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Fuzzy Based Approach for Monitoring the Mean and Range of the Products Quality

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ABSTRACT

Due to competition in the market, organization must have quality improvement program. Statistical quality control and especially control charts are proven quality improvement techniques. Control charts are based on the quality characteristics measurement in the course of time. There are some situations such as measurement error, sophisticated measurement instruments, costly skilled inspectors, environmental condition and imprecise specification limits that the quality characteristics of the products cannot be measured precisely. Fuzzy set theory is a well-known and proven technique in the case of imprecise, vague and uncertain situations. In the literature of control charts, there are also some research used fuzzy set theory that construct fuzzy control charts, determines the process condition by using transformation and defuzzification techniques (indirectly) which may reduce some useful information from the process. The purpose of this article is to develop a fuzzy Mean and Range ($\tilde{\bar{X}} - \tilde{R}$) control charts and monitor the process condition without any transformation techniques (directly). In this approach, observations and control limits are in case of triangular fuzzy numbers. The process condition is determined based on the percentage of area of the sample mean which remains outside the control limits. A numerical example in food industry is presented to illustrate the proposed approach. The result shows that the proposed approach is capable to detect even small shifts in the process quickly without any transformation techniques.

KEYWORDS: Statistical process control, control charts, fuzzy set theory, fuzzy control charts.

1. INTRODUCTION

Organizations, in today's competitive world, aim at maintaining their market shares and acquiring satisfaction of their customers and beneficiaries [1]. Statistical process control (SPC) is a well-known methodology for quality improvement and customer satisfaction through monitoring the process and identifying causes of variation. One of the basic quality improvement tools of SPC is control charts, also known as Shewhart charts. According to Montgomery [2], the control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time. In general, based on the quality characteristic, there are two broad categories of control charts, namely attribute and variable control charts. In practice, computing and measuring the quality characteristics are not accurate enough in many cases due to measurement error, sophisticated measurement instruments, costly skilled inspectors, environmental condition and imprecise specification limits. One may consider using fuzzy set theory to deal with incomplete and uncertain quality characteristics measurement. Fuzzy set theory was introduced by Zadeh [3] in 1965and developed rapidly in various filed[4-6]. The first application of fuzzy set theory in the area of SPC goes back to Bradshaw [7] who used fuzzy sets as a basis for the explanation of the measurement of the conformity of each product units with the specifications. Since then, several researchers attempt to use fuzzy set theory in the area of SPC and control charts.

Wang and Raz [8] and Raz and Wang [9] proposed probabilistic and membership approaches based on the fuzzy set theory for monitoring process average of the attribute quality characteristics which presented in form of linguistic data. They proposed four different transformation techniques to reduce the fuzzy subset associated to linguistic data to a crisp representative value. Probabilistic and membership approaches differ in interpretation of control limits and also in using the transformation techniques. Kanagawa et al. [10] proposed a control chart for process average and also process variability based on the estimation of probability distribution existing behind the linguistic data. This is different from the probability density function (normal distribution) employed by Wang and Raz [8] and Raz and Wang [9]. The difficulty of their method is related to determining this probability distribution. Laviolette et al [11] served the application of fuzzy set theory in control charts as an example to imply philosophical and practical problems of fuzzy methods. They proposed simpler alternatives based on traditional probability and statistical theory. Kandel [12] and Almond [13] claimed that the fuzzy set theory is capable and complete for dealing uncertain and vague situation but application of fuzzy set theory by Wang and Raz [8] and Raz and Wang [9] is restricted. Woodall et al. [14] reviewed the fuzzy quality control charts and proposed useful guideline to overcome the restriction and limitation of applying fuzzy set theory in case of control charts. They pointed out that using membership values of each items in each of the quality categories represented by linguistic variables might be a more useful

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approach. Taleb and Limam [15] compared fuzzy and probability approaches based on the average run length (ARL) and concluded that the choice of degree of fuzziness affected the sensitivity of control charts. Zabihinpour et al [16] reviewed fuzzy and statistical based control charts for monitoring attribute data and performed a simulation study to compare them based on average run length.

Gulbay and Kahraman [17-19] introduced α-level fuzzy control chart for attributes to reflect the vagueness of the data and tightness of the inspection. They also suggest an alternative approach known as a direct fuzzy approach for monitoring the vague number of nonconformities of the manufacturing processes. Although they did not transfer the linguistic data into representative value in order not to lose any information included in the fuzzy samples but unfortunately by using α-cut they transfer the fuzzy set to a crisp subset with membership grades of at least α.

In case of variable quality characteristics, Senturk and Erginel [20] introduced the framework of fuzzy $\tilde{\bar{X}}-\tilde{R}$ and $\tilde{\bar{X}}-\tilde{S}$ control charts. They transformed the traditional $\bar{X}-R$ and $\bar{X}-S$ control charts to fuzzy control charts and developed α-cut fuzzy $\tilde{\bar{X}}-\tilde{R}$ and $\tilde{\bar{X}}-\tilde{S}$ control charts by using α-cut approach and use α-level fuzzy midrange transformation techniques to determine the process conditions. They did not explain about the selection of α-level fuzzy midrange among different transformation techniques. Erginel[21] formulated the fuzzy control limits for individual measurements with α-cut and by using α-level fuzzy median transformation techniques. Alizadeh and Ghomi[22] developed mean and range control charts in fuzzy environment using different transformation methods. They defined the representative value for a triangular fuzzy number based on the transformation techniques proposed by Wang and Raz [8], but unfortunately, their definition in some cases does not match the original definition. Both of the aforementioned approaches transform the fuzzy observations and also fuzzy control limits to a crisp value to determine the process condition, but this procedure may reduce useful information from the process and it seems to be better to determine the process condition directly and without any transformation.

Shu and Wu [23] proposed fuzzy $\bar{X}-R$ control charts whose fuzzy control limits are obtained based on the result of the resolution identity. They utilized fuzzy dominance approach, which directly compares the fuzzy sample mean to the fuzzy control limits to determine the process condition. Faraz et al. [24] introduced a fuzzy control chart for variables based on a fuzzy acceptance region when uncertainty and randomness are put together. Faraz and Shapiro [25] proposed a control chart in an extension of Shewhart $\bar{X}-S^2$ control charts in fuzzy space without any defuzzification methods. In this approach, the out-of-control state is determined according to a fuzzy in-control area and a simple and exact graded exclusion measure that determines the degree to which fuzzy subgroups are excluded from the fuzzy in-control region. They used a fuzzy inference region instead of the basic structure of Shewhart control charts with upper and lower control limits and it may cause some difficulties for preliminary users.

The objective of this paper is to develop a fuzzy $\tilde{\bar{X}}-\tilde{R}$ control charts which monitor the process without any transformation techniques and also maintain the basic structure of Shewhart control charts.

This article is organized as follows. Basic concepts are presented in section 2. In section 3, the procedure of constructing fuzzy control chart and monitoring directly based on the percentage of area are introduced. In section 4, a numerical example in the food industry is used to validate the proposed approach and conclusion remarks are presented in section5.

2. Basic Concepts

In this section basic concepts of fuzzy numbers and traditional Shewhart control charts would be introduced.

2.1. Fuzzy numbers

A fuzzy number refers to an extension of a regular number in a way that it does not refer to one single value but rather to a connected set of possible values where each possible value has its own weight between 0 and 1. The weight is referred to as membership degree. Calculation with fuzzy numbers allows incorporation of uncertainty on parameters, properties, initial conditions and etc. Trapezoidal and triangular fuzzy numbers are the most famous shapes of fuzzy numbers. Suppose $\tilde{A}$ is a trapezoidal fuzzy number. It is represented by $\tilde{A}=(a,b,c,d;w)$, where a,b,c and d are real numbers and $0<w\le 1$ represents the degree of confidence of expert regarding $\tilde{A}$. If $w=1$, then $\tilde{A}$ is called a normal trapezoidal fuzzy number and denoted as $\tilde{A}=(a,b,c,d)$. If, $b=c$ then $\tilde{A}$ is called a triangular fuzzy number and denoted as $\tilde{A}=(a,b,c)$. In this study, the triangular fuzzy number would be considered. Eq. (1) represents the membership function of a triangular fuzzy number.

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & x<0 \\ \frac{x-a}{b-a} & a\le x<b \\ \frac{c-x}{c-b} & b\le x<c \\ 0 & x\ge c \end{cases} \tag{1}$$

2.2. Shewhart mean and range ($\bar{X}-R$) control charts

When dealing with a quality characteristic that is a variable, it is usually necessary to monitor both the mean value of the quality characteristic and its variability. Control of the process average or mean quality level is usually done with the control chart for mean ($\bar{X}$ control chart). Process variability can be monitored with either a control chart for the standard deviation, called the *S* control chart, or a control chart for the range, called the *R* control chart. According to Montgomery [2], the *R* chart is more widely used. This study is concentrated on the $\bar{X}-R$ control charts.

Control limits for the $\bar{X}$ and *R* could be calculated by means of Eq. (2) and (3), respectively.

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}, CL_{\bar{X}} = \bar{\bar{X}}, UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \quad (2)$$

$$LCL_R = D_3\bar{R}, CL_R = \bar{R}, UCL_R = D_4\bar{R} \quad (3)$$

where $\bar{\bar{X}}$, is the grand average of the subgroup mean, $\bar{R}$ is the average of the range of each subgroup and A_2, D_3 and D_4 are constants depending on sample size which has been tabulated in Montgomery [2] and in most standard references.

3. Fuzzy Mean and Range ($\tilde{\bar{\bar{X}}} - \tilde{R}$) Control charts

In this study, each observation is considered as a triangular fuzzy number $\tilde{X}_{ij} = (X_{a_{ij}}, X_{b_{ij}}, X_{c_{ij}})$; $i = 1,2,...,m$; $j = 1,2,...,n$ where *m* is the number of subgroup and *n* is the sample size in each subgroup. If $(X_{a_{i1}}, X_{b_{i1}}, X_{c_{i1}}),...,(X_{a_{in}}, X_{b_{in}}, X_{c_{in}})$ is a sample of *n* fuzzy observations in subgroup *i*, then $(\bar{X}_{a_i}, \bar{X}_{b_i}, \bar{X}_{c_i})$, the average of each sample, is

$$\bar{X}_{a_i} = \frac{\sum_{j=1}^{n} X_{a_{ij}}}{n}, \bar{X}_{b_i} = \frac{\sum_{j=1}^{n} X_{b_{ij}}}{n}, \bar{X}_{c_i} = \frac{\sum_{j=1}^{n} X_{c_{ij}}}{n} \quad (4)$$

And the range of the subgroup *i is*

$$R_{a_i} = \max(X_{a_{ij}}) - \min(X_{c_{ij}}); R_{b_i} = \max(X_{b_{ij}}) - \min(X_{b_{ij}}); R_{c_i} = \max(X_{c_{ij}}) - \min(X_{a_{ij}}); \ (j = 1,...,n) \quad (5)$$

To set up a fuzzy $\tilde{\bar{\bar{X}}}$ control chart, first; $\tilde{CL}_{\bar{X}} = (\tilde{CL}_a, \tilde{CL}_b, \tilde{CL}_c)$ should be calculated. $\tilde{CL}_{\bar{X}}$ is the fuzzy arithmetic mean of the observations and could be calculated as Eq. (6).

$$\tilde{CL}_{\bar{X}} = (\tilde{CL}_a, \tilde{CL}_b, \tilde{CL}_c) = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) \quad (6)$$

Where $\bar{\bar{X}}_k = \frac{\sum_{i=1}^{m} \bar{X}_{k_i}}{m}$; $k = a,b,c$

For calculating $\tilde{UCL}_{\bar{X}}$ and $\tilde{LCL}_{\bar{X}}$, first; the fuzzy average range $\tilde{\bar{R}} = (\bar{R}_a, \bar{R}_b, \bar{R}_c)$ should be calculated by using Eq. (7).

$$\bar{R}_k = \frac{\sum_{i=1}^{m} R_{k_i}}{m}; k = a,b,c \quad (7)$$

Then, fuzzy $\tilde{\bar{\bar{X}}}$ control limits could be obtained by using Eqs. (8), (9) and (10).

$$\tilde{UCL}_{\bar{X}} = (\tilde{UCL}_{\bar{X}_a}, \tilde{UCL}_{\bar{X}_b}, \tilde{UCL}_{\bar{X}_c}) = \tilde{CL}_{\bar{X}} + A_2\tilde{\bar{R}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) + A_2(\bar{R}_a, \bar{R}_b, \bar{R}_c) = (\bar{\bar{X}}_a + A_2\bar{R}_a, \bar{\bar{X}}_b + A_2\bar{R}_b, \bar{\bar{X}}_c + A_2\bar{R}_c) \quad (8)$$

$$\tilde{CL}_{\bar{X}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) \quad (9)$$

$$\tilde{LCL}_{\bar{X}} = (\tilde{LCL}_{\bar{X}_a}, \tilde{LCL}_{\bar{X}_b}, \tilde{LCL}_{\bar{X}_c}) = \tilde{CL}_{\bar{X}} - A_2\tilde{\bar{R}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) - A_2(\bar{R}_a, \bar{R}_b, \bar{R}_c) = (\bar{\bar{X}}_a - A_2\bar{R}_a, \bar{\bar{X}}_b - A_2\bar{R}_b, \bar{\bar{X}}_c - A_2\bar{R}_c) \quad (10)$$

And the fuzzy control limits of the $\tilde{R}$ chart are obtained by means of Eqs. (11), (12) and (13).

$$\tilde{UCL}_R = D_4\tilde{\bar{R}} = D_4(\bar{R}_a, \bar{R}_b, \bar{R}_c) = (D_4\bar{R}_a, D_4\bar{R}_b, D_4\bar{R}_c) \quad (11)$$

$$\tilde{CL}_R = \tilde{\bar{R}} = (\bar{R}_a, \bar{R}_b, \bar{R}_c) \tag{12}$$

$$L\tilde{C}L_R = D_3\tilde{\bar{R}} = D_3(\bar{R}_a, \bar{R}_b, \bar{R}_c) = (D_3\bar{R}_a, D_3\bar{R}_b, D_3\bar{R}_c) \tag{13}$$

Calculating the percentage of area (*PA*) of fuzzy observation ($\tilde{\bar{X}}_i$ in $\tilde{\bar{X}}$ chart and $\tilde{R}_i$ in $\tilde{R}$ chart) above the $U\tilde{C}L$ or below the $L\tilde{C}L$ is proposed to determine whether the process is *in-control* or *out-of-control*. If $\tilde{\bar{X}}_i$ or $\tilde{R}_i$ are completely between the fuzzy control limits then *PA* is equal to zero and the process is *in-control* (Fig. 1a). If $\tilde{\bar{X}}_i$ or $\tilde{R}_i$ are completely above or below the fuzzy control limits then *PA* is equal to 1 and the process is *out-of-control* (Fig. 1b). If $\tilde{\bar{X}}_i$ or $\tilde{R}_i$ are partially above or below the fuzzy control limits then *0<PA<1* and could be calculated by using Eq. (14). In this case, if *PA* is greater than a predefined percentage of area (β) then the process is*rather out-of-control* otherwise the process is *rather in-control* (Fig. 1c). The value of β directly affects the type I and type II error and should be determined carefully by the top management or quality engineers.

$$PA = \frac{\begin{array}{c}\text{The area of the sample mean which} \\ \text{remains outside the } U\tilde{C}L \text{ or } L\tilde{C}L\end{array}}{\text{Total area of sample mean}} \tag{14}$$

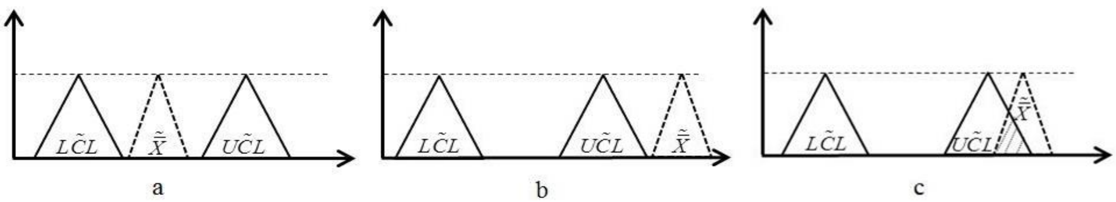

Figure 1. Process condition when the sample points and control limits are triangular fuzzy numbers; a: *in-control*, b: *out-of-control*, c: *rather-in-control* or *rather-out-of-control*

4. Numerical Example

In this section, a numerical example is presented to validate the proposed fuzzy direct $\tilde{\bar{X}} - \tilde{R}$ control charts. This example considers a case in food industry. The color of the food is one of the most important quality characteristics in food industry. It is an indication of ripeness or spoilage and depends on some features such as processing methods, packaging and storage. Colorimeter can be used for color measurement. The output from a colorimeter may be displayed as a linear scale of 100 values between the predefined clearest and the darkest color. Due to environmental condition the inspector expressed each measurement item in the form of a triangular fuzzy number (X_a, X_b, X_c). Twenty five samples with a sample size of four have been taken from the process when it is assumed to be *in-control*. Table 1 shows the observations. Table 2 shows the result of each subgroup fuzzy mean and fuzzy range calculation based on Eq. (4) and Eq. (5). Fuzzy control limits obtained from Eqs. (8)-(13) are shown in table 3. Table 2 also shows the overall percentage of area of each observation remains outside the fuzzy control limits. It shows that all samples are *in-control* as all *PA* is equal to zero. Therefore, the process is in the state of *in-control* and the obtained control limits could be used for monitoring the future productions.

To evaluate the performance of proposed control chart, 15 additional observations from the process were collected when new fruits as raw materials arrived. Table 4 shows these observations. The fuzzy mean, fuzzy range and *PA* of each subgroup are shown in table 5.

If β is considered to be 0.8, the *PA* indicates that the process is *rather-in-control* from the 27th sample since $PA_{\bar{X}}$ is greater than zero but not greater than β. In the 29th sample the process is *rather-out-of-control*. Generally, the *PA* of the last 15 observations indicates that there is a shift in the process mean.

Table 1. Fuzzy triangular observations

Subgroup	X_{a_1}	X_{b_1}	X_{c_1}	X_{a_2}	X_{b_2}	X_{c_2}	X_{a_3}	X_{b_3}	X_{c_3}	X_{a_4}	X_{b_4}	X_{c_4}
1	7.89	7.99	8.19	7.90	8.25	8.27	7.62	7.64	7.74	7.64	7.74	7.85
2	7.61	7.74	7.93	8.44	8.55	8.88	8.16	8.43	8.48	6.90	7.08	7.27
3	7.13	7.14	7.33	8.23	8.51	8.57	8.09	8.30	8.44	7.43	7.47	7.69
4	7.03	7.15	7.32	8.43	8.54	8.55	8.09	8.17	8.50	7.39	7.56	7.92
5	8.37	8.52	8.72	8.91	9.20	9.21	7.83	8.09	8.09	8.01	8.14	8.32
6	7.56	7.59	7.64	8.80	8.93	8.99	8.49	8.73	8.79	8.27	8.53	8.61
7	8.15	8.26	8.54	7.25	7.36	7.60	7.75	7.96	8.28	7.74	7.78	8.09

8	7.59	7.78	7.87	7.16	7.21	7.49	7.65	7.79	7.97	8.42	8.58	8.71
9	7.36	7.57	7.63	7.18	7.41	7.60	7.15	7.36	7.55	7.26	7.40	7.71
10	7.73	7.75	7.91	7.60	7.71	7.91	7.86	7.97	8.01	7.54	7.58	7.67
11	8.71	8.88	8.97	7.76	7.93	8.11	7.90	8.15	8.27	8.08	8.17	8.21
12	7.42	7.71	7.82	8.30	8.51	8.60	8.71	8.95	9.19	7.45	7.54	7.75
13	8.45	8.59	8.92	7.71	7.93	8.08	8.42	8.54	8.69	7.06	7.24	7.40
14	8.08	8.26	8.34	7.92	8.33	8.41	8.37	8.46	8.52	7.23	7.43	7.59
15	8.3	8.43	8.46	7.75	7.90	8.11	7.36	7.39	7.59	7.51	7.61	7.81
16	7.84	8.01	8.27	8.38	8.53	8.62	7.22	7.40	7.78	8.35	8.52	8.56
17	7.29	7.38	7.61	7.93	8.20	8.36	7.55	7.83	7.94	7.46	7.71	7.86
18	7.44	7.68	7.78	8.65	8.74	8.83	7.50	7.68	7.96	7.95	8.18	8.45
19	7.69	7.93	8.07	8.87	9.05	9.14	7.97	8.05	8.05	8.01	8.17	8.39
20	7.5	7.57	7.67	7.07	7.49	7.68	7.48	7.67	7.94	8.09	8.33	8.53
21	7.17	7.24	7.50	7.72	7.90	8.10	7.32	7.48	7.59	7.07	7.26	7.32
22	7.73	7.74	7.83	7.91	8.21	8.25	7.97	8.17	8.42	8.59	8.62	8.78
23	8.41	8.61	8.90	8.22	8.60	8.72	7.71	7.83	7.84	7.44	7.73	7.89
24	8.16	8.48	8.60	7.77	7.79	7.91	7.51	7.64	7.77	7.60	7.63	7.74
25	7.94	8.01	8.19	7.80	7.92	8.16	8.91	9.15	9.36	7.83	8.01	8.21

Table 2.Fuzzy mean, fuzzy range and *PA* for each subgroup

Subgroup	$\bar{X}_a$	$\bar{X}_b$	$\bar{X}_c$	$PA_{\bar{X}}$	R_a	R_b	R_c	PA_R
1	7.76	7.91	8.01	0	0.16	0.61	0.65	0
2	7.78	7.95	8.14	0	1.17	1.47	1.98	0
3	7.72	7.86	8.01	0	0.90	1.37	1.44	0
4	7.42	7.86	8.07	0	1.11	1.39	1.52	0
5	8.28	8.49	8.59	0	0.82	1.11	1.38	0
6	8.28	8.45	8.51	0	1.16	1.34	1.43	0
7	7.72	7.84	8.13	0	0.55	0.90	1.29	0
8	7.70	7.84	8.01	0	0.93	1.37	1.55	0
9	7.24	7.44	7.62	0	0	0.21	0.56	0
10	7.68	7.75	7.88	0	0.19	0.39	0.47	0
11	8.11	8.25	8.39	0	0.60	0.95	1.21	0
12	7.97	8.18	8.34	0	0.96	1.41	1.77	0
13	7.91	8.08	8.27	0	1.05	1.35	1.86	0
14	7.90	8.12	8.22	0	0.78	1.03	1.29	0
15	7.73	7.83	7.99	0	0.71	1.04	1.10	0
16	7.95	8.12	8.31	0	0.60	1.13	1.40	0
17	7.56	7.78	7.94	0	0.32	0.82	1.07	0
18	7.89	8.07	8.26	0	0.87	1.06	1.39	0
19	8.13	8.30	8.41	0	0.82	1.12	1.45	0
20	7.54	7.77	7.96	0	0.42	0.84	1.46	0
21	7.32	7.47	7.63	0	0.40	0.66	1.03	0
22	8.05	8.19	8.32	0	0.76	0.88	1.05	0
23	7.95	8.19	8.34	0	0.57	0.88	1.46	0
24	7.76	7.89	8.01	0	0.42	0.85	1.09	0
25	8.12	8.27	8.48	0	0.75	1.23	1.56	0

Table 3. Fuzzy triangular control limits

	$\tilde{\bar{X}}$	$\tilde{R}$
$U\tilde{C}L$	(8.224, 8.582, 8.9017)	(1.4392, 2.1487, 2.7448)
$C\tilde{L}$	(7.8312, 7.9955, 8.1525)	(0.6808, 1.0164, 1.2984)
$L\tilde{C}L$	(7.082, 7.409, 7.7597)	(0, 0, 0)

Table 4. Additional fuzzy triangular observations

Subgroup	X_{a_1}	X_{b_1}	X_{c_1}	X_{a_2}	X_{b_2}	X_{c_2}	X_{a_3}	X_{b_3}	X_{c_3}	X_{a_4}	X_{b_4}	X_{c_4}
26	8.37	8.37	8.56	8.1	8.29	8.36	8.01	8.02	8.20	9.45	9.62	9.82
27	8.54	8.72	9.10	8.12	8.25	8.34	8.83	9.00	9.12	8.97	9.02	9.18
28	8.44	8.75	8.95	8.65	8.66	8.66	8.86	8.94	9.07	8.04	8.21	8.38
29	9.19	9.52	9.77	9.15	9.19	9.46	8.16	8.17	8.64	8.72	8.80	8.96
30	8.62	8.68	8.76	8.76	8.91	8.93	8.26	8.4	8.56	7.63	7.81	8.11
31	7.65	7.85	8.13	8.63	8.84	8.94	8.52	8.55	8.69	8.28	8.37	8.66
32	8.37	8.59	8.77	8.1	8.14	8.14	8.66	8.78	9.01	8.44	8.46	8.61
33	9.02	9.2	9.33	8.16	8.43	8.68	8.41	8.43	8.49	8.77	8.81	8.92
34	8.50	8.58	8.67	8.44	8.53	8.55	8.96	9.01	9.27	8.35	8.47	8.75
35	8.67	8.78	8.84	8.2	8.39	8.62	8.57	8.67	8.85	8.87	8.98	9.27
36	8.22	8.46	8.71	8.53	8.67	8.91	7.59	7.74	7.9	8.77	8.83	8.84
37	7.12	7.33	7.58	8.03	8.42	8.51	7.85	7.99	8.15	8.82	8.99	9.06
38	7.79	7.88	7.9	8.87	9.01	9.04	8.77	8.83	9.11	8.5	8.67	8.74
39	8.75	8.91	9.08	8.27	8.48	8.58	8.47	8.73	8.86	8.84	8.93	9.1
40	8.09	8.35	8.55	9.19	9.33	9.48	8.51	8.8	8.89	7.53	7.81	8.15

Table 5. Fuzzy mean, fuzzy range and *PA* for each additional subgroup

Subgroup	$\bar{X}_a$	$\bar{X}_b$	$\bar{X}_c$	$PA_{\bar{X}}$	R_a	R_b	R_c	PA_R
26	8.48	8.58	8.74	0	1.25	1.60	1.81	0
27	8.62	8.75	8.94	0.4321	0.63	0.77	1.06	0
28	8.49	8.64	8.77	0.0374	0.48	0.73	1.03	0
29	8.80	8.92	9.21	0.9466	0.55	1.35	1.61	0
30	8.32	8.45	8.59	0	0.65	1.10	1.30	0
31	8.27	8.40	8.61	0	0.50	0.99	1.29	0
32	8.39	8.49	8.63	0	0.52	0.64	0.91	0
33	8.59	8.72	8.86	0.2254	0.53	0.77	1.17	0
34	8.56	8.65	8.81	0.0675	0.41	0.54	0.92	0
35	8.58	8.71	8.89	0.2610	0.25	0.59	1.07	0
36	8.28	8.43	8.59	0	0.87	1.09	1.32	0
37	7.96	8.18	8.33	0	1.24	1.66	1.94	0
38	8.48	8.60	8.7	0.0025	0.97	1.13	1.32	0
39	8.58	8.76	8.91	0.3679	0.26	0.45	0.83	0
40	8.33	8.57	8.77	0	1.04	1.52	1.95	0

5. Conclusion

Fuzzy control charts could be used to monitor the processes with uncertain, vague and/or imprecise observations. In this study, a fuzzy $\tilde{\bar{X}}-\tilde{R}$ control charts which their control limits are triangular fuzzy numbers are constructed based on fuzzy triangular observations. Instead of using transformation or defuzzification techniques to determine the process condition, a direct approach is proposed. It is based on the percentage of area (*PA*) of the sample mean which remains above the $U\tilde{C}L$ or below the $L\tilde{C}L$. The proposed approach shows that it has two advantages. Firstly, it maintains the process information hence using transformation techniques may lead to reducing some useful information from the process. Secondly, the information obtained from the *PA*, especially when the process is *rather-in-control*, can be used to make process modification, bringing it into control and reduce variability which is the goal of SPC. A comparison study based on average run length to evaluate the performance of proposed approach using different level of β is suggested for further research.

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