J. Basic. Appl. Sci. Res., 4(9)72-83, 2014
© 2014, TextRoad Publication

ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Positivity Preserving Using GC^1 Cubic Ball Interpolation

Samsul Ariffin Abdul Karim

Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS,
Bandar Seri Iskandar, 31750 Tronoh, Perak Darul Ridzuan, Malaysia

Received: July 21, 2013
Accepted: August 7, 2014

ABSTRACT

This paper studies the use of Cubic Ball interpolation for shape preserving of positive data sets. The parameters, r_i in the description of the cubic interpolant are subject to constraint hence its values will determine the existence of positive interpolant. The sufficient condition for preserving positive data will be developed. The degree smoothness attained is GC^1 (geometric continuity) which is more relaxed compared to the parametric continuity C^1 . Several numerical results with comparison to the existing scheme will be presented to test the applicability of the proposed scheme.

KEYWORDS: Shape preserving; Interpolation; Cubic Ball function; Continuity.

1. INTRODUCTION

Shape preserving data interpolation is an active research in Computer Graphics (CG) and Computer Aided Geometric Design (CAGD). For example when the given data are positive, then it is necessary the resultant interpolant also positive, indeed the positivity of the data sets will be preserved everywhere. This reason could also be applied to the convex and monotone data sets. One of the main objectives of shape preserving is to remove the noise or any oscillatory that exists in the original data which is normally being taken from experiment. The cubic spline interpolation is unable to preserve the shape of the data. There exist various papers in shape preserving interpolation for scalar data. For example Sarfraz (2000), Sarfraz (2002), Sarfraz (2003), Sarfraz et al. (2001), Hussain and Sarfraz (2008) studied the use of rational cubic interpolant (cubic numerator and cubic denominator) for preserving the positive, monotone and positive data meanwhile Sarfraz et al. (2010) studied positivity preserving for curves and surfaces by utilizing rational cubic spline with quadratic denominator. The authors derived the sufficient conditions for monotone, convex and positive on the shape parameters in the description of rational interpolant. The methods work well and maintained the geometric properties of the given scalar data sets. Schmidt and Hess (1988) have derived the necessary and sufficient conditions for positivity of cubic and quadratic polynomials. Butt and Brodlie (1993) have used cubic polynomials for positivity preserving by inserting one or two extra notes in interval where shape violation are found. Having inserting extra knots, the computation will be increased, hence is not the best way to assists user to control the shape of the data sets.

Sarfraz et al. (2005) has proposed shape preserving GC^1 cubic spline interpolation for positive, convex and monotone data. Motivated by Sarfraz et al. (2005), in this paper, the Cubic Ball function will be used in place of cubic spline function. One of main advantages of our proposed Cubic Ball interpolant compare to Sarfraz et al. (2005) is that the Cubic Ball interpolant could easily be reduced to quadratic spline interpolation that have been discussed in details by Schumaker (1983) and Lam (1990). The main difference between the proposed Cubic Ball interpolant with Fristch and Carlson (1980) and Fristch and Butland (1984) is that by using GC^1 continuity, there is no restriction to the first derivative for the shape preserving interpolation while in Fristch and Carlson (1980) and Fristch and Butland (1984), the first derivative need to be modified in the region in which the resulting cubic polynomial have C^1 continuity (positive and monotone). Similar to the work by Sarfraz et al. (2005), our proposed Cubic Ball interpolants, no extra knots need to be inserted in interval in which the shape violations are found. Another main feature of our purposed Cubic Ball interpolant is that the sufficient conditions proposed in this paper are different than the sufficient condition obtained in Safraz et al. (2005). Indeed, our proposed positive conditions give visual pleasing results as compared with Safraz et al. (2005) method. Results for monotonicity preserving using Cubic Ball interpolant with GC^1 continuous can be found in Karim (2013) and positivity preserving by using GC^1 rational quartic spline is discussed in details by Karim et al. (2013). This paper is devoted to the positivity preserving by using GC^1 cubic Ball interpolation. The main scientific contribution this paper is as follows:

(i) In this paper cubic Ball basis function has been used for positivity preserving while in Sarfraz et al. (2005) cubic spline was used and in Sarfraz et al. (2010) the rational cubic spline with quadratic denominator has been used for positivity preserving.
(ii) The sufficient condition for positivity is different from the sufficient condition in Sarfarz et al. (2005). Indeed, our proposed Ball cubic interpolation guarantee the existence of GC^1 cubic curves that interpolate all the given data sets.
(iii) Numerical comparison between the proposed Ball cubic interpolation and Sarfraz et al. (2005) also has been given.

***Corresponding Author:** Samsul Ariffin Abdul Karim, Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak Darul Ridzuan, Malaysia.

Both interpolant have GC^1 continuity. Whereas, rational cubic interpolant in Hussain and Sarfraz (2008) has C^1 continuity.

(iv) No extra knots are needed in order to preservers the positivity of the data, while in Butt and Brodlie (1993) the positivity is preserves by inserting extra knots between any two knots in the interval in which the shape violation are found.

(v) The work in this paper focusing on positivity preserving while in Shukri et al. (2012) the cubic Bézier curve were used for data constrained interpolation subject to circle, an ellipse and straight line.

For more details on shape preserving interpolation and approximation, the readers can refer to Goodman (2002), Kvasov (2000), Sarfraz (2008), Hussain and Sarfraz (2008) and Sarfraz et al. (2010) and the references cited therein. The remainder of the paper is organized as follows. Section 2 introduces the cubic Ball polynomial, Section 3 discusses the Cubic Ball interpolant and the determination of first derivatives and Section 4 discuss the proposed interpolant in the context of positivity preservation together with the implementation algorithm. Results and Discussion will be discussed in Section 5. A summary and conclusions are given in Section 6.

2. Cubic Ball Polynomial

Cubic Ball Curves

Given a set of points on the plane, $V_i, i = 0,1,2,3$ the Cubic Ball curve is defined by (Ball, 1974)

$$B(t) = \sum_{i=0}^{3} V_i \phi_i(t), \qquad 0 \le t \le 1 \tag{1}$$

where

$$\begin{cases} \phi_0(t) = (1-t)^2 . \phi_1(t) = 2t(1-t)^2 \\ \phi_2(t) = 2t^2(1-t), \phi_3(t) = t^2 \end{cases} \tag{2}$$

are the Cubic Ball basis functions (Ball, 1974).

For $0 \le t \le 1$ the basis function in (2) have the following two important properties:

- $\phi_i(t) \ge 0, \quad i = 0,1,2,3$ (positivity) (3)
- $\sum_{i=0}^{3} \phi_i(t) = 1$ (partition unity) (4)

The properties in eq. (3) and (4) implies that Cubic Ball curve, $B(t)$ is a convex combination of the control points V_i, meaning that the curve lies in the convex hull of the control polygon formed by joining the points V_i to V_{i+1} for $i = 0,1,2$. When $V_1 = V_2$, the Cubic Ball curve in eq. (1) reduced to following standard quadratic Bézier curve as shown below:

$$B_2(t) = \sum_{i=0}^{2} V_i B_i^2(t), \qquad 0 \le t \le 1 \tag{5}$$

where $B_i^2(t) = \binom{2}{i}(1-t)^{2-i} t^i$ is a Bernstein basis functions.

3. Cubic Ball Interpolant

Suppose that $\{ (x_i, f_i), i = 1,...,n \}$ is a given set of data points, where $x_1 < x_2 < ... < x_n$. Let $h_i = x_{i+1} - x_i$, $\Delta_i = \dfrac{(f_{i+1} - f_i)}{h_i}$ and a local variable, $\theta = \dfrac{(x - x_i)}{h_i}$ where $0 \le \theta \le 1$.

Now for $x \in [x_i, x_{i+1}], i = 1, 2, ..., n-1,$

$$s(x) = s(x_i + h_i\theta) \equiv S_i(\theta), \tag{6}$$

where

$$S_i(\theta) = A_0(1-\theta)^2 + 2A_1\theta(1-\theta)^2 + 2A_2\theta^2(1-\theta) + A_3\theta^2$$

To ensure the cubic function (6) be GC^1, the following interpolatory properties must be satisfied:

$$s(x_i) = f_i, \quad s(x_{i+1}) = f_{i+1},$$
$$s^{(1)}(x_i) = \frac{d_i}{r_i}, \quad s^{(1)}(x_{i+1}) = \frac{d_{i+1}}{r_{i+1}}, \tag{7}$$

where $s^{(1)}$ denotes derivative with respect to x and d_i denotes the derivative value which is given at the knot $x_i, i = 1,2,...,n$. The parameters r_i $(r_i > 0)$ will be constrained in order to generate the positive Cubic Ball interpolant on entire given interval $[x_i, x_{i+1}], i = 1,2,...,n-1$.

Hence, the unknowns $A_i, i = 0,1,2,3$ can be shown have the following values:

$A_0 = f_i, A_3 = f_{i+1},,$

$A_1 = f_i + \frac{h_i d_i}{2r_i}, A_2 = f_{i+1} - \frac{h_i d_{i+1}}{2r_{i+1}},$

Thus the Cubic Ball interpolant $S \in GC^1[x_1, x_n]$ in (6), can be rewritten as follows

$S(x) = S_i(\theta),$

where

$$S_i(\theta) = f_i(1-\theta)^2 + 2\left(f_i + \frac{h_i d_i}{2r_i}\right)\theta(1-\theta)^2 + 2\left(f_{i+1} - \frac{h_i d_{i+1}}{2r_{i+1}}\right)\theta^2(1-\theta) + f_{i+1}\theta^2 \tag{8}$$

Obviously when $r_i = 1$, the Cubic Ball interpolant (8) reduced to Cubic Ball polynomial in Hermite-like form with C^1 continuity which is in general not a shape preserving interpolant. The shape parameters r_i, $i = 1,2,...,n$ can be utilized in order to modify the shape of the interpolating curve.

In most applications, the first derivative $d_i, i = 1,2...,n,$ will not be given and it must be determined and estimated by using the method discussed in Delbourgo and Gregory (1985) and Sarfraz et al. (1997). In this paper, arithmetic mean method (AMM) will be used to estimate the first derivative value. This method is simple to use and provide a good results.

4. Positivity Preserving using Cubic Ball Interpolant

The Cubic Ball interpolant described in the previous section will not preserved the positivity of the data everywhere. This fact can be seen clearly in Figure 1 to Figure 3. In fact, the ordinary cubic spline also does not produce the positivity cubic interpolant as can be seen clearly in Figure 4 to Figure 6. In this section, the constraint will be derived into shape parameters r_i in which the positivity of the data will be preserved. The problem of positive interpolation can be summarized as follows: for a given sets of data points, $(x_i, f_i), i = 1,...,n$ with $x_1 < x_2 < ... < x_n$, and

$$f_1 > 0, f_2 > 0,..., f_n > 0 \text{ or } f_i > 0, i = 1,2,...,n, \tag{9}$$

Construct the cubic interpolant $S \in GC^1[x_1, x_n]$ which is positive on the whole given interval $[x_1, x_n]$. This idea can be expressed as follows

$$S > 0, \quad x_1 \le x \le x_n.$$

The main idea to use $S \in GC^1[x_1, x_n]$ cubic Ball interpolant for the positivity preservation of the data sets is to choose the suitable values of parameter $r_i, i = 1,2,...,n,$ in interval that the shape violation are found (in our case the positivity are not preserves). But how do we choose the right r_i's ? In the following paragraphs, sufficient conditions for the positivity of the cubic Ball interpolant will be derived by using the main result in Schmidt and Hess (1988). The Cubic Ball interpolant in (8) can be rewritten as follows

$$S_i(\theta) = a_i\theta^3 + b_i\theta^2 + c_i\theta + e_i, \tag{10}$$

where

$$a_i = \frac{h_i d_i + 2r_i f_i}{r_i} + \frac{h_i d_{i+1} - 2r_i f_{i+1}}{r_{i+1}},$$
$$b_i = \frac{-2h_i d_i + 3r_i f_i}{r_i} + \frac{-h_i d_{i+1} - 3r_i f_{i+1}}{r_{i+1}},$$
$$c_i = \frac{h_i d_i}{r_i},$$
$$e_i = f_i.$$

Now using the substitution $\theta = \frac{s}{1+s}, s \geq 0$, $S_i(\theta)$ can expressed as

$$U(s) = \alpha_i s^3 + \beta_i s^2 + \gamma_i s + \delta_i, \tag{11}$$

where

$$\begin{aligned} \alpha_i &= f_{i+1}, \\ \beta_i &= 3f_{i+1} - h_i d_{i+1}/r_{i+1},, \\ \gamma_i &= 3f_i + h_i d_i / r_i, \\ \delta_i &= f_i. \end{aligned}$$

From Proposition 2 in Schmidt and Hess (1988), for the strict inequality of positive data in (9), $S_i(\theta) > 0$ if $\left(S_i'(0), S_i'(1)\right) \in R_1 \cup R_2$

where

$$R_1 = \left\{ (a,b) : a > \frac{-3f_i}{h_i}, b < \frac{3f_{i+1}}{h_i} \right\},$$

$$R_2 = \left\{ \begin{array}{l} (a,b) : 36 f_i f_{i+1}\left(a^2 + b^2 + ab - 3(a+b)\Delta_i + 3\Delta_i^2\right) \\ +3\left(af_{i+1} - bf_i\right)\left(2abh_i - 3af_{i+1} + 3bf_i\right) \\ +4h_i\left(a^3 f_{i+1} - b^3 f_i\right) - a^2 b^2 h_i^2 > 0 \end{array} \right\} \tag{12}$$

where $a = S_i'(0)$, $b = S_i'(1)$.

Now, by differentiating $S_i(\theta)$, w.r.t. θ, the following results will be obtained:

$$S_i'(0) = \frac{-2f_i + 2\left(f_i + \frac{h_i d_i}{2r_i}\right)}{h_i},$$

$$S_i'(1) = \frac{2f_{i+1} - 2\left(f_{i+1} - \frac{h_i d_{i+1}}{2r_{i+1}}\right)}{h_i}$$

Thus the positivity of the Cubic Ball interpolant $S(\theta)$ required the following conditions:

$$\frac{-2f_i + 2\left(f_i + \frac{h_i d_i}{2r_i}\right)}{h_i} > \frac{-3f_i}{h_i},$$

$$\frac{2f_{i+1} - 2\left(f_{i+1} - \frac{h_i d_{i+1}}{2r_{i+1}}\right)}{h_i} < \frac{3f_{i+1}}{h_i}$$

Hence,

$$r_i > \frac{-h_i d_i}{3f_i}, \tag{13}$$

$$r_{i+1} > \frac{h_i d_{i+1}}{3f_{i+1}}. \tag{14}$$

Now, for $i = 2,3,...,n-1$ we will have two sets of parameter r_i that can be summarized as follows:

$$r_i > \text{Max}\left\{ -\frac{h_i d_i}{3f_i}, \frac{h_{i-1} d_i}{3f_i} \right\}, i = 2,3,...,n-1. \tag{15}$$

Equation (13), (14) and (15) can be summarized as Proposition 1 below.

Proposition 1 (Positivity of Cubic Ball Interpolant)

For a strictly positive data, the Cubic Ball interpolant defined over the interval $[x_1, x_n]$ is positive if in each subinterval $[x_i, x_{i+1}], i = 1,2,..., n-1$ the following sufficient conditions are satisfied:

$$r_1 > \frac{-h_1 d_1}{3 f_1},\ r_n > \frac{h_{n-1} d_n}{3 f_n}, \tag{16}$$

and

$$r_i > \text{Max}\left\{0, -\frac{h_i d_i}{3 f_i}, \frac{h_{i-1} d_i}{3 f_i}\right\}, i = 2,3,..., n-1. \tag{17}$$

Remarks 1: After the values of the shape parameters $r_i, i = 1, 2,..., n,$ is calculated using (16) and (17), it should be noted that if its values is equal to 1, then the interpolating curves C^1 continuous. Indeed, any C^1 curve is also GC^1 curve.

Remarks 2: The sufficient conditions in (16) and (17) are different from sufficient conditions in Safraz et al. (2005). This is one of the main finding in this paper. Furthermore, conditions in (16) and (17) can be rewritten as

$$r_1 = l_i + \text{Max}\left\{0, \frac{-h_1 d_1}{3 f_1}\right\},\ r_n = l_n + \text{Max}\left\{0, \frac{h_{n-1} d_n}{3 f_n}\right\}, \tag{18}$$

and

$$r_i = l_i + \text{Max}\left\{0, -\frac{h_i d_i}{3 f_i}, \frac{h_{i-1} d_i}{3 f_i}\right\}, i = 2,3,..., n-1. \tag{19}$$

Now by using (11) and applying the main results in Schmidt and Hess (1988), the following choices of the shape parameters r_i also provide the positive Cubic Ball interpolant.

Since $\alpha_i = f_{i+1} > 0,\ \delta_i = f_i > 0,$ the following holds

$$\beta_i > \alpha_i - 2\sqrt{\alpha_i \delta_i}, \text{ and } \gamma_i > \delta_i - 2\sqrt{\alpha_i \delta_i}. \tag{20}$$

This provides the following proposition.

Proposition 2: For a strictly positive data, the Cubic Ball interpolant defined over the interval $[x_1, x_n]$ is positive if in each subinterval $[x_i, x_{i+1}], i = 1,2,..., n-1$ the following sufficient conditions are satisfied:

$$r_1 > \text{Max}\left\{0, \frac{-h_1 d_1}{2 f_1 + 2\sqrt{f_1 f_2}}\right\},\ r_n > \text{Max}\left\{0, \frac{-h_{n-1} d_n}{2 f_n + 2\sqrt{f_{n-1} f_n}}\right\}, \tag{21}$$

and

$$r_i > \text{Max}\left\{0, \frac{-h_i d_i}{2 f_i + 2\sqrt{f_i f_i}}, \frac{h_{i-1} d_i}{2 f_i + 2\sqrt{f_{i-1} f_i}}\right\}, i = 2,3,..., n-1. \tag{22}$$

Similar to conditions (16) and (17) in Proposition 1, conditions in (21) and (22) can be rewritten as

$$r_1 = \lambda_1 + \text{Max}\left\{0, \frac{-h_1 d_1}{2 f_1 + 2\sqrt{f_1 f_2}}\right\},\ r_n = \lambda_n + \text{Max}\left\{0, \frac{-h_{n-1} d_n}{2 f_n + 2\sqrt{f_{n-1} f_n}}\right\}, \tag{23}$$

and

$$r_i = \lambda_i + \text{Max}\left\{0, \frac{-h_i d_i}{2 f_i + 2\sqrt{f_i f_i}}, \frac{h_{i-1} d_i}{2 f_i + 2\sqrt{f_{i-1} f_i}}\right\}, i = 2,3,..., n-1. \tag{24}$$

Thus, to preserve the positivity of the data sets, the sufficient conditions from Proposition 1 or Proposition 2 can be used.

Remarks 3: The results in Proposition 1 can also be obtained by using the fact that $\beta_i > 0$ and $\gamma_i > 0$.

Remarks 4: The sufficient condition in (23) and (24) provides smoother positivity interpolating curves as can be seen in the following section.

Remarks 5: Sufficient conditions in (23) and (24) are also valid for Sarfraz et al. (2005) method. Figure 10, Figure 14 and Figure 18 shows the results.

An algorithm to generate GC^1 positivity-preserving curves using the results in Proposition 1 or Proposition 2 is given as follows.

Algorithm for Positivity-preserving

1. Input the number of data points, n, and data points $\{x_i, f_i\}_{i=1}^{n}$.
2. For $i = 1,2,...,n$, estimate d_i using arithmetic mean method (**AMM**)
3. For $i = 1,2,...,n-1$
   - Define h_i and Δ_i
   - Calculate the shape parameter, r_i from Proposition 1 or Proposition 2
   - Calculate the inner control ordinates A_1 and A_2 and generate the piecewise interpolating positive curves using (8).

5. **RESULTS AND DISCUSSION**

In order to test the proposed Cubic Ball interpolant scheme with GC^1 continuity, three sets of data taken from Sarfraz et al. (2005; 2001) and Brodlie and Butt (1993) were used.

TABLE 1 a positive data from Sarfraz et al. (2005)

x	2	3	7	8	9	13	14
f	10	2	3	7	2	3	10

TABLE 2 A positive data from Butt and Brodlie (1993)

x	0	2	4	10	28	30	32
f	20.8	8.8	4.2	0.5	3.9	6.2	9.6

TABLE 3 A positive data from Sarfraz et al. (2001)

x	0	2	3	9	11
f	0.5	1.5	7	9	13

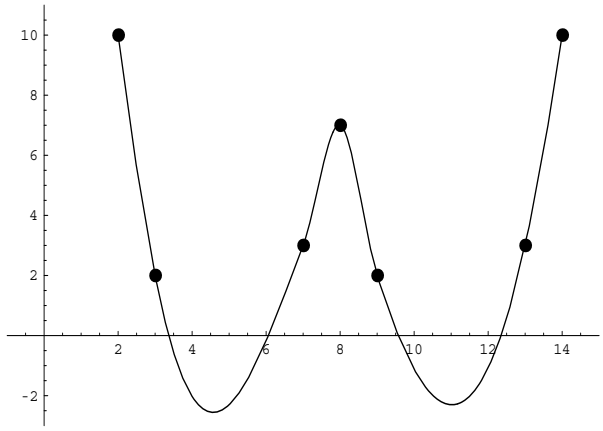

Figure 1- Default Cubic Ball interpolation for data in Table 1

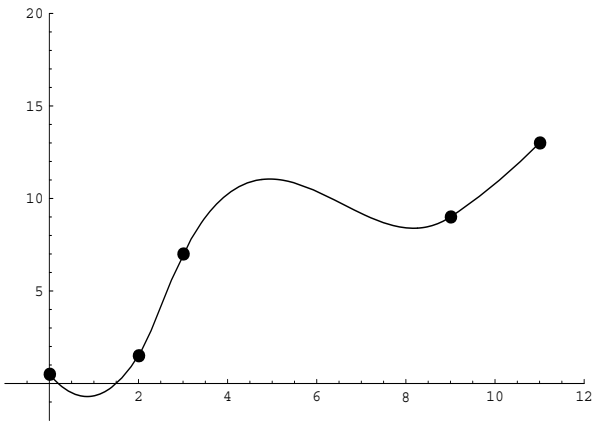

Figure 2- Default Cubic Ball interpolation for data in Table 2

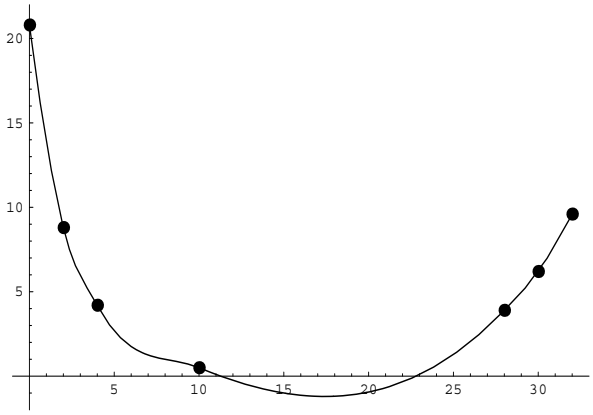

Figure 3- Default Cubic Ball interpolation for data in Table 3

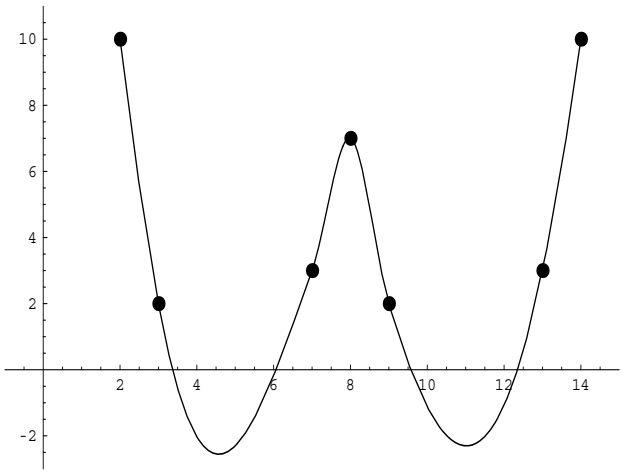

Figure 4- Default cubic spline interpolation for data in Table 1

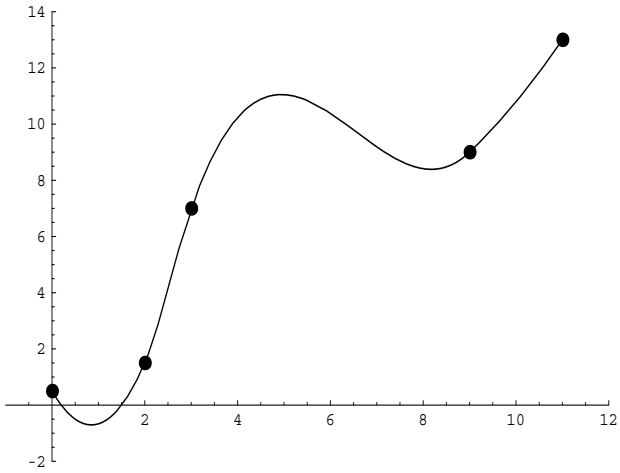

Figure 5-Default cubic spline interpolation for data in Table 2

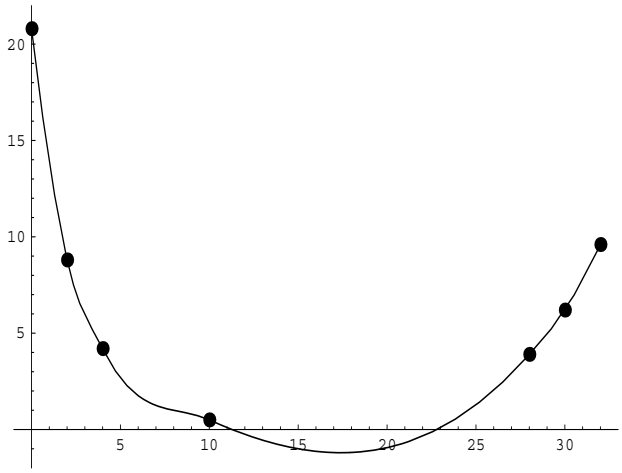

Figure 6- Default cubic spline interpolation for data in Table 3

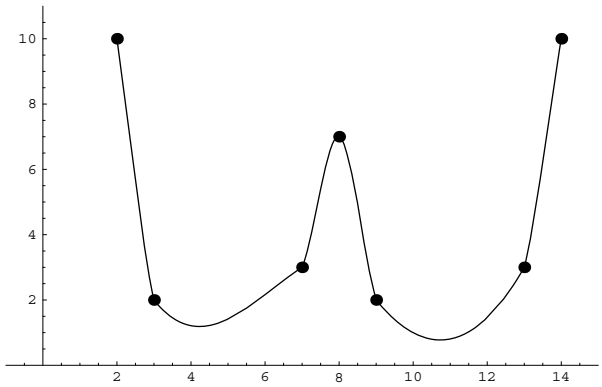

Figure 7- Shape preserving interpolation using Proposition 1 with $l_i = 1, 0.2, 1.2, 0.2, 1, 1, 1$ for data in Table 1

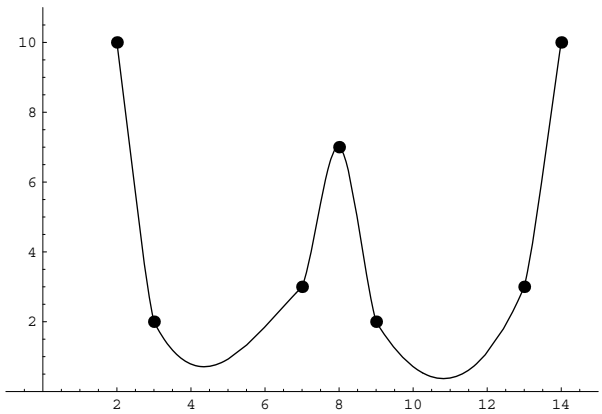

Figure 8- Shape preserving interpolation using Proposition 2 with $\lambda_i = 1, 0.2, 1.2, 1, 0.2, 1, 1$ for data in Table 1

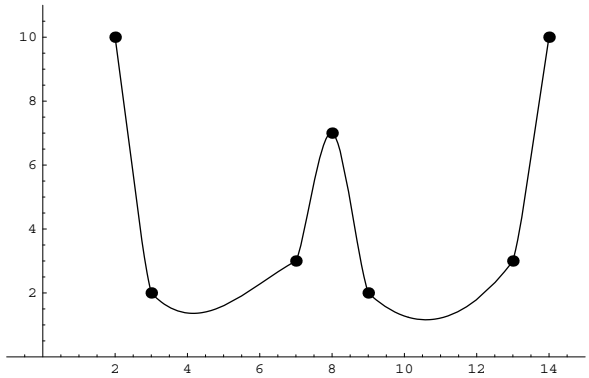

Figure 9- Shape preserving interpolation using Sarfraz et al. (2005) for data in Table 1

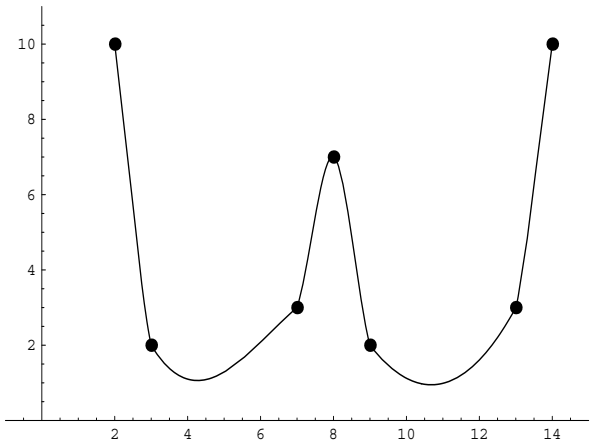

Figure 10- Shape preserving interpolation using Sarfraz et al. (2005) with proposed Proposition 2($\lambda_i = 1.1$) for data in Table 1

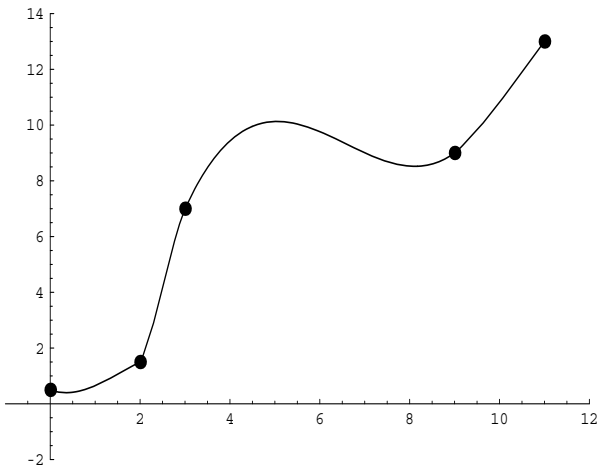

Figure 11- Shape preserving interpolation using Proposition 1 with $l_i = 1$, for data in Table 2

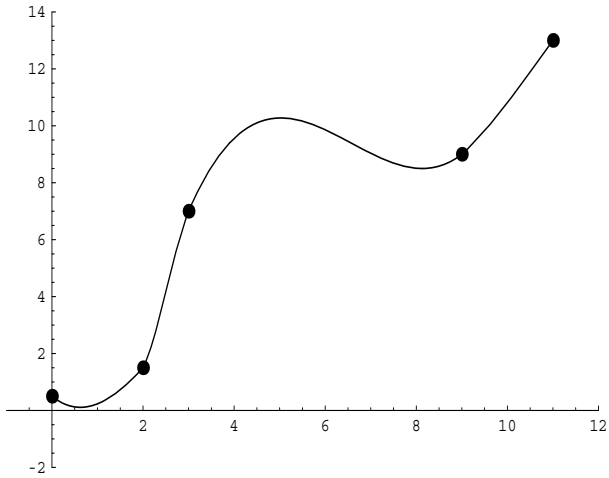

Figure 12- Shape preserving interpolation using Proposition 2 with $\lambda_i = 0.1,1,1,1,1$ for data in Table 1

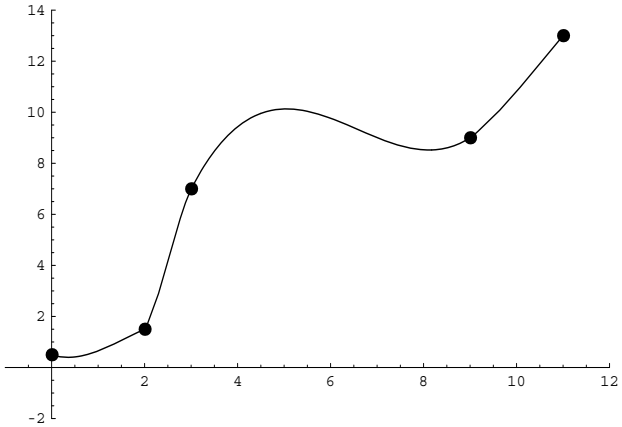

Figure 13- Shape preserving interpolation using Sarfraz et al. (2005) for data in Table 2

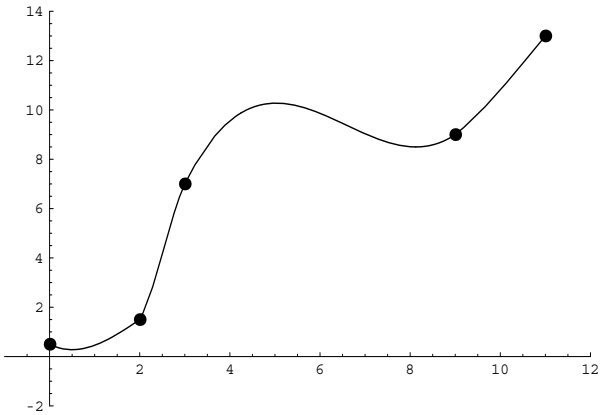

Figure 14- Shape preserving interpolation using Sarfraz et al. (2005) with proposed Proposition 2 ($\lambda_i = 1$) for data in Table 2

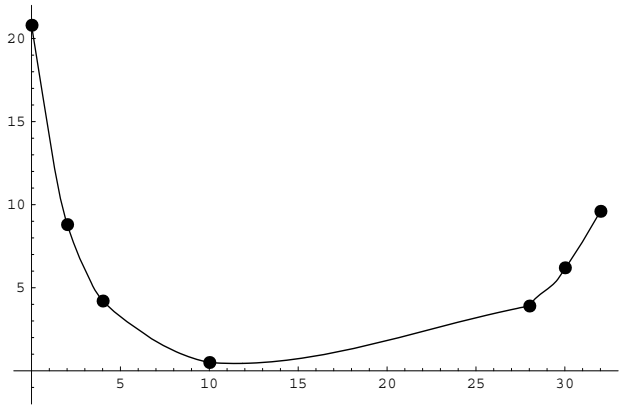

Figure 15- Shape preserving interpolation using Proposition 1 with $l_i = 1,1,1,0.2,0.5,1,1$ for data in Table 3

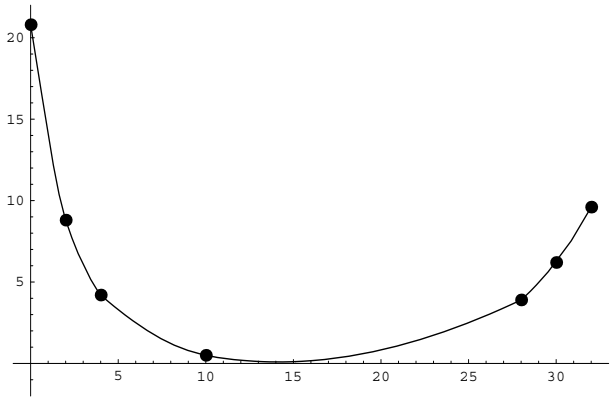

Figure 16- Shape preserving interpolation using Proposition 2 with $\lambda_i = 1,1,1,0.1,1,1,1$ for data in Table 3

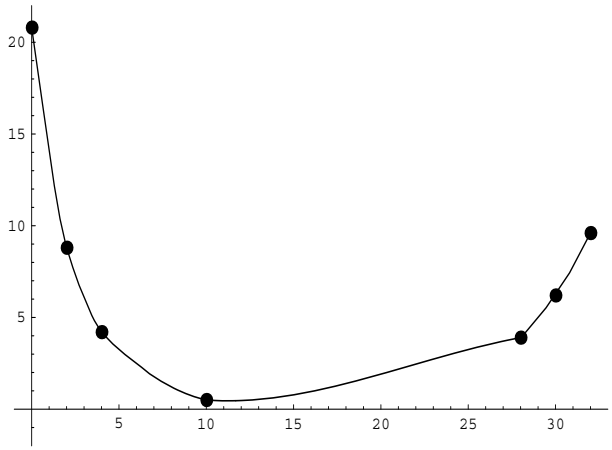

Figure 17- Shape preserving interpolation using Sarfraz et al. (2005) for data in Table 3

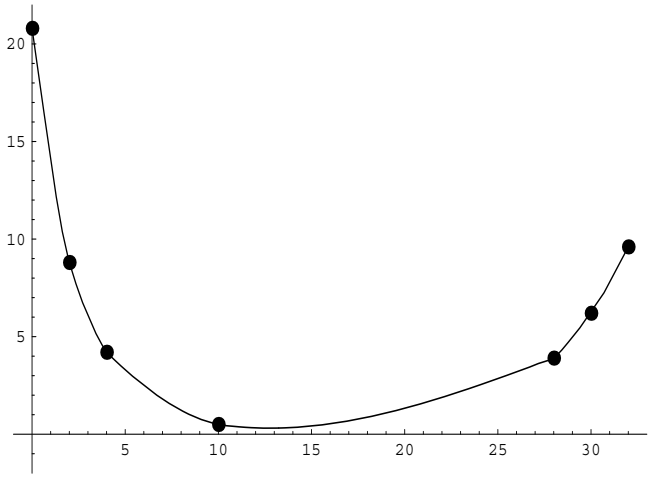

Figure 18- Shape preserving interpolation using Sarfraz et al. (2005) with proposed Proposition 2($\lambda_i = 1$) for data in Table 3

Figure 7 and Figure 8 shows shape preserving with proposed Cubic Ball interpolant with Proposition 1 and Proposition 2 respectively for data in Table 1. Figure 9 shows shape preserving using Sarfraz et al. (2005). Meanwhile Figure 10 shows shape preserving using Safraz et al. (2005) with our proposed Proposition 2. Figure 11 and Figure 12 show the results of shape preserving using Cubic Ball interpolant for data set in Table 2. Figure 13 and Figure 14 shows shape preserving using Sarfraz et al. (2005) for data sets in Table 2. Figure 15 and Figure 16 show shape preserving interpolation using Cubic Ball interpolant for data sets in Table 2. Finally, Figure 17 and Figure 18 show shape preserving using Sarfraz et al. (2005) for data sets in Table 3. Clearly from Figure 8, Figure 14 and Figure 16, shape preserving interpolation using Cubic Ball interpolant with Proposition 2 gives smoother results as compared to shape preserving interpolation using Proposition 1. Our proposed Proposition 2 also valid and it could be applied for shape preserving interpolation using Sarfraz et al. (2005) method. This fact can be seen in Figure 10, Figure 14 and Figure 18 respectively. Therefore, in this study, the main findings are given in Proposition 1 and Proposition 2 for Ball Cubic polynomial positivity. This differs from the sufficient condition in Sarfraz et al. (2005). Furthermore, Proposition 2 gives a smoother result and better than Proposition 1. Therefore, to preserves the shape of positive data, it is recommended that users use the scheme that proposed in this paper.

CONCLUSION

The work in this paper is concerned to the positivity-preserving for positive data sets using Cubic Ball interpolant. The degree of smoothness attained is GC^1. The shape constraints are restricted on shape parameters r_i to assure the positivity of the data are preserved everywhere. The results are comparable with works by Sarfraz et al. (2005). Proposition 1 and Proposition 2 guarantee the existence of positive GC^1 cubic Ball interpolant on $[x_1, x_n]$. Furthermore the sufficient conditions for positivity are different from sufficient condition in Sarfraz et al. (2005). The proposed method also provides greater flexibility to the user since there is no restriction to the first derivatives as compare to the works by Fristch and Carlson (1980) and no extra knots are required as appear in the work of Butt and Brodlie (1993). The work in this paper can be extended to the convexity preserving. The curve scheme can also be generalized to the surface cases. This will be the main focus for our future research.

Acknowledgments

The author would like to acknowledge Universiti Teknologi PETRONAS (UTP) for the financial support received in the form of a research grant: Short Term Internal Research Funding (STIRF) No. 76/10.11 and 35/2012.

REFERENCES

Ball, A.A. ,"CONSURF Part I: Introduction to Conic Lifting Title", Computer Aided Design, 6(4):243-249, 1974.

Butt, S. and Brodlie. K.W., "Preserving positivity using piecewise cubic interpolation", Computer and Graphics 17 (1):55-64, 1993.

Delbourgo, R. and Gregory, J.A. ,"The Determination of Derivative Parameters for a Monotonic Rational Quadratic Interpolant", IMA Journal of Numerical Analysis, 5:397-406, 1985.

Fritsch, F.N. & Carlson, R.E. ,"Monotone piecewise cubic interpolation" SIAM J. Numer. Anal. 17: 238-246, 1980.

Fritsch, F.N. & Butland, J. ,"A method for constructing local monotone piecewise cubic interpolants", SIAM J. Sci. and Statist. Comput. 5: 300-304, 1984.

Goodman, T.N.T. "Shape Preserving Interpolation by curves", in: J. Levesley, I.J. Anderson, J.C. Mason (Eds.), Algorithms for Approximation IV, University of Huddersfield Proceedings, Huddersfield, pp. 24-35, 2002.

Hussain, M.Z. and Sarfraz, M. "Positivity-Preserving interpolation of positive data by rational cubics", Journal of Computational and Applied Mathematics 218, pp. 446-458, 2008.

Karim, S.A.A. GC^1 Monotonicity Preserving using Cubic BallInterpolation. Australian Journal of Basic and Applied Sciences, 7(2): 780-790 (2013)

Karim, S.A.A., Kong, P.V. and Hashim, I. ,"Positivity Preserving using GC^1 Rational Quartic Spline", *AIP Conf. Proc.* 1522, 518-525 (2013); http://doi:10.1063/1.4801170.

Kvasov, B. I. 2000., "Methods of Shape-Preserving Spline Approximation", World Scientific, Singapore.

Lam, M.H. 1990. ,"Monotone and convex quadratic spline interpolation", Virginia Journal of Science. Volume 41(1): 1-13.

Sarfraz, M. "A rational cubic spline for visualization of monotonic data", Computers & Graphics 24(4):509-516, 2000..

Sarfraz, M. , Butt, S. and Hussain, M.Z. ,"Visualization of shaped data by a rational cubic spline interpolation", Computers & Graphics 25:833-845, 2001.

Sarfraz, M.,"Visualization of positive and convex data by a rational cubic spline interpolation", Information Sciences 146:1-4 pp. 239-254, 2002..

Sarfraz, M. ,"A rational cubic spline for the visualization of monotonic data: an alternate approach", Computers & Graphics 27: 107-121, 2003.

Sarfraz, M., "Interactive curve modeling: with applications to Computer Graphics", Vision and Image Processing. Springer-Verlag, London, 2008.

Sarfraz, M., Mulhem, M.A. and Ashraf, F., "Preserving Monotonic Shape of the Data Using Piecewise Rational Cubic Functions", Computer & Graphics 21(1):5-14, 1997.

Sarfraz, M., Hussain, M.Z. and Chaudary, F.S., "Shape Preserving Cubic Spline for Data Visualization", Computer Graphics and CAD/CAM 01 pp. 185-193, 2005.

Sarfraz, M., Hussain, M.Z. and Nisar. , "Positive data modeling using spline function", Applied Mathematics and Computation, 216, pp. 2036-2049, 2010.

Schmidt, J.W. and Hess, W. 1988., "Positivity of cubic polynomials on intervals and positive spline interpolation", BIT 28, 340-352.

Schumaker, L.L., "On shape preserving quadratic spline interpolant", SIAM J. Numerical Analysis 20:854-864, 1983.

Shukri, F.N.A., Abbas, M., Bashir, U., Awang, M.N.Hj., Jamal, E. and Ali, J.M., "Cubic Bézier Constrained Curve Interpolation", J.Basic. Appl. Sci. Res., 2(4), pp. 3682-3692, 2012