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Oham Solution of Thin Film Non-Newtonian Fluid on a Porous and Lubricating Vertical Belt

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ABSTRACT

In this paper, problem of thin layer third order fluid flow past a vertical lubricating and porous belt that modeled by a system of nonlinear differential equations have been studied in the presence of heat. The nonlinear differentials equations for the fields of velocity and temperature have been solved analytically by using Optimal Homotopy Asymptotic Method (OHAM). The influence of different physical parameters on the velocity and temperature field have been studied and discussed. The comparison of present work with published work is also mentioned graphically and numerically.
KEYWORDS: Lifting, Drainage, Third Grade, Slip Boundary Conditions, Porous, Medium

1. INTRODUCTION

In third grade thin film the transfer of heat are used largely in biological and geophysical science, mechanical engineering, civil and electrical engineering. The usage and importance of its application as seen in daily life in different food material like ketchup sauce and honey, wire coating, reactor fluidization etc. Taza Gul et al [1] examined the problem of thin film third grade fluid on a vertical belt with slip boundary condition. Nadeem and Awais [2] investigated a thin film flow through porous medium with variable viscosity. To find velocity and temperature they used ADM and OHAM method. The comparisons of these methods have been studied. Taza Gul et al [3] investigated thin film flow of an unsteady second order fluid over a vertical belt. They used OHAM and ADM methods for the fields of both velocity and temperature. The comparisons of these methods have been studied also. The combined effects of MHD and heat transfer have great importance in chemical processing. Related and attractive work may be found in [9,10]. Third grade fluid is a renowned model in non-Newtonian fluids which have their constitutive equations based on physically powerful theoretical foundations, because in this model the relation between stress and strain is not linear.

To solve actual world problems, varieties of approximate methods have been used in mathematics. We have used one of an efficient analytical method OHAM [11, 12].

During last few years, lots of studies have been done through porous medium [13, 14].

We have investigated the same problem on vertical porous belt in the presence of Heat, MHD and slip boundary conditions. The flow modeling is based on modified Darcy's Law for third grade fluid.

$$\nabla p = -\frac{\mu\phi U}{k}.$$

2. Basic Equations

The governing equation of incompressible is thermal electrically conducting and grade fluid is

$$\nabla.\mathbf{U} = 0 \tag{1}$$

$$\rho\frac{D\mathbf{U}}{Dt} = \nabla.\mathbf{T} + \rho g - \frac{\mu\phi U}{k}, \tag{2}$$

$$\rho c_p \frac{D\Theta}{Dt} = \kappa\nabla^2\Theta + tr(\boldsymbol{\tau}.\mathbf{L}), \tag{3}$$

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Where κ is the thermal conductivity, g is the body force per unit mass, **U** is velocity vector of the fluid , Θ is the temperature , **T** is the shear stress, ρ is the constant of density , c_p is the specific heat, $\mathbf{L} = \nabla\mathbf{U}$, τ is Cauchy stress tensor, and denote the material time derivative. The Lorentz force per unit volume is given by

Shear stress tensor **T** is given by

$$\mathbf{T} = -p\mathbf{I} + \boldsymbol{\tau}, \tag{4}$$

Where $-p\mathbf{I}$ denote spherical stress and $\boldsymbol{\tau}$ is define as

$$\tau = \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2\left(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1\right) + \beta_3\left(tr\mathbf{A}_1^2\right)\mathbf{A}_1 \tag{5}$$

Here α_i and β_j are the material constants, and $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ are the kinematical tensor given by

$$\mathbf{A}_n = \frac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}(\nabla u) + (\nabla u)^T\mathbf{A}_{n-1}, \qquad n \geq 1, \tag{6}$$

$$\mu \geq 0, \alpha_1 \geq 0, \left|\alpha_1 + \alpha_2\right| \leq \sqrt{24\mu\beta_3}, \beta_3 \geq 0 \tag{7}$$

3. Formulation of the Lift Problem

We suppose a third grade liquid in a wide box. A belt passes through the wide box in upward direction with uniform velocity U_0. A thin layer of third grade liquid of constant thickness δ carries by belt. We suppose that x-axis perpendicular to the belt and y-axis is parallel. The flow is steady and laminar on the surface of the belt. The external pressure is atmospheric all over.

We assume Third grade liquid in a wide container. A belt moving with uniform velocity U in the upward direction and passes through the container. Moving belt carries with itself a thin layer of third grade liquid of constant thickness δ. We assume x-axis perpendicular and y-axis parallel to the belt. Above the belt surface the flow is steady and laminar. The exterior pressure is atmospheric everywhere.

Velocity and temperature field are

$$\mathbf{U} = (0, u(x), 0), \qquad \Theta = \Theta(x), \tag{8}$$

Boundary conditions are

$$\begin{aligned} &u = U_0 - \gamma T_{xy} \quad \text{at} \quad x = 0 \\ &\frac{du}{dx} = 0, \qquad\qquad x = \delta, \\ &\Theta = \Theta_0, \text{ at } x = 0, \\ &\Theta = \Theta_1 \text{ at } x = \delta, \end{aligned} \tag{9}$$

Using the velocity field given in (9) in (1) and (5-7), the continuity equation (1) satisfies automatically and (5) gives the following components of stress tensor

$$\begin{aligned} T_{xx} &= -P + (2\alpha_1 + \alpha_2)\left(\frac{du}{dx}\right)^2, \\ T_{xy} &= \mu\frac{du}{dx} + 2(\beta_2 + \beta_3)\left(\frac{du}{dx}\right)^3, \\ T_{yy} &= -P + \alpha_2\left(\frac{du}{dx}\right), \\ T_{zz} &= -p \\ T_{xz} &= T_{yz} = 0, \end{aligned} \tag{10}$$

Using (11), the momentum and energy equation reduces to,

$$\mu\frac{d^2u}{dx^2}+6(\beta_2+\beta_3)\left(\frac{du}{dx}\right)^2\left(\frac{d^2u}{dx^2}\right)-\rho g-\sigma B_0^2u(x)-\frac{\mu\phi u}{k}=0,$$

$$\kappa\frac{d^2\Theta}{dx^2}+\mu\left(\frac{du}{dx}\right)^2+2(\beta_2+\beta_3)\left(\frac{du}{dx}\right)^4=0 \tag{11}$$

Introducing the following nondimensional variables:

$$\bar{u}=\frac{\delta}{\upsilon}U,\quad \bar{x}=\frac{x}{\delta},\quad \lambda=\frac{\phi\delta^2}{k},\quad \overline{\Theta}=\frac{\Theta-\Theta_0}{\Theta_1-\Theta_0},\quad \bar{\gamma}=\frac{\mu\gamma}{\delta},\quad \Lambda=\frac{\mu\gamma}{\delta},$$

$$B=\frac{\mu v^2}{k(\Theta_1-\Theta_0)\delta^2},\quad m=\frac{\delta^3 g}{\upsilon^2},\quad \beta=\frac{(\beta_2+\beta_3)\upsilon^2}{\mu\delta^4}, \tag{12}$$

$$\alpha=\frac{\delta U_0}{\upsilon},\quad R_e=\frac{U\delta}{\upsilon},\quad \upsilon=\frac{\mu}{\rho},$$

Where R_e is the local Reynolds number, α is nondimensional variable, Λ is slip parameter, λ is heat dimensionless number, m is the gravitational parameter, B is the heat parameter, using the above dimensionless variable in (10) and in (12) and dropping bars we obtain

$$\frac{d^2u}{dx^2}+6\beta\left(\frac{du}{dx}\right)^2\left(\frac{d^2u}{dx^2}\right)-m-\lambda u=0, \tag{13}$$

$$\frac{d^2\Theta}{dx^2}+B\left[\left(\frac{du}{dx}\right)^2+2\beta\left(\frac{du}{dx}\right)^4\right]=0 \tag{14}$$

$$u_n(0)=\alpha-\Lambda\left(\frac{du_n}{dx}+2\beta\left(\frac{du_n}{dx}\right)^3\right),\qquad \frac{du_n(1)}{dx}=0,\quad n=0, \tag{15}$$

$$u_n(0)=-\Lambda\left(\frac{du_n}{dx}+2\beta\left(\frac{du_n}{dx}\right)^3\right),\qquad \frac{du_n(1)}{dx}=0,\quad n>0, \tag{16}$$

$$\Theta_n(0)=0,\qquad \Theta_n(1)=1, \tag{17}$$

4. Basic Idea of OHAM

Here we discuss the concept of OHAM; we consider the boundary value problem as consider in [22]:

$$L(u(x))+N(u(x))+g(x)=0,\qquad B\left(u,\frac{\partial u}{\partial x}\right)=0, \tag{18}$$

Where L used for linear term, N show non-linear part of the differential equation, B is used as a boundary operator while g is an extra term. we construct OHAM as.

$$[1-p][L\varphi(x,p)+g(x)]=H(p)[L\varphi(x,p)+g(x)+N\varphi(x,p)],\quad B\left(\varphi(x,p),\frac{\partial\varphi(x,p)}{\partial x}\right)=0 \tag{19}$$

$p\in[0,1]$ used as embedding parameter, H(p), is an auxiliary function for $p\neq 0$ and $H(0)=0$. $\varphi(x,p)$ is an unknown function. When $p=0$ *and* $p=1$, it holds that:

$$\varphi(x,0)=u_0(x),\quad \varphi(x,1)=u(x), \tag{20}$$

When p varies from 0 *to* 1 then $\varphi(x,p)$ also varies from $u_0(x)$ *to* $u(x)$. Where the zero component solution $u_0(x)$ is obtained from equation (19) when $p=0$,

$$L(u_0(x))+g(x)=0, \qquad B\left(u_0(x),\frac{\partial u_0(x)}{\partial x}\right)=0, \tag{21}$$

Auxiliary function $H(p)$ is choosing as

$$H(p)=pc_1+p^2c_2+\cdots, \tag{22}$$

c_1,c_2 are auxiliary constants.

Marinca [22] uses a special procedure to expand $\varphi(x,p)$ with respect to p by using Taylor Series.

$$\varphi(x,p,c_i)=u_0(x)+\sum_{k\geq 1}u_k(x,c_i)p^k, \qquad i=1,2,\cdots \tag{23}$$

Inserting Eq. (23) into Eq. (19), collecting the same powers of p and equating each coefficient of p. The zero order problems given in equation (21) and the first order and second order given in equations (24, 25).

$$L(u_1(x))+g(x)=c_1N_0(u_0(x)), \qquad B\left(u_1(x),\frac{\partial u_1(x)}{\partial x}\right)=0, \tag{24}$$

$$L(u_2(x))-L(u_1(x))=c_2N_0(u_0(x))+c_1[L(u_1(x))+N_1(u_0(x),u_1(x))], \quad B\left(u_2(x),\frac{\partial u_2(x)}{\partial x}\right)=0 \tag{25}$$

The general governing equations for $u_k(x)$ are given by

$$L(u_k(x))-L(u_{k-1}(x))=c_kN_0(u_0(x))+\sum_{i=1}^{k-1}c_i[L(u_{k-i}(x))+N_{k-1}(u_0(x),u_1(x)\cdots u_{k-i}(x))]$$

$$,k=2,3,\cdots, \qquad B\left(u_k(x),\frac{\partial u_k(x)}{\partial x}\right)=0, \tag{26}$$

Here $N_m(u_0(x),u_1(x)\cdots u_{m-1}(x))$ is the coefficient of p^m, in the expansion of $N\varphi(x,p)$.

$$N(\varphi(x,p,c_i))=N_0(u_0(x))+\sum_{m=1}^{\infty}N_m(u_0(x),u_1(x)\cdots u_m(x))p^m \tag{27}$$

The convergence of the Series in equation (23) depend upon the auxiliary constants $c_1,c_2,\cdots$

If it converges at $p=1$, then the mth order approximation u is

$$u(x,c_1,c_2,\cdots c_m)=u_0(x)+\sum_{i=1}^{m}u(x,c_1,c_2,\cdots c_i). \tag{28}$$

Inserting Eq. (28) into Eq. (18), the residual is obtained as:

$$R(x,c_i)=L(u(x,c_i))+g(x)+N(u(x,c_i)), \quad i=1,2\cdots m \tag{29}$$

Methods like Ritz Method, Method of Least Squares, Galerkin's Methods are used to find the optimal values of $c_{i,}$ $i=1,2,3,4,\cdots$ We apply the

Method of Least Squares in our problem as given below:

$$J(c_1,c_2,\cdots c_n)=\int_a^b(x,c_1,c_2,\cdots c_m)dx, \tag{30}$$

Where a and b used as a domain.

Auxiliary constants $(c_1,c_2,\cdots c_n)$ can be identified from:

$$\frac{\partial J}{\partial c_1}=\frac{\partial J}{\partial c_2}=\cdots=0. \tag{31}$$

Auxiliary constants are used in the final solution.

4.1 The OHAM Solution :

Write equations (14, 15) in standard form of OHAM we obtained zero, first and second component problems of velocity and temperature as

Zero Components Problem of velocity and temperature:

$$\frac{d^2u_0}{dx^2} = m, \tag{32}$$

$$\frac{d^2\Theta_0}{dx^2} = 0, \tag{33}$$

$$u_0(0) = \alpha - \Lambda\left(\frac{du_0}{dx} + 2\beta\left(\frac{du_0}{dx}\right)^3\right), \quad \frac{du_0(1)}{dx} = 0. \tag{34}$$

Using the initial/boundary condition (32) and (33), the solution for the zero order problem (35) and (36) is

$$u_0(x) = \left(\alpha + m\Lambda + 2m^3\beta\Lambda - mx + \frac{m}{2}x^2\right). \tag{35}$$

$$\Theta_0(x) = x \tag{36}$$

First Components Problem of velocity and temperature:

The first component of velocity and temperature distribution is

$$\frac{d^2u_1}{dx^2} = m(1+C_1) + \lambda C_1 u_0[x] - \frac{d^2u_0}{dx^2}(1-C_1) - 6\beta C_1\left(\frac{du_0}{dx}\right)^2\frac{d^2u_0}{dx^2}, \tag{37}$$

$$\frac{d^2\Theta_1}{dx^2} = -BC_3\left(\frac{du}{dx}\right)^2 - 2\beta BC_3\left(\frac{du}{dx}\right)^4 - \frac{d^2\Theta_0}{dx^2}(1-C_3), \tag{38}$$

From (17) the boundary condition, for n=1, is

$$u_1(0) = -\Lambda\left(\frac{du_1}{dx} + 6\beta\left(\frac{du_0}{dx}\right)^2\frac{du_1}{dx}\right), \qquad \frac{du_1(1)}{dx} = 0, \tag{39}$$

The solution is

$$u_1(x) = \begin{bmatrix} \Lambda\left(m\lambda\left(\frac{1}{3}-\Lambda\right) + 2m^3\beta(1+\lambda-4\lambda\Lambda) + \alpha\lambda(1+6m^2\beta) + 12m^5\beta^2(1-\lambda\Lambda)\right) \\ +\left(m\lambda\left(\Lambda-\frac{1}{3}\right) + \lambda(2m^3\beta\Lambda+\alpha) - 2m^3\beta\right)x + \left(\lambda\left(\frac{\alpha+m\Lambda}{2}\right) + m^3\beta(3-\lambda\Lambda)\right)x^2 \\ +\left(\frac{m\lambda}{6} - 2m^3\beta\right)x^3 + \left(-\frac{m\lambda}{24} - \frac{m^3\beta}{2}\right)x^4 \end{bmatrix} C_1. \tag{40}$$

$$\Theta_1(x) = Bm^2\begin{pmatrix} -\left(\frac{1}{4}+\frac{1}{3}m^2\beta\right)x + \left(\frac{1}{2}+\beta m^2\right)x^2 - \frac{1}{3}(1+4\beta m^2)x^3 + \left(\frac{1}{12}+\beta m^2\right)x^4 \\ -\frac{2}{5}\beta m^2x^5 + \frac{1}{15}\beta m^2x^6 \end{pmatrix} C_3 \tag{41}$$

The second component of velocity and temperature:

The second component of velocity and temperature is too large that we cannot write it.

The values of C_i for velocity distribution are $C_1 = -0.93953067$, $C_2 = 1.51993996$

The values of C_i for temperature distribution are

$C_1 = -1.540614962$, $C_2 = -5.417489411$, $C_3 = -0.003010829$, $C_4 = -0.000066636$

6. Formulation of Drainage Problem

Geometric problem cannot be change. In this problem the vertical belt is stationary and liquid draining down the belt due to gravity. Therefore, in eq (14) m is taken positive.

Boundary condition for electrically conducting drainage problem is as follows:

$$u = -\gamma T_{xy} \quad \text{at } x = 0,$$

$$\frac{du}{dx} = 0 \quad \text{at } x = \delta, \tag{42}$$

Using nondimensional variable the slip boundary condition for drainage problem become

$$u_n(0) = -\Lambda\left(\frac{du_n}{dx} + 2\beta\left(\frac{du_n}{dx}\right)^3\right), \quad \frac{du_n(1)}{dx} = 0, \quad n \geq 0. \tag{43}$$

6.1: Solution of the Drainage Problem by OHAM.

Zero Component Problems: The velocity and temperature distribution are

$$\frac{d^2u_0}{dx^2} = -m, \tag{44}$$

$$\frac{d^2\Theta_0}{dx^2} = 0, \tag{45}$$

$$u_0(0) = -\Lambda\left(\frac{du_0}{dx} + 2\beta\left(\frac{du_0}{dx}\right)^3\right), \frac{du_0(1)}{dx} = 0, \tag{46}$$

The solution is

$$u_0(x) = m\Lambda - 2m^3\beta\Lambda + mx - \frac{m}{2}x^2, \tag{47}$$

$$\Theta_0(x) = x \tag{48}$$

First Component Problem: Consider

$$\frac{d^2u_1}{dx^2} = -m(1+C_1) + \lambda C_1 u_0[x] - \frac{d^2u_0}{dx^2}(1+C_1) - 6\beta C_1\left(\frac{du_0}{dx}\right)^2\frac{d^2u_0}{dx^2} \tag{49}$$

$$\frac{d^2\Theta_1}{dx^2} = -BC_3\left(\frac{du_0}{dx}\right)^2 - 2B\beta C_3\left(\frac{du_0}{dx}\right)^4 - \frac{d^2\Theta_0}{dx^2}(1+C_3) \tag{50}$$

$$u_1(0) = -\Lambda\left(\frac{du_1}{dx} + 6\beta\left(\frac{du_0}{dx}\right)^2\frac{du_1}{dx}\right), \qquad \frac{du_1(1)}{dx} = 0, \tag{51}$$

$$u_1(x)=\begin{pmatrix}\left(-\frac{m\Lambda\lambda}{3}-2m^3\beta\Lambda(1+\lambda-4\lambda\Lambda)-12m^5\beta^2\Lambda(1-\lambda\Lambda)+m\Lambda^2\lambda\right)\\+\left(m\lambda\left(\frac{1}{3}-\Lambda\right)+2m^3\beta(1-\lambda\Lambda)\right)x+\left(\frac{m\Lambda\lambda}{2}+m^3\beta(3+\Lambda\lambda)\right)x^2\\+\left(\frac{m\lambda}{6}+2m^3\beta\right)x^3+\left(\frac{m\lambda}{24}-\frac{m^3\beta}{2}\right)x^4\end{pmatrix}C_1 \tag{52}$$

$$\Theta_1(x)=\begin{bmatrix}-B\left(\frac{1}{4}m^2+\frac{1}{3}m^4\beta\right)x+B\left(\frac{1}{2}m^2+m^4\beta\right)x^2-\frac{B}{3}\left(m^2+4m^4\beta\right)x^3\\+B\left(\frac{1}{12}m^2+m^4\beta\right)x^4-Bm^4\beta\left(\frac{2}{5}x^5-\frac{1}{15}x^6\right)\end{bmatrix}C_3 \tag{53}$$

Second Component Problem. The boundary condition as

$$u_2(0)=-\Lambda\left(\frac{du_2}{dx}\left(1+6\beta\left(\frac{du_0}{dx}\right)^2\right)+6\beta\left(\frac{du_1}{dx}\right)^2\frac{du_0}{dx}\right),\qquad \frac{du_2(1)}{dx}=0 \tag{54}$$

The second component of velocity and temperature as too long which we cannot write it.

The values of C_i for velocity distribution are

$C_1=-0.4182785283,\qquad C_2=0.731930077.$

The values of C_i for temperature distribution are

$C_1=-3.594968907,\quad C_2=-17.767338517,\quad C_3=-0.049334554,\quad C_4=0.08273455.$

Table 1: The comparison between present work and published work for lift profile when the other parameters are $\alpha=0.1,\ \beta=0.2,\ m=0.3,\ \Lambda=0.1,\ and\ \ \lambda=0$

x	Published	Present	Absolute Error
0	1.0303212	1.0303212	2.168404×10^{-19}
0.1	1.0026012	1.0026012	3.252606×10^{-19}
0.2	0.9776635	0.9776635	4.878909×10^{-19}
0.3	0.9555563	0.9555563	5.421011×10^{-19}
0.4	0.9363129	0.9363129	7.589415×10^{-19}
0.5	0.9199697	0.9199697	8.673617×10^{-19}
0.6	0.9065563	0.9065563	1.084202×10^{-18}
0.7	0.8960975	0.8960975	1.192622×10^{-18}
0.8	0.8886129	0.8886129	2.276824×10^{-18}
0.9	0.8841164	0.8841164	2.168404×10^{-18}
1	0.8826166	0.8826166	1.734723×10^{-18}

Table 2: The comparison between present work and published work for drainage profile when the other parameters are $\alpha=1,\ \beta=0.2,\ m=0.1,\ \Lambda=0.01,\ and\ \ \lambda=0$

X	Published	Present	Absolute Error
0	-0.00099956	-0.00099956	1.270549×10^{-21}
0.1	0.008466489	0.008466489	6.776264×10^{-21}
0.2	0.016942105	0.016942105	0
0.3	0.024425309	0.024425309	1.3552527×10^{-20}
0.4	0.030914344	0.030914344	1.3552527×10^{-20}
0.5	0.036440767	0.036440767	0
0.6	0.040904006	0.040904006	1.3552527×10^{-20}
0.7	0.044402264	0.044402264	2.7105054×10^{-20}
0.8	0.046901617	0.046901617	1.3552527×10^{-20}
0.9	0.048401467	0.048401467	5.4210109×10^{-20}
1	0.048401467	0.048401467	2.7105054×10^{-20}

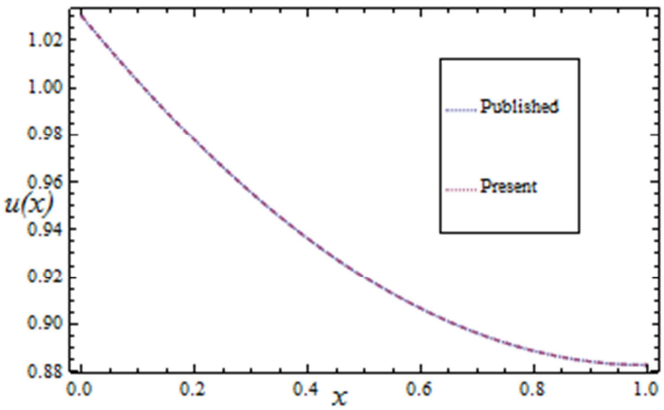

Figure 1: Comparison of lift profile for present work with published work in [1] when the other parameters are $\alpha = 0.1, \beta = 0.2, m = 0.3, \ \Lambda = 0.1, and \ \ \lambda = 0$

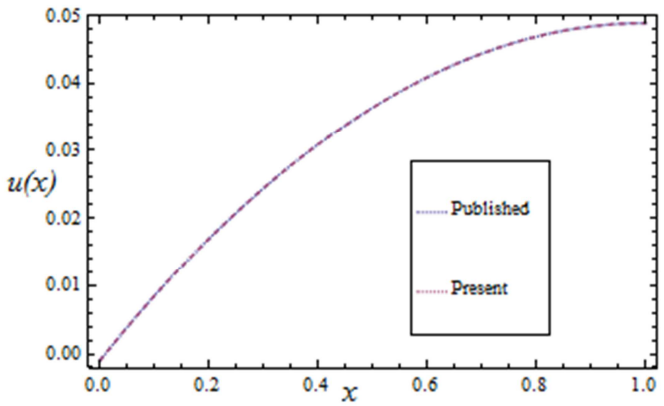

Figure 2: For drainage profile the comparison of present work with published work in [1] when $\alpha = 1, \beta = 0.2, m = 0.1, \ \Lambda = 0.01, and \ \ \lambda = 0$

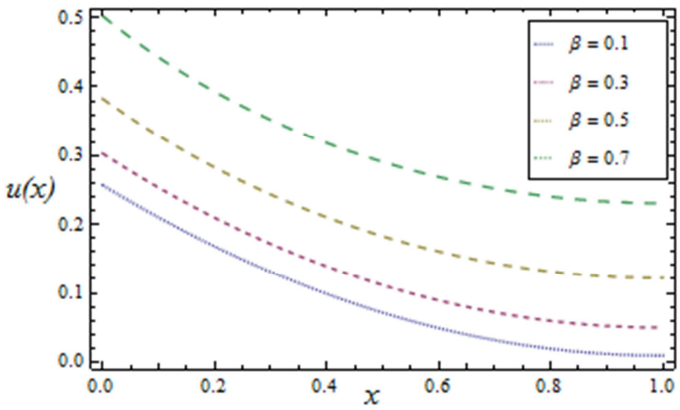

Figure 3: The variation of non-Newtonian "β" on velocity profile for lift problem by keeping the parameter fixed $\alpha = 0.1, \ m = 0.5, \ \Lambda = 0.3, and \ \ \lambda = 0.4$

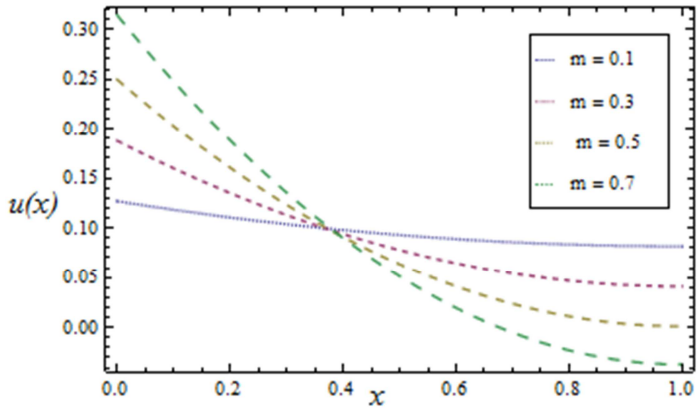

Figure 4: This figure shows the influence of the gravitational parameter "m" on the velocity profile "u(x)" for lift problem where the other parameters are $\beta = 0.02,\ \alpha = 0.1,\ \Lambda = 0.3, and\ \lambda = 0.2$

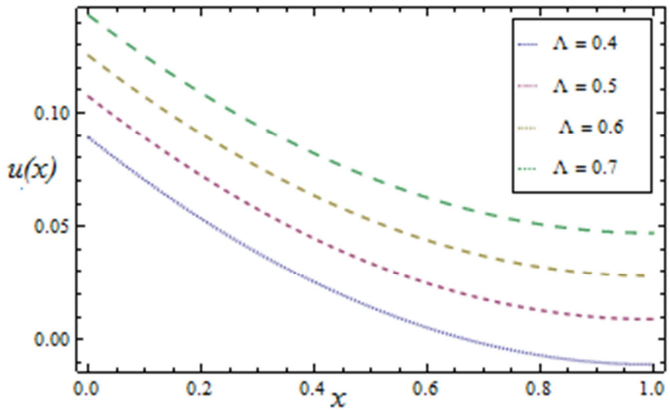

Figure 5: Shows the slip parameter " Λ " of lift velocity profile by keeping the other parameter fixed $\alpha = 0.01,\ \beta = 0.03,\ m = 0.2, and\ \lambda = 0.3$

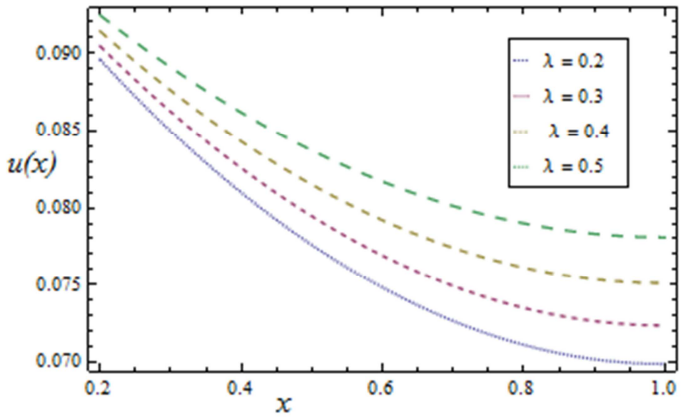

Figure 6: Effect of lift velocity profile on porosity "λ" by keeping the other parameter $\beta = 0.5,\ \alpha = 0.1, M = 0, m = 0.07, and\ \ \Lambda = 0.01$

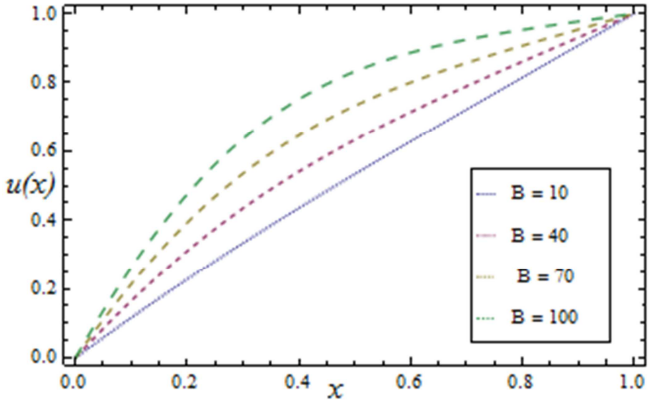

Figure 7: The figure shows the heat parameter "B" on temperature profile for lift problem by keeping other fixed $\alpha = 0.1, \beta = 0.6, m = 0.7, \Lambda = 0.8, and \ \lambda = 0.7$

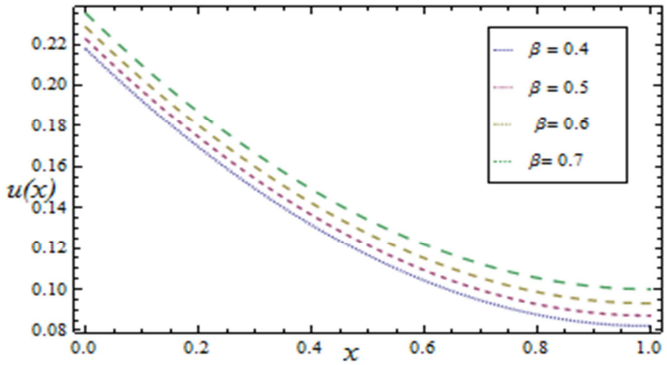

Figure 8: The influence of non-Newtonian β on velocity profile u(x) for drainage problem keeping $m = 0.3, \ \alpha = 0.1, \ \Lambda = 0.4, and \ \lambda = 0.6$

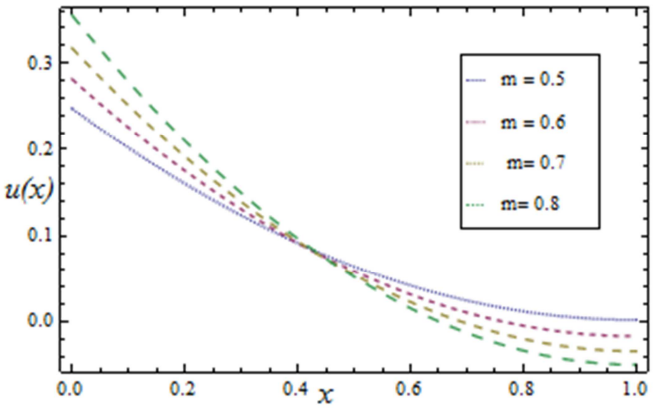

Figure 9: The figure shows the gravity effect of "m" on velocity profile for drainage problem, where $\beta = 0.04, \ \alpha = 0.1, \ \Lambda = 0.3, \ and \ \lambda = 0.5$

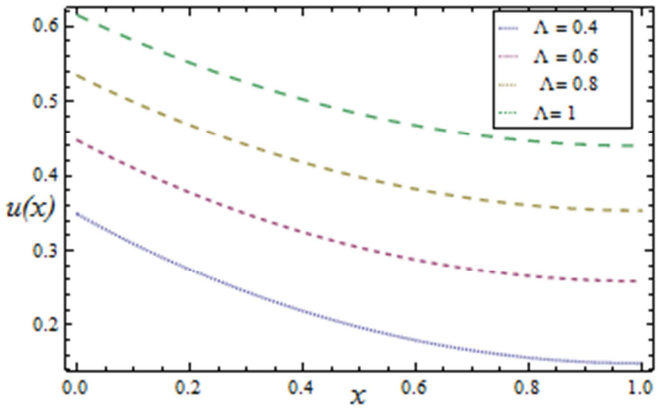

Figure 10: The effect of slip parameter " Λ " for drainage problem on velocity profile and by keeping other parameter $\beta = 0.6, \quad m = 0.4, \ \alpha = 0.1, and \ \ \lambda = 0.3.$

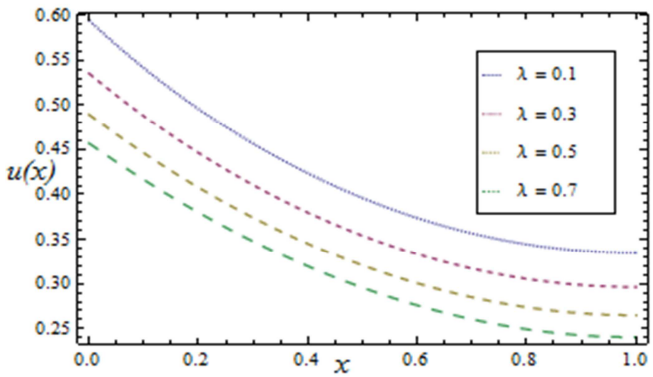

Figure 11: The effect of porosity "λ" on velocity profile for drainage problem and the other parameter are fixed which are $\beta = 0.4, \quad m = 0.5, \ \alpha = 0.1, \ and \ \ \Lambda = 0.6$

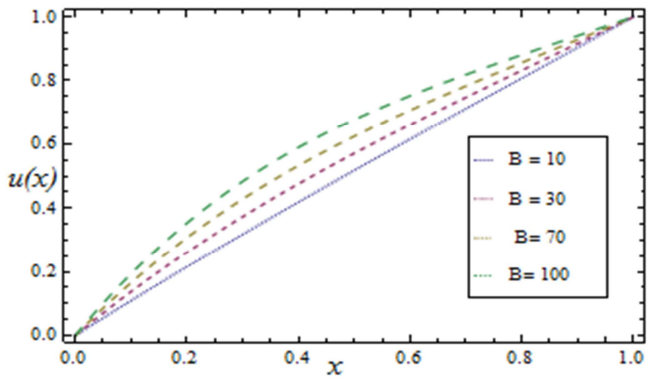

Figure 12: The figure shows the heat parameter "B" on temperature profile for drainage problem by keeping other fixed $\alpha = 0.1, \beta = 0.6, \ m = 0.6, \ \Lambda = 0.7, and \ \ \lambda = 0.9$

9. RESULTS AND DISCUSSION

Table 1 and 2 shows the absolute error in the comparison of present work with published work in [1]. The effects of non-Newtonian Parameter α and β, gravitational parameter *m*, with porosity λ, slip parameter Λ, B is the heat parameter for both lift and drainage Velocity profiles are plotted and discussed in Figures 1–12. Figure 1 and 2 shows the geometry of comparison of present work with published work in [1]. Figure 3 shows that the increase in

non-Newtonian parameter β for lift profile speeds up the velocity of the flow as we increase the value of parameter β. Figure 4 shows the gravitational parameter m for lift profile. When the value of m increases the velocity is decreases, while they intersect in the domain (0.2, 0.4) for some extant after that the velocity is decreases because gravitational force is small near the surface of belt. As we increase the value of slip parameter Λ, the friction goes on decreasing and the velocity of the fluid increases as shown in Figure 5. From figure 6 we see that the velocity of the fluid increases as we increase the value of the porosity parameter λ. Figure 7 shows that the motion of flow raises up is we increases the non-Newtonian parameter B. Figure 8 shows the non-Newtonian parameter β for drainage profile. The velocity profile differs little from the Newtonian one; however when β is increased, these profiles become more compressed showing the shear-thinning effect. The gravitational parameter m for drainage profile is shown in figure 9. The velocity decreases when we increases the gravitational parameter m and they intersect in the interval (0.4, 0.6). From figure 10 we see that is we increase the slip parameter, the friction force is less and the velocity increases. Figure 11 shows that in increase in porosity parameter λ decrease in velocity for drainage profile. From figure 12 if we increase the heat parameter there is increase in the heat distribution.

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