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Design of LQR Controller for a Nonlinear Maglev System

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ABSTRACT

This paper exhibits the design of a Linear Quadratic Regulator (LQR) controller for magnetic levitation system. The principle plan goal is to guarantee that the controller accomplishes steadiness and tracks a reference input. The design of controller includes obtaining state space model of the Magnetic Levitation system, which results in a nonlinear model. The open loop response of the system is simulated using MATLAB and it results in an unstable system. The LQR controller is designed which shows improved performance for different tracks. Furthermore, the difference among the different realization techniques has been analyzed in detail for rounding off error or truncation error and an optimal non fragile controller design has been presented. Different disturbances were imposed upon the simulated model. All the results are analyzed in open and closed loops.
KEY WORDS: Magnetic Levitation System, Linear Quadratic Regulator, Realization Techniques.

1. INTRODUCTION

MAGLEV is an arrangement of transportation that suspends, manages and propped vehicle, prevalently trains, utilizing magnetic levitation from a very large number of magnets for lift and drive. This technique can possibly be quicker, calmer and smoother than wheeled mass travel framework [1]. It is exceedingly nonlinear and open loop unstable system. This unstable aspect of Maglev and its innate non-linearity make the modeling and control issues extremely difficult. In the course of recent decades, the few control techniques utilizing both traditional and cutting edge plan strategies have been utilized and executed as a part of Maglev [2], [3], [4]. Magnetic levitation system not just displays testing issues for control building research, additionally have numerous pertinent applications, for example, fast transportation frameworks (Maglev trains) and attractive course [4]. From an instructive perspective, this procedure is exceedingly inspiring and reasonable' for research facility tests and classroom exhibits, as reported in the building training writing [3]. On the off chance that a question is set too far from the attractive source, the attractive field is excessively feeble, making it impossible to help the heaviness of the protest. On the off chance that set excessively near the attractive source, the attractive field turns out to be excessively solid and causes the protest, making it impossible to move towards the source until the point when it reaches the magnet [5], [6], [7].
The forces acting on an object under magnetic levitation are gravitational, electrostatic and magneto static fields which makes the system unstable. Effect of gravitational force on the object is countered by electromagnetic force. Maglev device is an example of an inherently unstable system. The major components of a MAGLEV system are shown in Fig. 1.

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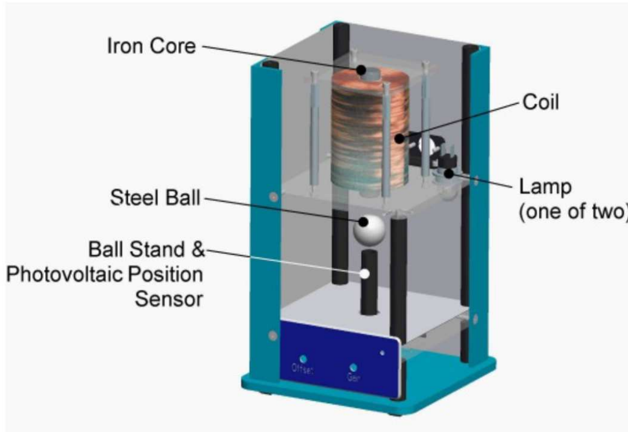

Fig. 1. MAGLEV System

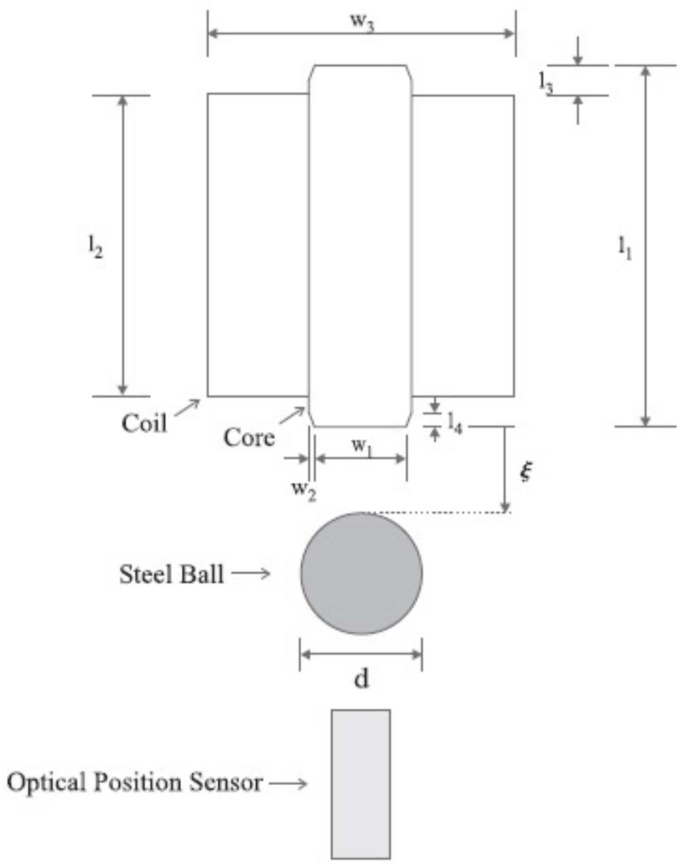

Fig. 2. MAGLEV Schematic

The MAGLEV system consists of iron core, coil, lamp, steel ball with stand and a photo voltaic position sensor as shown in Figure 1 and shown schematically in Figure 2. Using MATLAB, the system is modeled and operated to levitate the ball. Signals are sampled at a rate of 1ms. The system is a one-positional degree of freedom setup designed to levitate the stainless steel ball in between the free space of ball stand and iron core when a magnetic field which is created by supply of energy in the iron core. Space provided for this levitation is 14mm with a reference of 0mm on top of ball stand and 14mm at the bottom of iron core. Light sensor placed inside the ball stand measures the ball position for this setup. Here ξ represents the distance between

the coil face and ball with positive ξ values assumed in downward direction. This distance parameter has a range of ξ ∈ 2 [0, 14] mm.
There are certain techniques to stabilize such unstable system. One of the popular linearization technique is feedback linearization which has also been used to linearize the MAGLEV systems [8], [9]. In [10], the nonlinear dynamics were approximated by the use of Taylor's series expansion. A robust feedback linearization controller for an electromagnetic suspension system was presented in [11].
Valer and Lia manufacture a nonlinear model for attractive levitation framework and proposes frameworks linearization standard (the development in Fourier arrangement and the conservation of the main request terms) keeping in mind the end goal to linearize the procured nonlinear model [12].
In this paper, our main objective is to design an LQR controller for the MAGLEV system. The nonlinear and unstable nature of the system makes it a real challenge to control the system. These systems have unstable open loop response, to make the response of the system stable feedback path was used. Linear Quadratic Regulator Controller (LQR) was used to make the closed loop response of the system stable.

2. MODELLING OF THE SYSTEM

Dynamic equations describing the system are given by Faradays law of electromagnetics and forces balancing the part of electromagnetics. Let F (ξ, i) and mg denote the forces from magnetic field and gravity, respectively, then according to Faradays law of electromagnetics, we can write the following:

$$\frac{d\lambda(\xi,i)}{dt} = v - ir \tag{1}$$

where λ(ξ , i) is the magnetic flux linkage of the coil, v is the applied voltage and r is the total resistance. The total resistance can be expressed as $R = R_c + R_m$, where R_c is the resistance of the coil and R_m is the measurement resistance. The force balancing the electromagnetic can be expressed by the equation given as follows:

$$m\ddot{\xi} = F(\xi,i) + mg \tag{2}$$

We assume the system has no magnetic saturation and let W(ξ , i) denote the co-energy of the magnetic field, then we can write the following:

$$F(\xi,i) = \frac{\partial W(\xi,i)}{\partial \xi} = \int_0^i \frac{\partial \lambda(\xi,i)}{\partial x} \partial \zeta \tag{3}$$

For the experiment, no magnetic saturation is assumed and λ(ξ , i) = L(ξ) i with inductance

$$L(\xi) = \alpha + \frac{2\beta}{\xi + \kappa} \tag{4}$$

$$Li + Lm(\xi) \tag{5}$$

Here α, β and κ are positive constants and L1 and Lm are the leakage inductance and magnetizing inductance respectively. Co-energy and coil force are given as follows:

$$W(\xi,i) = L(\xi)i^2 \tag{6}$$

$$F(\xi,i) = \frac{\partial W}{\partial \xi}(\xi,i) = \frac{L'(\xi)i^2}{2} = -\frac{\beta i^2}{(\xi+\kappa)} \tag{7}$$

where $L^{/}$ is the derivative of L with respect to position. The state-space model of the system is as follows:

$$\dot{x} = f(x) + h(x)u \tag{8}$$

We can write the following

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{L(\xi)}(u - L'(\xi)(\bar{\xi})i \quad (9)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{\alpha\xi + 2\beta}\left(\bar{\xi}u + \frac{2\beta\dot{\xi}i}{\xi}\right) \quad (10)$$

$$\frac{\mathrm{d}^2\bar{\xi}}{\mathrm{d}t^2} = g - \frac{\beta i^2}{m\bar{\xi}^2} \quad (11)$$

$\xi = \bar{\xi} + \kappa$ and the input $u = v - ir$

Let $x = [x_1, x_2, x_3]^T = [i, \bar{\xi}, \dot{\bar{\xi}}]^T$ then the system modeled in Eqn.8 has the following values:

$$f(x) = \begin{bmatrix} \frac{2\beta x_1 x_3}{2\beta x_2 + \alpha x_2^2} \\ x_3 \\ g - \frac{\beta x_1^2}{m x_2^2} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \frac{x_2}{\alpha x_2 + 2\beta} \\ 0 \\ 0 \end{bmatrix}. \quad (12)$$

3. PARAMETER IDENTIFICATION

In parameter identification, the main task was to identify the values of the parameters α, β and κ. The remaining parameters e.g m, R and g are used as available from Quanser MAGLEV manual. In order to proceed with the parameter identification, calibration of the system was done carefully. The system was calibrated covering the physical unit to prevent light entering from outside. For identification of the parameter, a sine wave was given as input for the system to track. Frequency of the wave was 0.15Hz. By using least square fit around the equilibrium point, the values of β and κ were calculated in MATLAB using the function lsqcurvefit.m. This function solves a nonlinear problem to identify coefficients β and κ. The identified and typical values obtained from manual are given in Table I.

Table I
PARAMETER VALUES USED IN THE MODEL

Parameter	Value
m	0.068 kg
g	9.8 m/sec2
β	74000
κ	3.4 mm
α	0.4 H
R	11 Ω

4. LINEARIZATION

In order to perform the linearization of the system the Jacobian were calculated which is expressed by the Eqn.13

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{2\beta x_3}{2\beta x_2 + \alpha x_2^2} & -\frac{2\beta x_1(2\beta + 2\alpha x_2)x_3}{(2\beta x_2 + \alpha x_2^2)^2} & \frac{2\beta x_1}{2\beta x_2 + \alpha x_2^2} \\ 0 & 0 & 1 \\ -\frac{2\beta x_1}{m x_2^2} & \frac{2\beta x_1^2}{m x_2^3} & 0 \end{bmatrix}, \quad \frac{\partial g(x)}{\partial x} = [0 \quad 1 \quad 0] \quad (13)$$

Performing the linear approximation of the system modeled by the equation given above, about the equilibrium point,

$$i = i_0, \xi = \bar{\xi}, \dot{\bar{\xi}} = 0, \bar{\xi_0} = i_0\sqrt{\frac{\beta}{mg}}$$

The linearized model of the system is given below,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Where,

$$A = \begin{bmatrix} 0 & 0 & \frac{2\sqrt{mg\beta}}{\xi_0 L(\xi_0)} \\ 0 & 0 & 1 \\ -\frac{2}{\xi_0}\sqrt{\frac{g\beta}{m}} & \frac{2g}{\xi_0} & 0 \end{bmatrix} B = \begin{bmatrix} \frac{1}{L(\xi_0)} \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} D = [0]$$

This model is used for the LTI Controller design

FEEDBACK CONTROL WITH VOLTAGE AS INPUT

5.1 *Feedback Tracking Controller for the Nonlinear Model*

The nonlinear model of the system in control affine from is expressed as follows:

$$\dot{x} = f(x) + g(x)uy = h(x) \quad (15)$$

$$f(x) = \begin{bmatrix} \frac{2\beta x_1 x_3}{2\beta x_2 + \alpha x_2^2} \\ x_3 \\ g - \frac{\beta x_1^2}{m x_2^2} \end{bmatrix}, g(x) = \begin{bmatrix} \frac{x_2}{2\beta + \alpha x_2} \\ 0 \\ 0 \end{bmatrix}, h(x) = x_2 \quad (16)$$

In order to calculate the relative degree of the system, the following calculations were made

$$h(x) = x_2 \quad (17)$$

$$L_g h = 0 \quad (18)$$

$$L_f h = x_3 \quad (19)$$

$$L_g L_f h = 0 \quad (20)$$

$$L_g L_f^2 h = -\frac{2\beta x_1}{m x_2(\alpha x_2 + 2\beta)} \quad (21)$$

From the above equation, it can be seen that the relative degree of the system is $\rho = 3$ provided that $x2 \neq 0$ and $x1 \neq 0$. To write the system in tracking form according to M.T. Lemma the following calculations were performed

$$h(x) = x_2 \quad (22)$$

$$L_f h = x_3 \quad (23)$$

$$L_f^2 h = g - \frac{\beta x_1^2}{m x_2^2} \quad (24)$$

$$L_f^3 h = \frac{2\beta x_1^2 x_3}{m x_2^2} - \frac{4\beta^2 x_1^2 x_3}{m x_2^2(\alpha x_2^2 + 2\beta x_2)} \quad (25)$$

So, the system in tracking form can be expressed as

$$\dot{z_1} = z_2 \quad (26)$$

$$\dot{z_2} = z_3 \quad (27)$$

$$\dot{z_3} = L_f^3 h + u L_g L_f^2 h \quad (28)$$

$$y = z_1 \quad (29)$$

Where

$$z_1 = h(x) = y = x_2 \quad (30)$$

$$z_2 = L_f h = x_3 \quad (31)$$

$$z_3 = L_f^2 h = g - \frac{x_1^2 \beta}{m x_2^2} \quad (32)$$

$$y = z_1 \quad (33)$$

The state feedback can be expressed as

$$u = \frac{-L_f^3 h + v}{L_g L_f^2 h} = \frac{x_1 x_3 \alpha}{x_2} + \frac{mvx_2(x_{2\alpha+2\beta})}{2x_1\beta} \quad (34)$$

Where

$$v = -k_1(z_1 - yr) - K_2(z_2 - \dot{y}_r) - K_3(z_3 - \ddot{y}_r) + \dddot{y}_r \quad (35)$$

$$v = -k_1(x_2 - y_r) - k_2(x_3 - \dot{y}_r) - k_3(g - \ddot{y}_r - \frac{x_1^2\beta}{mx_2^2}) + \dddot{y}_r \quad (36)$$

The domain of the diffeomorphism is the whole operating range of Maglev.

5.2 *Feedback Tracking Controller for the Linearized Model*

For the linearized model of the system with ξ_0= 7mm, the following values were obtained as given by

$$\dot{x} = \begin{pmatrix} 0 & 0 & 0.0052 \\ 0 & 0 & 1 \\ -3.255*10^4 & 0.28028*10^4 & 0 \end{pmatrix} x + \begin{pmatrix} 2.348*10^{-6} \\ 0 \\ 0 \end{pmatrix} u \quad (37)$$

The poles of the linearized model expressed by the above equation are located at [-51.31,0, 51.31] the tracking control law for the system is given by the equation as follow

$$u = -\frac{2x_3\sqrt{gm\beta}}{\xi_0} + \frac{L(\xi_0)(2gx_3 - v\xi_0)}{2\sqrt{\frac{g\beta}{m}}} \quad (38)$$

A state feedback controller was designed to put the closed loop poles at [-50,-2,-49] and the feedback gains were calculated as shown by the equation given below

$$K = 10^7 * \begin{bmatrix} 4.2997 & -0.37658 & -0.00690 \end{bmatrix} \quad (39)$$

5. CONTROLLER DESIGN

When we give voltage without the designing of a controller the steel ball will either fall down or stick to the electromagnet, so in this paper we have designed a LQR controller which successfully control the voltage given to the system and also control the ball. The open loop vs closed loop response of the system is given in the Figure 3.

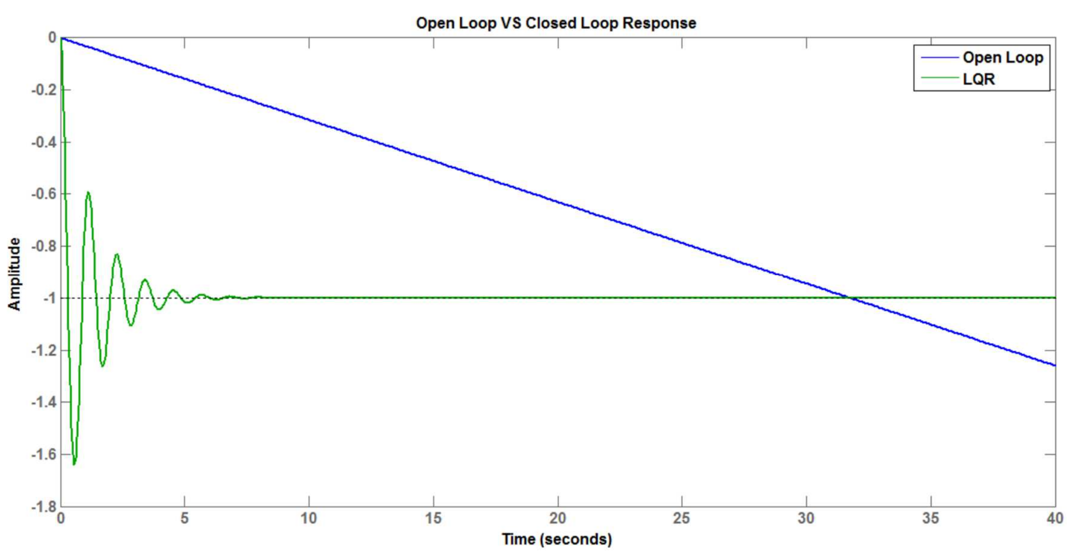

Fig. 3. Open loop VS Close loop Response

The close loop response of the system consists of the oscillation in order to reduce the oscillation and to make the system stable, the gain of the system is increased.
By increasing the gain of the system the system gives better response and settled the oscillations quickly as shown in the Figure 4.

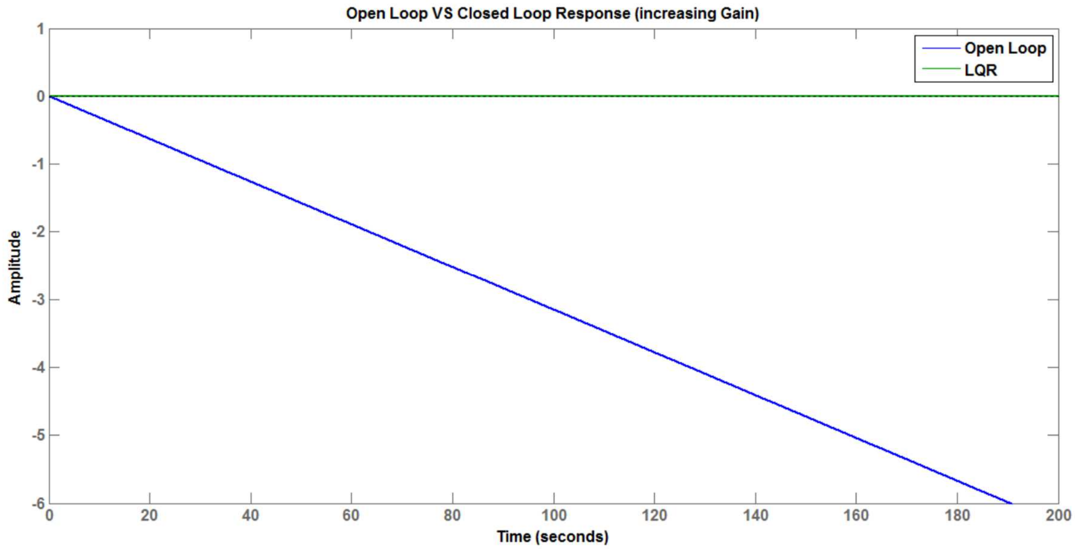

Fig. 4. Open loop VS Close loop Response with Increase Gain

In order to obtain a non-fragile optimal controller different realization techniques are used. Minimal realization (The realization is known as "minimal" as it defines the system with least number of states) Balanced realization, Modal realization and Observer based canonical realization are the other different techniques used to obtain a reduced and non-fragile model.

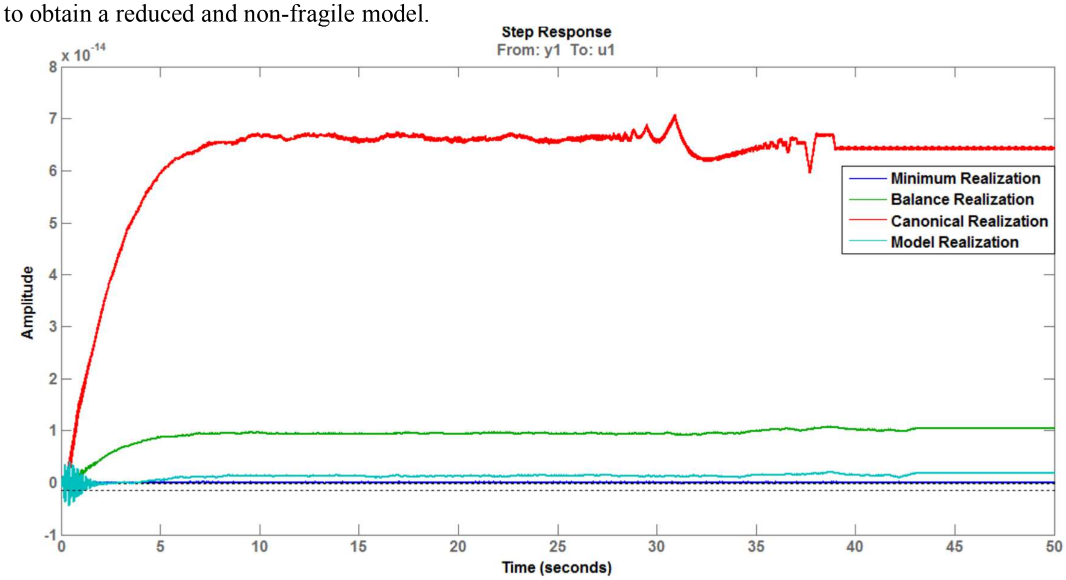

Fig. 5. Realization Technique

In the Figure 5 different types of realization techniques are compared, by applying these techniques the controllers action is made more efficient and the system is made highly stable and non-fragile.

TABLE II
REALIZATION ANALYSIS FOR DIFFERENT CONTROLLERS

Realization Type	LQR Controller
Minimal Realization	$0.2 * 10^{-16}$
Balanced Realization	$8.29 * 10^{-16}$
Observer based canonical Realization	$3.9 * 10^{-16}$
Modal Realization	$3.84 * 10^{-16}$

A brief summary of all types of realization techniques is shown in the Table II. This table shows that Minimal realization gives the least error to the controller.

6. CONCLUSION

The LQR Controller gives the better reaction for various Input unsettling influences. The LQR controller settles the motions all the more rapidly, diminishing the swaying and overshoot as appeared in the Figure along these lines composed LQR controller gives better dealing with capacity to extensive variety of unsettling influences. Also Minimal realization technique gives the least error to the controller which represents the most optimal and most non-fragile optimal controller technique.

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