \cup A^*$ (see [11]). Also, throughout the paper The family $\tau_\mu = \{\mu \cap \lambda : \lambda \in \tau\}$ is the τ structure induced over $\mu \subset X$ by τ is called τ_μ- weak structure [15] (by short $\tau_\mu S$) . The elements of τ_μ are called τ_μ-open set ; a set υ is a τ_μ-closed set if $\mu - \upsilon \in \tau_\mu$. We note τ^c_μ the family of all τ_μ-closed set. Let $\upsilon \in \tau_\mu$ structure we define the τ_μ-interior (by short i_{τ_μ} of τ as the finest τ_μ- pen sets contained in υ that is, $i_{\tau_\mu}(\upsilon) = \bigcup_{\xi\in\tau_\mu}\{\xi : \xi \subseteq \upsilon\}$. Let $\upsilon \in \tau_\mu$ structure we define the τ_μ-closure (by short c_{τ_μ}) of τ as the smallest τ_μ- closed sets which contained in υ that is , $c_{\tau_\mu}(\upsilon) = \bigcap_{\xi\in\tau_\mu}\{\mu - \xi : \upsilon \subseteq \mu - \xi\}$. A subset λ of X is said to be τ_μ-sso(λ) (resp . τ_μ-so(λ) , τ_μ-$po(\lambda)$, τ_μ-spo(λ)) if $\lambda \subset i_{\tau_\mu}\left(c_{\tau_\mu}\left(i_{\tau_\mu}(\lambda)\right)\right)$ (resp . $\lambda \subset c_{\tau_\mu}\left(i_{\tau_\mu}(\lambda)\right)$, $\lambda \subset i_{\tau_\mu}\left(c_{\tau_\mu}(\lambda)\right)$, $\lambda \subset c_{\tau_\mu}(i_{\tau_\mu}(c_{\tau_\mu}(\lambda)))$. We will denote the family of all τ_μ- sso (resp. τ_μ-so , τ_μ-po , τ_μ-spo) sets in a space (X,τ) , $\mu \subseteq X$ by τ_μ- sso(X) (resp . τ_μ-so(X) , τ_μ- po(X) , τ_μ- spo(X)) . The complement of the elements of τ_μ- sso(X) (resp . τ_μ- so(X) , τ_μ- po(X) , τ_μ-spo(X)) ae called τ_μ- ssc(X) (resp . τ_μ- sc(X) , τ_μ-pc(X) , τ_μ-spc(X) sets . Let τ_μ be a $\tau_\mu S$, Let A and B be subsets of $\tau_\mu S$. If either A or B is τ_μ- so(X), then $i_{\tau_\mu}(c_{\tau_\mu}(A \cap B)) = i_{\tau_\mu}(c_{\tau_\mu}(A)) \cap i_{\tau_\mu}(c_{\tau_\mu}(B))$.
For more details about weaker forms of τ_μ-open set see [15].

Definition 1.1. let (X,τ) be a , *weak structure of* X, $\lambda \subseteq X$ is called

(1) η_{τ_μ}-set if $\lambda = U \cup B$ where $U \in \tau_\mu$ and B is $\tau_\mu ssc$- set

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(2) β_{τ_μ}-set if $\lambda = U \cup B$ where $U \in \tau_\mu$ and B is $\tau_\mu sc$- set

We denote the family of all η_{τ_μ}-set (resp. β_{τ_μ}-set) by $\eta_{\tau_\mu}(X)$ (resp. $\beta_{\tau_\mu}(X)$)

Remark 1.1. (1) Since X is τ_μ-closed, as well as τ_μ-semiclosed. and so every τ_μ-open set is η_{τ_μ}-sets, as well as β_{τ_μ}-set

(2) If $X \in \tau_\mu$ every $\tau_\mu ssc$-set is an η_{τ_μ}-set. And every $\tau_\mu sc$-set, is β_{τ_μ}-set

(3) $\eta_{\tau_\mu}(X) \subset \beta_{\tau_\mu}(X)$

Example 1.1. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}\}$. and $\mu = \{a, b, c\}$, then $\tau_\mu = \{\phi, \{a\}, \{b\}, \{c\}\}$. If $\lambda = \{a\}$, then $i_{\tau_\mu}(\lambda) = \{a\}$, then $i_{\tau_\mu}\, c_{\tau_\mu}(\lambda) = \{a\} \subset \lambda$, and so λ is τ_μ-sc. Since $\lambda = \lambda \cap \{a\}, \{a\} \in \tau_\mu$. then $\lambda \in \eta_{\tau_\mu}(X)$.

Theorem 1.1. Let (X, τ) be a , $weak\ structure\ of\ X$ and A, μ ∈ X. , then the following stamens are equivalent:

(1) $\lambda\ is\ an\ \eta_{\tau_\mu}\text{-}set$

(2) $\lambda = U \cap c_{\tau_\mu}(\tau_\mu - sso(\lambda)\ for\ some\ \tau_\mu\text{-}open\ set\ U\ and\ \tau_\mu sso(X)\ is\ generalized$ topology on X

Proof

(1) ⇒ (2). Suppose λ is an η_μ-set. Then $\lambda = U \cap F$ where $U \in \tau_\mu$ and F is $\tau_\mu ssc$-set. Now $\lambda \subseteq F$ implies that $c_{\tau_\mu}(\tau_\mu - sso(\lambda)) \subseteq F$, since $\tau_\mu - sso\ is\ a$ generalized space by Lemma 1.1 and so $\lambda \subseteq U \cap c_{\tau_\mu}(\tau_\mu - sso(\lambda)) \subseteq U \cap F = \lambda$ which implies that
$\lambda = U \cap c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right)$.

(2) ⇒ (3). By Lemma 1.1, since $\tau_\mu - sso(\lambda)$ is a generalized space and so $c_{\tau_\mu}(\lambda)$ is $\tau_\mu - ssc$. Hence $\lambda \in \eta_{\tau_\mu}(X)$

Theorem 1.2. Let (X, τ) be a $weak\ structure\ of\ X$, $\lambda, \mu \subset X$, then the following stamens are equivalent:

(a) $\lambda\ is\ an\ \eta_{\tau_\mu}\text{-}set$

(b) $c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right)\ is\ \tau_\mu - ssc$

(c) $\lambda \cup \left(X - \left(\tau_\mu - sso(\lambda)\right)\right)\ is\ \tau_\mu - sso\text{-}set$

(d) $\lambda \subseteq i_{\tau_\mu}(\lambda \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda)))$

$Then\ \ (a) \Rightarrow (b) \Rightarrow (c) \Rightarrow (d)$

Proof

(a) ⇒ (b) . If λ is an $\eta_{\tau_\mu}\text{-}set$, by Theorem 1.1 , $\lambda = U \cap c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right)$, for some $\tau_{\mu-}$open set U. Now $c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - \lambda = c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - (U \cap\ c_{\tau_\mu}(\tau_\mu - sso(\lambda))) = c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) \cap ((X - U) \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda))) =$
$c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) \cap ((X - U)$ and so $c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - \lambda$ is $\tau_\mu - ssc$

(b) ⇒ (c) $c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - \lambda$ is $\tau_\mu - ssc$, implies that $X - (c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - \lambda)$ is $\tau_\mu - sso$, and so $\lambda \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda)))$ is $\tau_\mu - sso$.

(c) ⇒ (d). Since $\lambda \subseteq \lambda \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda)))$, by (c), $\lambda \subseteq i_{\tau_\mu}(\lambda \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda))))$.

Example 1.2 Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}\}$, $\lambda = \mu = \{a, b, c\}$, and $\tau_\mu = \{\phi, \{a\}, \{b\}, \{c\}\}$. If $\lambda = \{a, b, c\}$, then $c_{\tau_\mu}\left(\tau_\mu - sso(\lambda)\right) - \lambda = \{d\}$ is $\tau_\mu - ssc(\lambda)$, and $\lambda \cup (X - c_{\tau_\mu}(\tau_\mu - sso(\lambda))) = \{a, b, c\} \cup \{X - X\} = \{a, b, c\} \cup \emptyset = \{a, b, c\} = \lambda$ is $\tau_\mu - sso\text{-}\ set$ but λ is not an η_{τ_μ}-set

Definition 1.2 Let (X, τ) be a $weak\ structure\ of\ X$, $\lambda, \mu \subseteq X$. Then λ is said :-
τ_μ-locally closed if $\lambda = G \cap F$ where G is τ_μ-open and F is τ_μ-closed

Remark 1.2 Since X is τ_μ-closed, every τ_μ-open set is τ_μ-locally closed .The following theorem is gives some properties of τ_μ-locally closed sets.

Theorem 1.3 Let (X, τ) be a $weak\ structure$ space and $\lambda, \mu \subset X$. Then the following hold :
(a) *If λ is τ_μ-locally closed, then $\lambda = U \cap c_{\tau_\mu}(\lambda)$ for some τ_μ-open set U*
(b) *If τ is a generalized topology and $\lambda = U \cap c_{\tau_\mu}(\lambda)$ for some τ_μ-open set U, then λ is τ_μ-locally closed.*

proof
(a) $\Rightarrow$ (b). Suppose λ is a τ_μ-locally closed set. Then $\lambda = U \cap F$ where $U \in \tau_\mu$ and F is τ_μ-closed. Now $\lambda \subseteq F$ implies that $c_{\tau_\mu}(\lambda) \subset F$, by Lemma and so $\lambda \subset U \cap c_{\tau_\mu}(\lambda) \subset U \cap F = \lambda$ which implies that $\lambda = U \cap c_{\tau_\mu}(\lambda)$.
(b) $\Rightarrow$ (a). If τ is a generalized topology, then $c_{\tau_\mu}(\lambda)$ is τ_μ-closed and so λ is τ_μ-locally closed.
Corollary 1.1 ([12], *Theorem* 2.8) . Let (X, τ) be a generalized *space and* $\lambda, \mu \subset X$.
Then the following are equivalent.
(a) λ is τ_μ-locally closed.
(b) $\lambda = U \cap c_{\tau_\mu}(\lambda)$ *for some* τ_μ-open *set* U.

Theorem 1.4. *Let* (X,τ) *be a WS space and* $\lambda, \mu \subseteq X$. *If λ is τ_μ-open, then λ is an $\tau_\mu - sso$-set and an* η_{τ_μ}-set
Proof . If λ is τ_μ-open, then clearly, λ is $\tau_\mu - sso$-set and an η_{τ_μ}-set.
Theorem 1.5. Let (X,τ) be a generalized space and $\lambda, \mu \subset X$. Consider the following statements.
(a) λ is τ_μ-open.
(b) λ is $\tau_\mu - $ sso-set and an η_{τ_μ}-set.
(c) λ is τ_μ-open and τ_μ-locally closed.
(d) λ is τ_μ-preopen and τ_μ-locally closed.
(e) λ is τ_μ-preopen and an η_{τ_μ}-set.
(f) λ is τ_μ-preopen and a $\tau_\mu -$ spo-set.

Then $(a) \Rightarrow (b) \Rightarrow (c) \Rightarrow (d) \Rightarrow (e) \Rightarrow (f)$. *If τ_μ is a quasi-topology, then* $(f) \Rightarrow (a)$.
proof
(a) $\Rightarrow (b)$. The proof follows from Theorem 1.3
(b) $\Rightarrow$ (c). Since λ is an η_{τ_μ} τ-set, $\lambda = U \cap\ c_{\tau_\mu}((\tau_\mu - sso(X)(\lambda))$ for some τ_μ-open set U. Since λ is $\tau_\mu - spo$ set, $c_{\tau_\mu}(c_{\tau_\mu}\left(\tau_\mu - sso(X)(\lambda)\right) = c_{\tau_\mu}(\lambda \cup\ c_{\tau_\mu}\left(i_{\tau_\mu}\left(c_{\tau_\mu}(\lambda)\right)\right)$ by Lemma 2.2 of [3] and so $= c_{\tau_\mu}\left(c_{\tau_\mu}\left(i_{\tau_\mu} c_{\tau_\mu}(\lambda)\right)\right) = c_{\tau_\mu}\left(i_{\tau_\mu}\left(c_{\tau_\mu}\right)(\lambda)\right) = c_{\tau_\mu}\left(\tau_\mu - sso(X)\right)(\lambda)$ which implies that $c_{\tau_\mu}\left(\tau_\mu - sso(X)\right)(\lambda)$ is τ_μ-closed. Hence λ is τ_μ-locolly closed
The proofs of (c)⇒(d), (d)⇒(e) and (e)⇒(f) are clear.

(f) $\Rightarrow$ (a). Conversely, suppose λ is both a τ_μ-preopen set and a $\tau_\mu - sso$- set. Then $\lambda = U \cap F$ where U is τ_μ-open and $i_{\tau_\mu}\left(c_{\tau_\mu}(F)\right) = i_{\tau_\mu}(F)$. Since λ is τ_μ-preopen , $\lambda \subset i_{\tau_\mu}\left(c_{\tau_\mu}(\lambda)\right) = i_{\tau_\mu}(c_{\tau_\mu}(U \cap F)) \subset i_{\tau_\mu}\left(c_{\tau_\mu}(U)\right) \cap i_{\tau_\mu}\left(c_{\tau_\mu}(F)\right) =$
$i_{\tau_\mu}(c_{\tau_\mu}(U)) \cap\ i_{\tau_\mu}$. Hence $\lambda = \lambda \cap U \subset i_\mu c_{\tau_\mu}(U) \cap\ i_{\tau_\mu}(F) \cap U = U \cap i_{\tau_\mu}(F) \subset$
$U \cap F = \lambda$. Therefore , $\lambda = U \cap i_{\tau_\mu}(F)$. Since τ_μ is a quasi-topology , λ is τ_μ-open

Definition 1.3. Let (X,τ) and (Y,γ) be a weak structures, $\mu \subset X$, $\mu' \subset Y$. A function $f\colon X \to Y$ is said to be (τ_μ -continuous [2]) (resp., (η_{τ_μ} -continuous, β_{τ_μ}-Continuous, $\tau_\mu s$-continuous, $\tau_\mu ss$-continuous, $\tau_\mu p$-continuous, $\tau_\mu lc$-continuous) if $f^{-1}(V) \cap \mu \in \tau_\mu(\text{resp.}, f^{-1}(V) \cap \mu \in \eta_{\tau_\mu}, f^{-1}(V) \cap \mu \in \beta_{\tau_\mu}, \quad f^{-1}(V) \cap \mu \in \tau_\mu - so(X)$, $f^{-1}(V) \cap \mu \in \tau_\mu - sso(X),\ f^{-1}(V) \cap \mu \in \tau_\mu - po(X), f^{-1}(V) \cap \mu \in \tau_\mu - lc(X))$ for every $V \in \gamma_{f(\mu)}$
The following theorem gives decompositions of $(\tau_\mu, \gamma_{f(\mu)})$-continuous functions, the proof of which follows from Theorem 1.5

Theorem 1.6. *Let (X,τ) and (Y,γ) be a weak structure spaces and $\mu \subset X$, $\mu' \subset Y$ where (X,τ) is a quasi topological space, and let $f: X \to Y$ be a function. Then the following are equivalent*

(a) f *is* τ_μ*-continuous.*
(b) f *is* η_{τ_μ} *- continuous and* τ_μs *- continuous*
(c) f *is* $\tau_\mu lc$*- continuous and* $\tau_\mu ss$ *-continuous*
(d) f *is* $\tau_\mu lc$*- continuous and* $\tau_\mu p$*-continuous.*
(e) f *is* η_{τ_μ} *- continuous and* $\tau_\mu p$ *- continuous*
(f) f *is* β_{τ_μ} *- continuous and* $\tau_\mu p$ *- continuous*

Theorem 1.7. *Let (X,τ) and (Y,γ) be $\tau S's, \mu \subset X$*, and $f: X \to Y$. Then f is τ_μ*- continuous* if for each $\upsilon \in \gamma_{f(\mu)}$, we have $\mu \cap f^{-1}(\upsilon) \in \tau_\mu$

Definition 1.4 Let $f: (X,\tau) \to (Y,\gamma)$ be a mapping from a weak structure (X,τ) to another (Y,γ) , and $\mu \subset X$. Then f is called :

(i) an τ_μ –semicontinuous (briefly, $\tau_\mu sc$) mapping if for each $\upsilon \in \gamma_{f(\mu)}$, we have $\mu \cap f^{-1}(\upsilon) \in \tau_\mu - so;$

(ii) an τ_μ –precontinuous (briefly, $\tau_\mu pc$) mapping if for each $\upsilon \in \gamma_{f(\mu)}$, we have $\mu \cap f^{-1}(\upsilon) \in \tau_\mu - po;$

(iii) an τ_μ –strongly semi continuous (briefly , $\tau_\mu ssc$) mapping if for each $\upsilon \in \gamma_{f(\mu)}$, We have $\mu \cap f^{-1}(\upsilon) \in \tau_\mu - sso;$

(iv) an τ_μ – semi precontinuous (briefly, $\tau_\mu spc$) mapping if for each $\upsilon \in \gamma_{f(\mu)}$, we have $\mu \cap f^{-1}(\upsilon) \in \tau_\mu - spo;$

Remark 1.3. The implications between these different concepts are given by the following diagram:

$$\begin{tikzcd} & \tau - c \arrow[d] & \\ & \tau_\mu\text{- c} \arrow[d] & \\ & \tau_\mu\text{- ssc} \arrow[dl] \arrow[dr] & \\ \tau_\mu\text{- pc} \arrow[dr] & & \tau_\mu\text{- sc} \arrow[dl] \\ & \tau_\mu\gamma\text{- c} \arrow[d] & \\ & \tau_\mu\text{- spc} & \end{tikzcd}$$

Examples 1.3. Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b\}\}$, $\mu = \{b, c\}, \tau_\mu = \{\phi, \{b\}\}$, $Y = \{x, y\}, \gamma = \{\phi, \{x\}\}, f : X \to Y$, and $f(a) = f(b) = x$, $f(c) = y$, then $f(\mu) = \{x, y\}$, then $\gamma_{f(\mu)} = \{\emptyset, \{x\}\}$, since: $f^{-1}(\emptyset) = \emptyset \cap \mu = \emptyset \in \delta_\mu$, then $f^{-1}(x) = \{a, b\} \cap \mu = \{b\} \in \delta_\mu$, $then\ \tau$ is continuous . $and\ f^{-1}(a) = \emptyset$, $f^{-1}(x) = \{a, b\} \notin \tau$, then f is not τ-continuous mapping

2. IDEAL $*$- μ- WEAK STRUCTURE SPACE AND μ-WEAK LOCAL FUNCTION

For a subset $\mu \subseteq X$. Let τ_μ and I a μ- weak structure space and an ideal μ- weak structure space (denoted by $(\tau_\mu)_I$ on a set X

Let τ_μ is a $\mu -$ weak structure space and $\mathfrak{A}\tau_\mu(x) = \{U : x \in U, U \in \tau_\mu\}$ be the family of τ_μ-open sets which contain a point $x \in X$.

Definition 2.1. An ideal I on a μ- weak structure space (X, τ_μ) is a non-empty collection of subsets of X which Satisfies the following properties: (1) $A \in I$ and $B \subseteq A$ implies $B \in I$; (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$

Definition 2.2. Let $(X, (\tau_\mu)_I)$ be an ideal μ- weak structure space . For a subset $A, \mu \subseteq X$, $A^*_{\tau_\mu}(I, \tau_\mu) = \{x \in X : U \cap A \notin I\ for\ every\ U \in \mathfrak{A}\tau_\mu(x)\}$ is called the μ-weak local function of A with respect to I and τ_μ. We will simply write $A^*_{\tau_\mu}$ for $A^*_{\tau_\mu}(\tau_\mu, I)$

Theorem 2.1. Let $(\tau_\mu$ be a μ- weak structure on a set $X, I, \mathcal{J}$ ideals on X and A, B, μ be subsets of X. The following properties hold:

1. If $A \subseteq B$, then $A^*_{\tau_\mu} \subseteq B^*_{\tau_\mu}$
2. If $I \subseteq \mathcal{J}$, then $A^*_{\tau_\mu}(\mathcal{J}) \subseteq A^*_{\tau_\mu}(I)$
3. $A^*_{\tau_\mu} = c_{\tau_\mu}(A^*_{\tau_\mu}) \subseteq c_{\tau_\mu}(A)$
4. $A^*_{\tau_\mu} \cup B^*_{\tau_\mu} \subseteq (A \cup B)^*_{\tau_\mu}$
5. $\left(A^*_{\tau_\mu}\right)^*_{\tau_\mu} \subseteq A^*_{\tau_\mu}$
6. If $A \in I$, then $A^*_{\tau_\mu} = \emptyset$

Proof. (1) Let $A \subseteq B$. let $x \notin B^*_{\tau_\mu}$. implies that $U \cap B \in I$ for some $U \in \mathfrak{A}\tau_\mu(x)$. Since $U \cap A \subseteq U \cap B$ and $U \cap B \in I$. Then $U \cap A \in I$ from the definition of ideals. Thus, we have $x \notin A^*_{\tau_\mu}$. Hence we have $A^*_{\tau_\mu} \subseteq B^*_{\tau_\mu}$.

(2) Let $I \subseteq \mathcal{J}$ and $x \in A^*_{\tau_\mu}(\mathcal{J})$. Then $U \cap A \notin \mathcal{J}$ for every $U \in \mathfrak{A}\tau_\mu(x)$. By hypothesis, $U \cap A \notin I$. So$x \in A^*_{\tau_\mu}(I)$.

(3) Since $A^*_{\tau_\mu} \subseteq c_\tau(A^*_{\tau_\mu})$. Let $x \in c_{\tau_\mu}(A^*_{\tau_\mu})$. Then $A^*_{\tau_\mu} \cap U \neq \emptyset$ for every $U \in \mathfrak{A}\tau_\mu(x)$. Therefore, there exists some $y \in A^*_{\tau_\mu} \cap U$ and $U \in \mathfrak{A}\tau_\mu(y)$. Since $y \in A^*_{\tau_\mu}$, $A \cap U \notin I$ and hence $x \in A^*_{\tau_\mu}$. Hence $c_{\tau_\mu}(A^*_{\tau_\mu}) \subseteq A^*_{\tau_\mu}$ and $c_{\tau_\mu}(A^*_{\tau_\mu}) = A^*_{\tau_\mu}$. Again, let $x \in c_{\tau_\mu}(A^*_{\tau_\mu}) = A^*_{\tau_\mu}$, then $A \cap U \notin I$ for every $U \in \mathfrak{A}\tau_\mu(x)$. This implies $A \cap U \neq \emptyset$ for every $U \in \mathfrak{A}\tau_\mu(x)$. Therefore, $x \in c_{\tau_\mu}(A)$. This proves $A^*_{\tau_\mu} = c_{\tau_\mu}(A^*_{\tau_\mu}) \subseteq c_{\tau_\mu}(A)$.

(4) This follows from (1).

(5) Let $x \in \left(A^*_{\tau_\mu}\right)^*_{\tau_\mu}$. Then, for every $U \in \mathfrak{A}\tau_\mu(x)$, $U \cap A^*_{\tau_\mu} \notin I$ and hence $U \cap A^*_{\tau_\mu} \neq \emptyset$. Let $y \in U \cap A^*_{\tau_\mu}$. Then $U \in \mathfrak{A}\tau_\mu(y)$ and $y \in A^*_{\tau_\mu}$. Hence we have $U \cap A \notin I$ and $x \in A^*_{\tau_\mu}$. This shows that $\left(A^*_{\tau_\mu}\right)^*_{\tau_\mu} \subseteq A^*_{\tau_\mu}$.

(6) Suppose that $x \in A^*_{\tau_\mu}$. Then for any $U \in \mathfrak{A}\tau_\mu(x)$, $U \cap A \notin$ I. But, since $A \in I$, $U \cap A \in I$. This is a contradiction. Hence $A^*_{\tau_\mu} = \emptyset$

The converses of Theorem 2.1 need not be true as seen in the following examples

Example 2.1. Let $X = \{a, b, c, d\}$, $\mu = \{a, b, c\}$ and $\tau_\mu = \tau = \{\emptyset, \{a\}, \{b\}, \{a, b, c\}, \{b, c\}, \{a, c\}\}$ be a μ-weak structure on the set X with $I = \{\emptyset, \{a\}\}$. For $A = \{a, c\}$ and $B = \{b, c\}$, we have $A^*_{\tau_\mu} = \{c, d\} \subseteq B^*_{\tau_\mu} = \{b, c, d\}$ but $A \nsubseteq B$.

Example 2.2. Let $X = \{a,b,c,d\}$, $\mu = \{a,b,c\}$ and $\tau_\mu = \tau = \{\emptyset,\{a\},\{b\},\{a,b,c\},\{b,c\},\{a,c\}\}$ be a μ- weak structure on the set X with $I = \{\emptyset,\{b\}\}$ and $\mathcal{J} = \{\emptyset,\{a\}\}$. It is easily seen that $I \nsubseteq \mathcal{J}$ but for $A = \{a,c\}$ we have $A^*_{\tau_\mu}(\mathcal{J}) = \{c, d\} \subseteq A^*_{\tau_\mu}(I) = \{a,c,d\}$.

Example 2.3. Let $X = \{a,b,c\}$, $\mu = \{a,c\}$ and $\tau_\mu = \tau = \{\emptyset,\{a\},\{c\}\}$ be a μ- weak structure on the set X with $I = \{\emptyset,\{a\}\}$. For $A = \{a,c\}$, we have $c_{\tau_\mu}(A) = X \neq A^*_{\tau_\mu}(I) = \{b,c\} = c_{\tau_\mu}(A^*_{\tau_\mu}(I))$.

Example 2.4. Let $X = \{a,b,c\}$, $\mu = \{a,b,c\}$ and $\tau_\mu = \tau = \{\emptyset,\{a,b\},\{b,c\}\}$ be a μ- weak structure on the set X with $I = \{\emptyset,\{b\}\}$. For $A = \{a\}$ and $B = \{c\}$ we have $A^*_{\tau_\mu}(I) = \{a\}$, $B^*_{\tau_\mu}(I) = \{c\}$ and $(A \cup B)^*_{\tau_\mu}(I) = X$. Hence, we have $(A \cup B)^*_{\tau_\mu} \neq A^*_{\tau_\mu} \cup B^*_{\tau_\mu}$.

Definition 2.3. Let $(X,(\tau_\mu)_I)$ be an ideal μ- weak structure space . The set operator $c^*_{\tau_\mu}$ is called a μ- weak structure $*$-closure and is defined as follows: $c^*_{\tau_\mu}(A) = A \cup A^*_{\tau_\mu}$ for $A,\mu \subseteq X$. We will denote by $\tau^*{}_\mu(\tau_I)$ the μ- weak structure determined by $c^*_{\tau_\mu}$, that is, $\tau^*{}_\mu(\tau_I) = \{U \subseteq X : c^*_{\tau_\mu}(\mu - U) = \mu - U\} = \{U \subseteq X : i^*_{\tau_\mu}(U) = U\}$. $\tau^*{}_\mu(\tau_I)$ is called an $*$- μ- weak structure which is finer than τ_μ. The elements of $\tau^*{}_\mu(\tau_I)$ are said to be $\tau^*{}_\mu(I)$ -open and the complement of a $\tau^*{}_\mu(I)$–open set is said to be $\tau^*{}_\mu(I)$ $-$closed Throughout the paper we simply write $\tau^*{}_\mu(I)$ for $\tau^*{}_\mu(\tau_I)$. If I is an ideal on X, then $(X,\tau^*{}_I)$ is called an ideal $*$- μ- weak structure space.

Proposition 2.1. The set operator $c^*_{\tau_\mu}$ satisfies the following conditions:

1. $A \subseteq c^*_{\tau_\mu}(A)$
2. $c^*_{\tau_\mu}(\emptyset) = \emptyset$ and $c^*_{\tau_\mu}(X) = X$
3. If $A \subseteq B$, then $c^*_{\tau_\mu}(A) \subseteq c^*_{\tau_\mu}(B)$
4. $c^*_{\tau_\mu}(A) \cup c^*_{\tau_\mu}(B) \subseteq c^*_{\tau_\mu}(A \cup B)$
5. $c^*_{\tau_\mu}(A \cap B) \subseteq c^*_{\tau_\mu}(A) \cap c^*_{\tau_\mu}(B)$

Proof. The proofs are clear from Theorem 2.1 and the definition of $c^*_{\tau_\mu}$

3. IDEAL μ- WEAK STRUCTURE SPACE

In this section let a μ- weak structure τ_μ have the property any finite intersection of τ_μ-open sets is τ_μ-open and (X,τ_I) is called an ideal μ- weak structure space with this property

Proposition 3.1. Let (X,τ_I) be an ideal μ- weak structure space and $A,\mu \subseteq X$, then $U \cap A^*_{\tau_\mu} = U \cap (U \cap A)^*_{\tau_\mu} \subseteq (U \cap A)^*_{\tau_\mu}$ for every $U \in \tau_\mu$

Proof. Suppose that $U \in \tau_\mu$ and $x \in U \cap A^*_{\tau_\mu}$. Then $x \in U$ and $x \in A^*_{\tau_\mu}$. Let $V \in \mathfrak{A}\tau_\mu(x)$. Then $V \cap U \in \mathfrak{A}\tau_\mu(x)$ and $V \cap (U \cap A) = (V \cap U) \cap A \notin I$. This shows that $x \in (U \cap A)^*_{\tau_\mu}$ and hence we obtain $U \cap A^*_{\tau_\mu} \subseteq (U \cap A)^*_{\tau_\mu}$. Moreover, $U \cap A^*_{\tau_\mu} \subseteq U \cap (U \cap A)^*_{\tau_\mu}$ and by Theorem 2.3 (1) $(U \cap A)^*_{\tau_\mu} \subseteq A^*_{\tau_\mu}$ and $U \cap (U \cap A)^*_{\tau_\mu} \subseteq U \cap A^*_{\tau_\mu}$. Therefore, $U \cap A^*_{\tau_\mu} = U \cap (U \cap A)^*_{\tau_\mu}$

Theorem 3.2. Let (X,τ_I) be an ideal μ- weak structure space $A, B,\mu \subseteq X$. Then $A^*_{\tau_\mu} \cup B^*_{\tau_\mu} = (A \cup B)^*_{\tau_\mu}$

Proof. It follows from Theorem 2.1 that $A^*_{\tau_\mu} \cup B^*_{\tau_\mu} \subseteq (A \cup B)^*_{\tau_\mu}$. To prove the reverse inclusion, let $x \notin A^*_{\tau_\mu} \cup B^*_{\tau_\mu}$. Then x belongs neither to $A^*_{\tau_\mu}$ nor to $B^*_{\tau_\mu}$. Therefore there exist $U, V \in \mathfrak{A}\tau_\mu(x)$ such that $U \cap A \in I$ and $V \cap B \in I$. Since I is additive, $(U \cap A) \cup (V \cap B) \in I$. Moreover, since I is hereditary and

$$\begin{aligned}(U \cap A) \cup (V \cap B) &= [(U \cap A) \cup V] \cap [(U \cap A) \cup B] \\ &= (U \cup V) \cap (A \cup V) \cap (U \cup B) \cap (A \cup B) \\ &\supseteq (U \cap V) \cap (A \cup B),\end{aligned}$$

$(U \cap V) \cap (A \cup B) \in I$. Since $(U \cap V) \in \mathfrak{A}\tau_\mu(x)$, $x \notin (A \cup B)^*_{\tau_\mu}$. Hence $(A \cup B)^*_{\tau_\mu} \subseteq A^*_{\tau_\mu} \cup B^*_{\tau_\mu}$. Hence we obtain $A^*_{\tau_\mu} \cup B^*_{\tau_\mu} = (A \cup B)^*_{\tau_\mu}$.

Theorem 3.3. Let (X,τ_I) be an ideal μ- weak structure space and $A, B,\mu \subseteq X$. Then the following properties hold:

1. $c^*_{\tau_\mu}(A \cup B) = c^*_{\tau_\mu}(A) \cup c^*_{\tau_\mu}(B)$
2. $c^*_{\tau_\mu}(A) = c^*_{\tau_\mu}(c^*_{\tau_\mu}(A))$

Proof. By Theorem 3.2, we obtain

(1) $c^*_{\tau_\mu}(A \cup B) = (A \cup B)^*_{\tau_\mu} \cup (A \cup B) = (A^*_{\tau_\mu} \cup B^*_{\tau_\mu}) \cup (A \cup B) = c^*_{\tau_\mu}(A) \cup c^*_{\tau_\mu}(B)$

(2) $c^*_{\tau_\mu}\left(c^*_{\tau_\mu}(A)\right) = c^*_{\tau_\mu}\left(A^*_{\tau_\mu} \cup A\right) = (A^*_{\tau_\mu} \cup A)^*_{\tau_\mu} \cup \left(A^*_{\tau_\mu} \cup A\right) = ((A^*_{\tau_\mu})^*_{\tau_\mu} \cup A^*_{\tau_\mu}) \cup \left(A^*_{\tau_\mu} \cup A\right) = A^*_{\tau_\mu} \cup A = c^*_{\tau_\mu}(A)$

Corollary 3.1. Let (X, τ_I) be an ideal μ- weak structure space , $A, \mu \subseteq X$ and $c^*_{\tau_\mu}(A) = A \cup A^*_{\tau_\mu}$. Then $\gamma^*_{\tau_\mu} = \{U \subseteq X : c^*_{\tau_\mu}(\mu - U) = \mu - U\}$ is a topology for X

Proof. By Proposition 2.1 and Theorem 3.3, $c^*_{\tau_\mu}(A) = A \cup A^*_{\tau_\mu}$ is a Kuratowski closure operator. Therefore, $\gamma^*_{\tau_\mu}$ a topology for X.

Lemma 3.1. Let (X, τ_I) be an ideal μ- weak structure space and $A, B, \mu \subseteq X$. *Then* $A^*_{\tau_\mu} - B^*_{\tau_\mu} = (A - B)^*_{\tau_\mu} - B^*_{\tau_\mu}$

Proof. By Theorem 3.2, $A^*_{\tau_\mu} = [(A - B) \cup (A \cap B)]^*_{\tau_\mu} = (A - B)^*_{\tau_\mu} \cup (A \cap B)^*_{\tau_\mu} \subseteq (A - B)^*_{\tau_\mu} \cup B^*_{\tau_\mu}$. Thus $A^*_{\tau_\mu} - B^*_{\tau_\mu} \subseteq (A - B)^*_{\tau_\mu} - B^*_{\tau_\mu}$. By Theorem 2.3, $(A - B)^*_{\tau_\mu} \subseteq A^*_{\tau_\mu}$ and hence $(A - B)^*_{\tau_\mu} - B^*_{\tau_\mu} \subseteq A^*_{\tau_\mu} - B^*_{\tau_\mu}$ Hence $A^*_{\tau_\mu} - B^*_{\tau_\mu} = (A - B)^*_{\tau_\mu} - B^*_{\tau_\mu}$

Corollary 3.2. Let (X, τ_I) be an ideal μ- weak structure space and $A, B, \mu \subseteq X$ with $B \in I$. Then $(A \cup B)^*_{\tau_\mu} = A^*_{\tau_\mu} = (A - B)^*_{\tau_\mu}$

Proof. Since $B \in I$, by Theorem 2.1, $B^*_{\tau_\mu} = \emptyset$. By Lemma 3.1, $A^*_{\tau_\mu} = (A - B)^*_{\tau_\mu}$ and by Theorem 3.2 $(A \cup B)^*_{\tau_\mu} = A^*_{\tau_\mu} \cup B^*_{\tau_\mu} = A^*_{\tau_\mu}$

Theorem 3.7. Let (X, τ_I) be an ideal μ- weak structure space Then $\beta(\tau_I) = \{V - l : V \in \tau_\mu, l \in I\}$ *is a basis* for $(\gamma)^*_{\tau_\mu}$

Proof. Let (X, τ_I) be an ideal μ- weak structure space , $\mu \subseteq X$. It is obvious that A is $(\gamma)^*_{\tau_\mu}$ -closed if and only if $A^*_{\tau_\mu} \subseteq A$. Now we have $U \in (\gamma)^*_{\tau_\mu}$ if and only if $(\mu - U)^*_{\tau_\mu} \subseteq \mu - U$ if and only if $U \subseteq \mu - (\mu - U)^*_{\tau_\mu}$ Therefore $x \in U \in (\gamma)^*_{\tau_\mu}$ implies that $x \notin (\mu - U)^*_{\tau_\mu}$. This implies that there exists $V \in \mathfrak{U}\tau_\mu(x)$ such that $V \cap (\mu - U) \in I$. Now let $l = V \cap (\mu - U)$ and we have $x \in V - l \subseteq U$, where $V \in \mathfrak{U}\tau_\mu(x)$ and $l \in I$. Now we need only show that β is μ- closed under finite Intersection. Let $A, B \in \beta$, then $A = H - l$ and $B = K - j$, where $H, K \in \tau_\mu$ and $l, j \in I$. Now, we have

$$\begin{aligned}(H - l) \cap (K - j) &= [H \cap (X - l)] \cap [K \cap (\mu - j)]\\ &= [H \cap K] \cap [\mu - l) \cap (\mu - j)]\\ &= [H \cap K] \cap [\mu - (l \cup j)]\\ &= [H \cap K] - (l \cup j).\end{aligned}$$

Since $(l \cup j) \in I$ and $[H \cap K] \in \tau_\mu$, $A \cap B \in \beta$. Therefore β is μ- closed under finite intersection. Thus $\beta = \{V - l : V \in \tau_\mu, l \in I\}$ is a basis for $(\gamma)^*_{\tau_\mu}$.

4. APPLICATIONS

Definition 4.1. Let (X, τ_μ) be μ -weak structure, $A, \mu \subseteq X$. A point $x \in X$ is called accumulation points of a subset A iff for every τ_μ-open sets λ containing x , $(X - \lambda) \cap A \neq \emptyset$.

Remark 4.1. The set of all τ_μ-accumulation points of a subset A of a μ -weak structure (X, τ_μ) is $A^{\tau_\mu} = \{x \in X : U \cap A$ is infinite for every $U \in \mathcal{N}(x)\}$ where $\mathcal{N}$ is the ideal of all nowhere dense sets.

Definition 4.2. Let (X, τ_μ) be μ -weak structure, $A, \mu \subseteq X$. A point $x \in X$ is called condensation points of A iff for every $U \in \mathcal{N}(x)$, $U \cap A$ is uncountable

The set of all condensation points of A is $A^k = \{x \in X : U \cap A$ is uncountable for every $U \in \mathcal{N}(x)\}$. It is interesting to note that $A^*_{\tau_\mu}(I)$ is a generalization of closure points, τ_μ-accumulation points and condensation points

We call a class $\Omega \subseteq P(X)$ a generalized topology [2] (briefly, GT) if $\phi \in \Omega$ and the arbitrary union of elements of Ω belongs to Ω.

A set X with a GT Ω on it is called a generalized topological space (briefly, $GT\ S$) and is denoted by (X, Ω).

The proofs of the following theorem is clear.

Theorem 4.1. For a μ -weak structure space (X, τ_μ), $\mu \subseteq X$. the following properties are equivalent:

1. $\tau_\mu = \Omega$ i.e. τ is a generalized topology in the sense of Császár.
2. $i_{\tau_\mu}(A)$ is τ_μ-open for every subset A of X
3. $c_{\tau_\mu}(A)$ is τ_μ-closed for every subset A of X

Remark 4.2. For a μ -weak structure space (X, τ_μ), $\mu \subseteq X$, and $\tau_\mu{}^* = \{A \subset X : A = i_{\tau_\mu}(A)\}$. Then:

1. $\tau_\mu{}^*$ is a GT containing τ_μ
2. $\tau_\mu \subseteq \tau_\mu{}^* \subseteq \tau_\mu{}^*(I)$

Theorem 4.2. Let (X, τ_I) be an ideal μ- weak structure space. Then, $\tau_\mu{}^*(I)$ *is a* GT containing $\tau_\mu{}^*$

Proof. If $A \in \tau_\mu{}^*$, $A = i_{\tau_\mu}(A) \subseteq i^*_{\tau_\mu}(A)$ and hence $A \in \tau_\mu{}^*(I)$. Therefore, $\tau_\mu{}^*(I)$ contains $\tau_\mu{}^*$. Let $A_\alpha \in \tau_\mu{}^*(I)$. for each $\alpha \in \Delta$. Then $A_\alpha = i^*_{\tau_\mu}(A_\alpha) \subseteq i^*_{\tau_\mu}(\cup A\alpha)$ for each $\alpha \in \Delta$. Hence $\cup A\alpha \subseteq i^*_{\tau_\mu}(\cup A\alpha)$ and $\cup A\alpha = i^*_{\tau_\mu}(\cup A\alpha)$. Therefore, $\cup A\alpha \in \tau_\mu{}^*(I)$. And $\tau_\mu{}^*(I)$ is a GT

Definition 4.3. Let (X, τ_I) be an ideal μ- weak structure space and $A, \mu \subseteq X$. Then

1. $A \in \tau_\mu -$ sso(τ_I) if $A \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(i_{\tau_\mu}(A)))$
2. $A \in \tau_\mu -$ so(τ_I) if $A \subseteq c^*_{\tau_\mu}(i_{\tau_\mu}(A))$
3. $A \in \tau_\mu$-po(τ_I) if $A \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$
4. $A \in \tau_\mu$-spo(τ_I) if $A \subseteq c^*_{\tau_\mu}(i_{\tau_\mu}(c^*_{\tau_\mu}(A)))$

Lemma 4.5. Let (X, τ_I) be an ideal μ- weak structure space , we have the following

1. $\tau_\mu \subseteq \tau_\mu -$ sso $(\tau_I) \subseteq \tau_\mu -$ so$(\tau_I) \subseteq \tau_\mu$-spo$(\tau_I)$
2. $\tau_\mu \subseteq \tau_\mu -$ sso $(\tau_I) \subseteq \tau_\mu$-po$(\tau_I) \subseteq \tau_\mu$-spo$(\tau_I)$

Definition 4.4 Let (X, τ_I) be an ideal μ- weak structure space . The ideal μ- weak structure space is said to be τ_μ-extermally disconnected if $c^*_{\tau_\mu}(A) \in \tau_\mu$ for $A, \mu \subseteq X$ and $A \in \tau_\mu$

Theorem 4.3. Let (X, τ_I) be an ideal μ- weak structure space. Then the implications $(1) \Rightarrow (2), (3) \Rightarrow (4)$ and $(5) \Rightarrow (6) \Rightarrow (7)$ hold. If $\tau_\mu = \tau_\mu{}^*$ then the following statements are equivalent:

1. (X, τ_I) is τ_μ-externally disconnected.
2. $i^*_{\tau_\mu}(A)$ is τ_μ-closed for each τ_μ-closed set $A, \mu \subseteq X$
3. $c^*_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$ for each $A, \mu \subseteq X$
4. $A \in \tau_\mu$-po(τ_I) for each $A \in \tau_\mu -$ so(τ_I)
5. $c^*_{\tau_\mu}(A) \in \tau_\mu$ for each $A \in \tau_\mu$-spo(τ_I)
6. $A \in \tau_\mu$-po(τ_I) for each $A \in \tau_\mu$-spo(I)
7. $A \in \tau_\mu -$ sso (τ_I) if and only if $A \in \tau_\mu -$ so(τ_I)

Proof. $(1) \Rightarrow (2)$. Let A be a τ_μ-closed set. Then $\mu - A$ is τ_μ-open. By using (1), $c^*_{\tau_\mu}(\mu - A) = \mu - i^*_{\tau_\mu}(A) \in \tau_\mu$. Thus $i^*_{\tau_\mu}(A)$ is τ_μ-closed

$(2) \Rightarrow (3)$. Let μ, $A \subseteq X$. Then $\mu - i_\tau(A)$ is τ-closed and by (2) $i^*_{\tau_\mu}(\mu - i_{\tau_\mu}(A))$ is τ_μ-closed. Therefore, $c^*_{\tau_\mu}(i_{\tau_\mu}(A))$ is τ_μ-open and hence $c^*_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$

$(3) \Rightarrow (4)$. Let $A \in \tau_\mu -$ so(τ_I). By (3), we have $A \subseteq c^*_{\tau_\mu}(i_{\tau_\mu}(A)) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$. Thus, $A \in \tau_\mu$-po(τ_I)

(4) ⇒ (5). Let $A \in \tau_\mu$-spo(τ_I) Then $c^*_{\tau_\mu}(A) = c^*_{\tau_\mu}(i_{\tau_\mu}(c^*_{\tau_\mu}(A)))$ and $c^*_{\tau_\mu}(A) \in \tau_\mu - \mathrm{so}(\tau_I)$. By (4), $c^*_{\tau_\mu}(A) \in \tau_\mu$-po$(\tau_I)$. Thus $c^*_{\tau_\mu}(A) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$ and hence $c^*_{\tau_\mu}(A)$ is τ_μ-open

(5) ⇒ (6). Le t $A \in \tau_\mu$-spo(τ_I). By (5), $c^*_{\tau_\mu}(A) = i_{\tau_\mu}\left(c^*_{\tau_\mu}(A)\right)$. Thus, $A \subseteq c^*_{\tau_\mu}(A) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(A))$ and hence $A \in \tau_\mu$-po(τ_I)

(6) ⇒ (7). Let $A \in \tau_\mu - \mathrm{so}(\tau_I)$ then $A \in \tau_\mu$-spo(τ_I). Then by (6), $A \in \tau_\mu$-po(τ_I). Since $A \in \tau_\mu - \mathrm{so}(\tau_I)$ and $A \in \tau_\mu$-po(I) then $A \in \tau_\mu - sso\ (\tau_I)$

(7) ⇒ (1). Let A be a τ_μ-open set. Then $c^*_{\tau_\mu}((A) \in \tau_\mu - \mathrm{so}(\tau_I)$ and by using (7) $c^*_{\tau_\mu}(A) \in \tau_\mu - \mathrm{sso}\ (\tau_I)$ Therefore, $c^*_{\tau_\mu}((A) \subseteq i_{\tau_\mu}(c^*_{\tau_\mu}(\,(i_{\tau_\mu}(c^*_{\tau_\mu}((A))))) = i_{\tau_\mu}(c^*_{\tau_\mu}((A))$ and hence $c^*_{\tau_\mu}(A) = i_{\tau_\mu}(c^*_{\tau_\mu}(A))$. Hence $c^*_{\tau_\mu}(A)$ is τ_μ-open and (X, τ_I) is τ_μ-externally disconnected

5. RESULTS AND DISCUSSION

The purpose of this paper is to define $i^*_{\tau_\mu}$ and $c^*_{\tau_\mu}$ under more general conditions and to show that the important properties of these operations remain valid under these conditions, Also we define and study weaker form of τ_μ-open sets , τ_μ- continuity and an ideal $*$- μ- weak structure space and μ -weak local function with some applications on a set X are defined and their properties are discussed.

6. CONCLUSION

We can add many applications in ideal μ- weak structure space like if (X, τ_I) be an ideal μ- weak structure space. If $\tau_\mu = \tau_\mu{}^*\ (I)$. Then if (X, τ_I) is τ_μ-externally disconnected then $c^*_{\tau_\mu}(A) \cap c_{\tau_\mu}(B) \subseteq c_{\tau_\mu}(A \cap B)$ for each $A \in \tau_\mu$, $B \in \tau_\mu{}^*$ and $c^*_{\tau_\mu}(A) \cap c_{\tau_\mu}(B) = \emptyset$ for each $A \in \tau_\mu$, $B \in \tau_\mu{}^*$ with $A \cap B = \emptyset$.

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