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Comparison between Stress Invariant and Fatigue Critical Plane Theories in Non Proportional Loading via Time Variant Critical Planes and Cycle Counting by Rain Flow Method

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ABSTRACT

Steel that is under cyclic stress is failed in lower stress than required continues stress for failing. A failure that is occurred under dynamic loading is named as fatigue failure. Up to now, comprehensive comparison is done between fatigue theories. In this paper results of these theories are compared and analyzed. Theories based on static failure criteria such as maximum shearing stress or tereska theory, octagon shearing stress or vonmisses theory and etc. are be reliable for proportional loading but the results have salient error for Nonproportional loading. Also theories based on stress invariant such as crossland theory, sines theory are evaluated, after that findley, McDiarmid, dang van and fatemi theories are analysis. In all cases Matlab codes are prepared. Many of the components are subjected to cyclic loading, the load range may be fixed or may vary relative to time. In cases where the stress range is fixed, counting the number of applied is straightforward, but the major problem occurs with variable amplitude stress.To estimate the fatigue life of a component, the stress and strain applied to the component must be decomposed to the cycle of stress and strain with fixed ranges in order to be able to use them in S-N diagram. To correlate S-N diagram of a material and stress variation history, it is required to use the appropriate method for counting the number of cycles corresponding to different levels of stress.Finally Critical plane methods with 1.17 average safety factor are better than Equivalent stress methods with 1.03 average safety factor, they show that theories based on static failure criteria and sines theory with Nonproportional loading are not inefficient and models such as dang van are complex but they are used in complex loading actually. Findley model have good accuracy in each loading.

KEYWORDS: Non Proportional Loading, Static Failure Criteria, Critical Plane, Cyclic Loading, Variable Amplitude Stress, Rain Flow Method.

INTRODUCTION

In most applicable cases, a static failure criterion isn't good choosing for fatigue manner description. The main defects of these theories are that for Nonproportional loading, they aren't useful and their results have significant errors [1].

Also, calculated equivalent stress mostly is positive. In other words, one dimensional negative stress is replaced with equal tensile stress. The basic assumptions in two parametric theories are experimental and consist of invariant stress and critical plane theories [2]. Crossland's results was presented in 50s last century. Sines was presented his results in same years [3]. Failure process in multi- axial fatigue conditions has the highest importance in automobile and airplane industries.

Fatigue manner in theories that based on invariant stress is described by using hydrostatic stress and second invariant tensor of deviatoric stress [4]. In critical plane theories the plane that its possibility of fatigue is higher, is considered, so the planes that their component of stress or combination of stress components are maximum, are evaluated. Mentioned theories are usually based on a stress or a strain definition [5].

With the development of fatigue life models, the different types of classification are introduced. In these methods, physical properties such as stress, strain, etc. must be evaluated during testing. Usually to calculate the normal stresses, an external load on the structure is considered during the time interval, but systems that have varying vibrational modes, amplitude and frequency should also be considered. The most simple laboratory test is stress - life calculation, which is produced by a sinusoidal load. However in reality, loading is happening quite arbitrarily [6, 7]. Therefore we have diverse information, which do not abide by any order.

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In some practical applications, a constant amplitude alternating loading is involved, but more often we are faced with, a history of irregular loading in time. For extreme changes of load with time, it is not clear how these specific cases should be distinguished, and it is not clear that how these cases could be defined as cycles. In cases where the stress level is fixed between a minimum and maximum, which is called fixed amplitude stress, there is no problem to count the cycles. The distance from a peak to the next peak is a cycle. But if one does this method for an irregular alternative loading, large errors may exist in the result.

In fact, there are small cycles, which cause most fatigue damages. Thus, not considering those cycles, results in inaccurate estimates. Cycles could be identified with a comprehensive approach which is called cycle counting.

In fact, the goal of cycle counting is the computation of damages magnitude, the Palmgren-Miner rule is used to estimate life time. palmgren stated the rule in Sweden in the 1920s to estimate the lifetime of spherical bearings, and later was adopted extensively by BF Langer in 1937. However, until this rule introduced in MA Miner article in 1945, it was completely unknown.

Matsuishi and Endo (1968) originally developed the rainflow cycle counting method based on the analogy of aindrops falling on a pagoda roof and running down the edges of the roof. A number of variations of this original scheme have been published for various applications. Among these variants are the three-point cycle counting techniques (including the SAE standard [Rice et al., 1997], the ASTM standard [ASTM E1049, 1985], and the range-pair method [Rice et al., 1997]) and the four-point cycle and counting rule (Amzallag et al., 1994)[8, 9].

After examining in a variety of studies, it presented to be used in diverse fatigue analysis [10]. Later it used by different researchers in automotive, aviation, energy, steel and etc.

In this study, equation forms in the highest cyclic fatigue are presented with higher effort. The problem with these theories is that they are used to determined limited or unlimited fatigue life. In this paper, the new approach is obtained for fatigue life determination under multi- axial loading to define equivalent stress. In previous researches, critical plane situation was fixed but in this paper, variability of critical plane angle is presented in each code.

The aim of the project was to count the cycles of an alternative stress loading with variable amplitude. After reception of tensions history, it will be studied and cycle count obtained as a result. Each of the calculated cycles can be inspected separately and another scope of this study was to compromised fatigue theories for Nonproportional loading. Finally according to sorting the most appropriate category of theories is introduced.

MATERIALS AND METHODS

The theories and its formulation:

Sines theory:

The theory that is based on stress invariant criteria, is suggested for loads that are consist of bending and torsion. Octahedral shearing stress was used as fatigue damage criteria [11].

Using this theory, static torsion didn't influence on fatigue limitations in cyclic bending or cyclic torsion states. Static tension and pressure affected on fatigue limitations in those states linearly. Furthermore average of hydrostatic stress had relation with fatigue limitation during cycle (Figure 1).

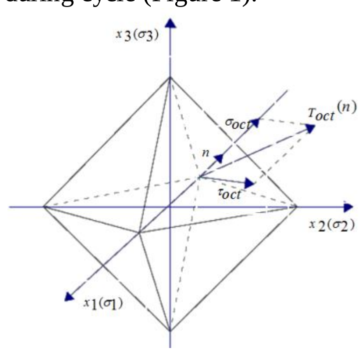

Figure 1 Octahedral plane

Finally, a failure criterion is expressed as follow (Eq. 1):

$$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta \quad \text{or} \; (\tau_{oct})_a + \alpha(3\sigma_h) \tag{1}$$

$\Delta\tau_{oct}$ is range of octahedral shearing stress, $(\tau_{oct})_a$ is amplitude of octahedral shearing stress and σ_h is hydrostatic stress that is calculated as follows (Eq. 2):

$\sigma_h = \frac{\sigma_X^{mean} + \sigma_y^{mean} + \sigma_z^{mean}}{3}$ (2)

α and β coefficients are obtained from experience.
These equations can be calculated as follows (Eq. 3):

$a_s \cdot (\sqrt{J_2})_a + b_s \cdot \sigma_{H,m} \le f_{-1}$ (3)

a_s And b_s coefficients are introduced from Eq. 4:

$a_s = \frac{f_{-1}}{t_{-1}}$ $b_s = 6 \cdot \frac{f_{-1}}{f_0} - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}}$ (4)

Crossland theory:
This theory also is based on invariant stress criteria. In this criterion, maximum of hydrostatic stress and second invariants of the stress deviator tensor are desired [12]. A failure criterion is introduced as follows (Eq. 5):

$a_c \cdot (\sqrt{J_2})_a + b_c \cdot \sigma_{H,\max} \le f_{-1}$ (5)

a_c and b_c are calculated form Eq. 6:

$a_c = \frac{f_{-1}}{t_{-1}}$ $b_c = (3 - \frac{f_{-1}}{t_{-1}})$ (6)

Fatemi theory:
Fatemi theory is one of the critical plane theory but it based on equivalent strain [13]. According to this theory, failure is begin when Eq. 7 is satisfied:

$\frac{\Delta\gamma}{2}\left(1 + k\,\frac{\sigma_{n,\max}}{\sigma_{yield}}\right) = \frac{\tau_f'}{G}(2 \cdot N)^{b\gamma} + \gamma_f'\,(2 \cdot N)^{c\gamma}$ (7)

According to done experiments, appropriate range for c and b are suggested as follows (Eq. 8) [14]:

$-0.14 < b < -0.06$ (8)
$-0.7 < c < -0.5$

Dang van theory:
This model is observed from this science that fatigue crack is begun from seeds with plastic deformation and shear stress is important parameter in crack beginning of seeds [15]. The simplest equation for damage value evaluation is suggested as follows (Eq. 9):

$\tau(t) + a\sigma_h(t) = b$ (9)

$\tau(t)$ and $\sigma_h(t)$ are shearing and hydrostatic microscopic stresses. a and b are equation constants.
Dang van equation can be written as follows (Eq. 10), (Eq. 11):

$a_{DV} \cdot C_a + b_{DV} \cdot \sigma_{H,\max} \le f_{-1}$ (10)

$a_{DV} = \frac{f_{-1}}{t_{-1}}$ $b_{DV} = 3 - \frac{3}{2} \cdot \frac{f_{-1}}{t_{-1}}$ (11)

Faindly theory:
The theory express that normal stress σ_n on a shearing plane can be affected on allowable shearing stress amplitude $\frac{\Delta\tau}{2}$. (Eq. 12)

$(\frac{\Delta\tau}{2} + k\,\sigma_n)_{\max} = f$ (12)

k and f parameters are obtained from experience. f parameter was named as damage parameter. Form of this criterion in 1 dimensional shearing stress is as follows (Eq. 13):

$\frac{\Delta\tau}{2} + k\,\sigma_n = \tau_f^*(2N_f)^b$ $\tau_f^* = \sqrt{1+k^2}\,\tau_f'$ (13)

The correction factor value $\sqrt{1+k^2}$ approximately is 1.04.

In this theory, applied stress on specific plane is considered inside part of material. These planes are named as critical planes that are one or more material inside planes with maximum damage criterion value [16].

Fatigue life time is controlled by the combination of applied stress and strain on critical planes. σ_θ and τ_θ stresses on a plane that are under θ angle can be calculated from Eq. 14 and Eq. 15:

$$\sigma_\theta = \frac{\sigma_X + \sigma_y}{2} + \frac{\sigma_X - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta) \quad (14)$$

$$\tau_\theta = -\frac{\sigma_X - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta) \quad (15)$$

θ Angle of critical plane is determined from $\sigma_n = \sigma_\theta$ and $\sigma_a = (\tau_a)_\theta$ replacement and phrase derivation $(\tau_a = k \cdot \sigma_n)$ equal to zero toward θ. Finally in pure torsion state (Eq. 16):

$$\sqrt{1+k^2}\,\frac{\Delta\tau}{2} = f \quad (16)$$

For bending or pure axial load it can be calculated similar to above (Eq. 17), (Eq. 18):

$$\sigma_\theta = \frac{\sigma}{2} + \frac{\sigma}{2}\cos(2\theta) = \sigma \cdot \cos^2\theta \quad (17)$$

$$\tau_\theta = -\frac{\sigma}{2}\sin(2\theta) \quad (18)$$

Generally, Eq. 19 can be obtained:

$$\sqrt{(\sigma_a)^2 + k^2(\sigma_{\max})^2} + k\,\sigma_{\max} = 2f \quad (19)$$

For ductile material, K is between 0.2 to 0.3 generally.

It expected that failure is occurred in plane that had high value of $\frac{\Delta\tau}{2} + k\sigma_n$. For specific states such as proportional loads with the same phases and critical plane determination with using maximum amplitude, principal stress values can be able in loading cycle.

McDiarmid theory:

In this theory, applied stress on specific planes is important. These planes are named as critical planes one or more material inside planes with maximum damage criterion value [17].

The criterion was based on shearing stress amplitude $\frac{\Delta\tau_{\max}}{2}$ on a plane that has maximum shearing stress, also normal stress $\sigma_{n,\max}$ is suggested on that plane (Eq. 20).

$$\frac{\Delta\tau_{\max}}{2\tau_{A_\vee B}} + \frac{(\sigma_n)_{\max}}{2\sigma_u} = 1 \quad \text{or} \quad \frac{(\tau_{\max})_a}{\tau_{A_\vee B}} + \frac{(\sigma_n)_{\max}}{2\sigma_u} = 1 \quad (20)$$

$t_{A_\vee B} \equiv (t_A \text{ or } t_B)$ that are related to shearing fatigue strength for A and B crack states. A type cracks grow on body area and length of those is along outside boundary. B type cracks are crack that grow from outside area to depth. σ_u is the ultimate tensile strength of material. This model is similar to findley method.

In findley equation, if k parameter replace with $\frac{t_{A_\vee B}}{2\sigma_u}$ outward form will similar with canonic form. The critical plane not only isn't a plane that has the highest damage parameter (f) but also is a plane that have the highest shearing stress. This plane is different from findley criterion plane in some cases.

In Figure 2, details of calculation process are described:

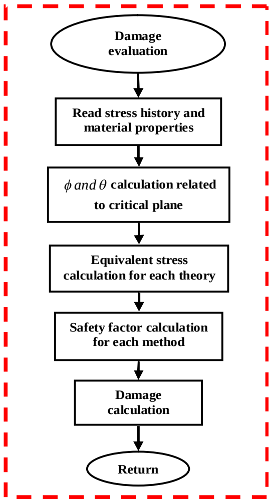

Figure 2. Details of algorithm and calculation process

Stress history that was used in this paper, are listed as follows (Table 1):

Table 1. Stress history in each period of time

	$\sigma_x(MPa)$	$\sigma_y(MPa)$	$\sigma_z(MPa)$	$\tau_{xy}(MPa)$	$\tau_{xz}(MPa)$	$\tau_{yz}(MPa)$
1^s	**20.1**	**11**	**-3.33**	**-23.48**	**41**	**-22**
2^s	**-3.91**	**-2**	**-27.44**	**14**	**-9**	**16**
3^s	**-15.3**	**-19**	**20**	**4.58**	**-13.2**	**33**
4^s	**10**	**37**	**14.22**	**-6.1**	**19**	**7**
5^s	**27.2**	**17**	**12.2**	**35**	**24**	**-13.8**

Critical plane determination

In findley method, critical plane is introduced as a plane that is maximized linear combination from shearing stress amplitude and maximum effective normal stress. From McDiarmid theory, critical plane is a plane that is maximized shearing stress amplitude. Direction of plane can be determined from its normal vector in spherical coordinate (θ_c, ϕ_c) .

If multi- axial stress is applied on a material element, according to critical plane definition in each theory, a critical plane will exist in each period of time but it didn't exist in specific plane, so in arbitrary loading, critical plane situation was not constant and it was function of time.

According to this subject, presented computer program calculated normal stress and shearing stress values using spherical coordinate with $0 \le \theta \le 360, 0 \le \phi \le 180$, so equivalent stress history can be obtained. Normal stresses in different planes are as follow (Eq. 21):

$$\sigma_n = \sigma_{yp} =$$

$$\sin^2\phi(\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta) \quad (21)$$

$$+\sin^2\phi(\tau_{xz}\cos\theta+\tau_{yz}\sin\theta)+\sigma_z\cos^2\phi$$

Effective shearing stress is calculated as follows (Eq. 22):

$$\tau = \tau_{\theta\phi} = \left\{ \left[\sin\phi(\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta) + \cos\phi(\tau_{xy}\sin\theta - \tau_{yz}\cos\theta) \right]^2 + \left[\begin{array}{l} \sin\phi\cos\phi(\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\cos 2\theta - \sigma_z) \\ + \cos 2\phi(\tau_{xz}\cos\theta + \tau_{yz}\sin\theta) \end{array} \right]^2 \right\}^{1/2} \quad (22)$$

Hydrostatic stress basically expressed that effects of tensile mean stress was applied on 1 direction, it can be naturalized by a compressive mean stress, so effect of mean stress wasn't considered.

Rain flow method

An irregular stress history is a sequence of peaks and valleys. The peaks and valleys are locations where loading orientation changes. Stress history is divided into smaller parts called simple and general intervals, then the stress differences between the peaks and valleys computed.

A simple interval is between a peak and following valley or vice versa. General interval considered as a distance between a peak and a valley which is not right after the peak or vice versa, level crossing counting [18].

As it apparent from Figure 3, a combination of X-Y-Z (Peak- valley- peak or valley-peak- valley) assumed to be a cycle, if in second interval $\Delta\sigma_{yz}$ be greater or equal to $\Delta\sigma_{xy}$ in the first interval [19]. If the second interval was actually greater than or equal, then a cycle equal first interval ($\Delta\sigma_{xy}$) will be counted.

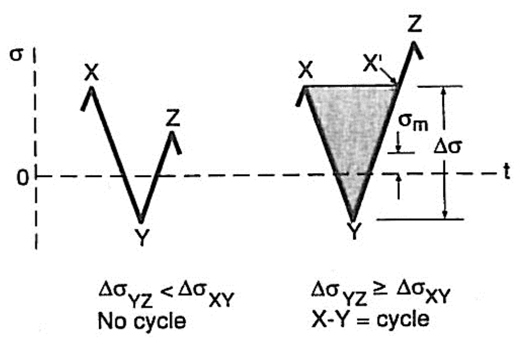

Figure 1 Condition of cycle counting in rain flow method

Three-point and four-point cycle counting lead to the identical range-mean rainflow matrix. There are some unique features of the four-point cycle counting method. First, this technique is very easy to use in conjunction with as-recorded data acquisition and data reduction, because it does not require rearrangement of the load-time history. Second, this method can be easily implemented for cycle extrapolation and load-time history reconstruction.

Finally, this cycle counting method is very generic, because the three-point rainflow matrix can be deduced from the four-point rainflow matrix and its residue.

RESULTS AND DISCUSSION

For the best results comparing, Sines, Crossland and Dang van theories were programmed in matlab, finally the results are reported as follow (Figure 4 to Figure 7):

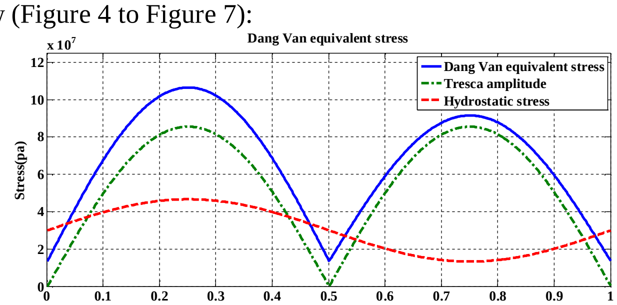

Figure 4 Equivalent stress in Dang van model

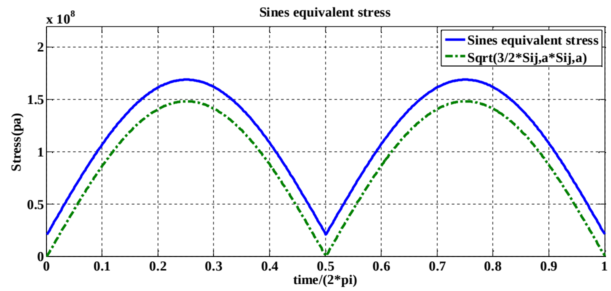

Figure 5 Equivalent stress in Sines model

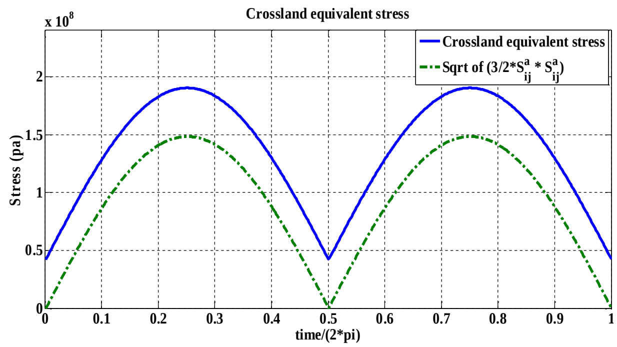

Figure 6 Equivalent stress in Crossland model

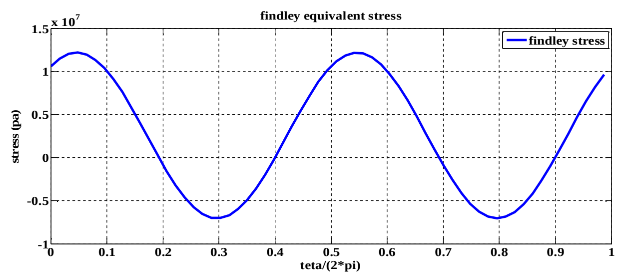

Figure 7Equivalent stress in Findley model

According to Table 1 for input stress history, results for each critical plane theory are obtained (Table 2).

Table 2. Equivalent stress and safety factor for Dang van, Fatemi,McDiarmid and findley theories

	McDiarmid	Findley	Fatemi	Dang van
Equivalent stress(MPa)	**0.7**	**0.95**	**0.31**	**0.94**
Safety factor	**1.4085**	**1.0205**	**1.003**	**1.2302**

According to applied stresses, critical plane details are determined as follows:

$\phi = 120$

$\theta = 5$

Also another history was used to compare Dang van theory with theories based on stress invariant, so the results are obtained as follow part (Table 3):

Table 3. Equivalent stress and safety factor for Sines, Crossland and Dang van theories

	Dang van	Crossland	Sines
Equivalent stress(MPa)	**106.52**	**190.32**	**169.2**
Safety factor	**0.9764**	**1.0929**	**0.9466**

According to results and comparison were occurred among Dang van and stress invariant theories, it can be seen that Crossland theory had 11.93 % error and its safety factor was higher than one. Sines theory had 3.05 % error (error was calculated in comparison with Dang van theory) but safety factor was lower than one unlike Crossland method.

Maximum stress related to Dang van theory was lower than other theories so this method have lower safety factor. It can be said that Dang van theory operated more cautious sines and crossland theories approximately had same results. Stresses in these theories were more close together and had very different with Dang van theory. According to obtained results, reliability factor in Dang van theory was lower than 1 but in Crossland theory was higher than one.

Findley theory result was same as Dang van theory because these theories were based on critical planes. In cases, loading process was applied one axial with R ratio, difference between principal plane definition in McDiarmid and Findley models was appeared.

In presented works with other scientists, among the moment critical planes, the plane with the highest failure criterion value was selected as critical plane and life time prediction was based on stress components changing on mentioned plane. In this study, stress components changing was checked on different planes and according to presented equivalent stresses definition, plane that have maximum stored damage, is considered as critical plane.

Rain-flow cycle counting method, in the first step usually turns initial history stress – time, to the form of peaks and valleys, which were made to be fully identifiable.

As the procedure used in this research, all the information converted into a matrix. Each component of the stress and it's time, was considered an element of the matrix. Afterward the peaks and valleys are connected to each other in the form of a matrix.

It should be noted that the interval between the peaks and valleys is not important, and only the position of the peaks and valleys is important. Counting cycles starts from the beginning of history, since generally periodic history is at hand, it is better to adjust input history in such a way, that it starts from a maximum or minimum stress in the entire history. This can be done to simplify the procedure and to get more precise results.

Figure 8 shows the stress history, which is expressed as a matrix. Stress values rounded to integers, if they are not already, then an element of matrix assigned to the value. It should be noted that all the numbers are rounded in the same manner in order to get smooth results in cycle counting.

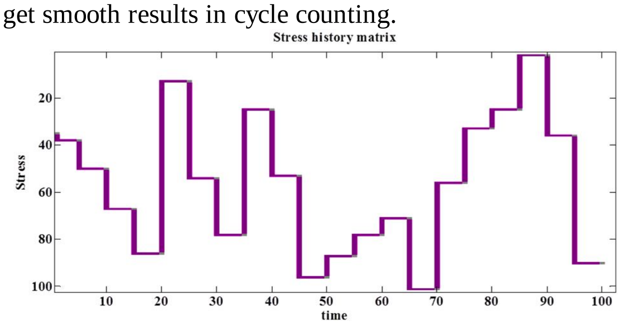

Figure 8 Matrix form of stress history

Figure 9 is the matrix of said stress, which some parts of it have been removed. In this figure, first cycle calculated and after counting the corresponding cycle is removed. It can be seen from Figure 9, in contrast to the continuous and distinct cycles in simple loading, there is absolutely no reason that in this particular case, the cycles be continuous, and it can contain initial and end parts of stress history. It can be identified as a separate cycle. As described earlier, these cycles cover the general intervals.

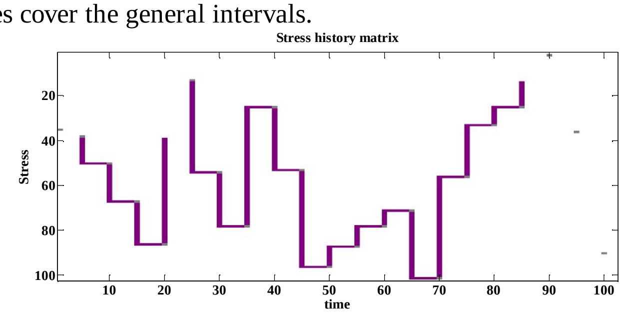

Figure 9 Counting and removing the first cycle from cycle history

This trend continues, and cycles are counted one after another, and then be removed. Figure 10 shows last cycle calculated from the history. As it can be seen this cycle is a simple interval.

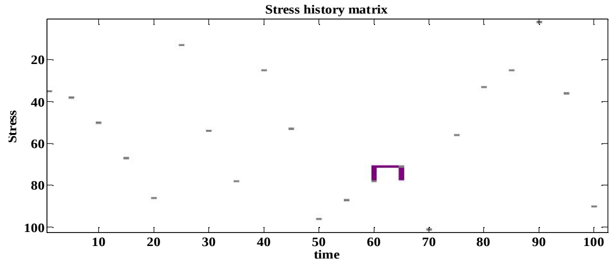

Figure 10Counting last cycle

After counting and removing all cycles, as shown in Figure 11, only the peaks and valleys (the relative maximum and minimum points) stay, that in fact are input data to the program.

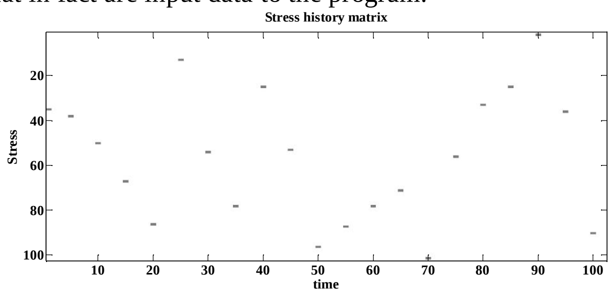

Figure 2 Relative maximum and minimum points (input data)

CONCLUSION

By looking on obtained results, it can be observed that critical plane theories had better results, because critical plane situation may depend on principal plane situation and critical plane situation under Nonproportional loading will be time function. So complexity of problem is higher in invariant loading and it is the advantage of these theories.

Models such as Dang van seem complex but in actual these can be used in complex loading. Findley model has high accuracy and good acceptability in complex loading.

The theories based on static failure criterion and sines theory weren't used for invariant loading.

From presented theories, equivalent stress and strain can be introduced. These stresses or strains were considered as criteria so they were used in last theories.

List of Symbols

C_a	Amplitude of shearing stress on critical plane
f_0	Failure limitation in reversed tension
f_{-1}	Fatigue limitation in reversed axial loading
G	Shearing module
J_2	Second invariant of the stress deviator tensor
k	Shearing coefficient
N	Number of Cycles to first crack occurrence
t_{-1}	Fatigue limitation in reversed torsion

Greek symbols

$\sigma_{H,m}$	Average of hydrostatic stress
$\sigma_{H,\max}$	Maximum of hydrostatic stress
$\sigma_{n,\max}$	Maximum of normal stress

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