Facility Location with the Disruption Risk in the Supply Chain Network Design

M. Ghomi-Avili1, A. Makui1, S. Baharak Naghibi2, E. Yadegari3

1Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
2Department of Industrial Engineering, Islamic Azad University, Tehran, Iran
3Department of Industrial Management, Shahid Beheshti University, Tehran, Iran

ABSTRACT

In a classical facility location problem (FLP), parameters are regarded as deterministic; however, stochastic parameters (e.g., demand variation and transportation costs) have been addressed in the FLP recently, and less attention is paid to facility failure and breakdown. When one or more distribution center incurs failure, the needs of customers are met by other distribution centers leading to an increase in transportation costs. Besides, establishment of more distribution centers to reduce transportation costs imposes additional costs. As a result, the aim of this paper is to make a balance between transportation costs and the costs of establishing distribution centers. The presented model then is solved by a simulated annealing (SA) algorithm. Finally, numerical examples and experimental results are presented.

KEYWORDS: Supply chain network design; Facility location; Reliability; Disruption; Simulated Annealing Algorithm.

1. INTRODUCTION

Designing a supply chain network, consisting of several strategic decisions dependent on the construction of a supply chain, is the most fundamental issue in supply chain management (SCM), which highly affects other tactical and operational decisions of a company. Strategic decisions require a great amount of investment to make and execute. Designing a supply chain network therefore plays a key role in SCM. In real world, organizations cope with numerous stochastic conditions, all of which severely impact the short, medium, and long-term decisions of the company. These uncertainties stem from various factors such as the exact amount of customer demands and transportation costs, among others. Although each organization does its best to lessen the side effects of the factors, many unexpected events occur persistently. An example of these is the malfunction of the facilities. Even though any component of a system incurs face failure or breakdown in the long time, in classical modeling of a facility location problem (FLP), it is always assumed that facilities never malfunction or break down, and the analyzed system always works properly. However, in real world, facilities are constantly prone to malfunctioning or failing, reliability and Floods, earthquakes, and worker strikes are some instances of this situation. This fact testifies to the absolute necessity of developing more practical location models and considering the malfunction of the facilities. As a result, considering facility failure and breakdown in the proposed logistics network model is the aim of this paper. After the literature review of supply chain network design problems and simulated annealing algorithms in Section 3, a mathematical model for a facility location problem with the disruption risk in the supply chain network design will be proposed with the specification of its properties in Section 4, the solving method for the proposed model is elaborated and experimental results and detailed analysis are discussed. Finally, in Section 6, the conclusion will be drawn.

2. LITERATURE REVIEW

This section aims to examine the most crucial studies that have directly addressed the issue of reliability in location problems. The first study was carried out by Snyder [1] who directly examined the location problems and facility assignment considering the issue of reliability. He was a pioneer in directly addressing the problem of facility breakdown or failure in his dissertation titled “Facility Reliability in Location and Facility Assignment”. He modeled two problems of facility location titled “Facility location problems with constant cost and p-median” in two ways: (1) with a single-objective minimizing the maximum cost of breakdown, and (2) with bi-objectives minimizing the overall costs of operation (i.e., the cost of daily transportation, the time which all the facilities are operating without failure) and expected failure costs (i.e., expected transportation costs which concerning facility failure). He calculated upper and lower bounds by applying the Lagrangian relaxation (LR) method to the problem and determined the optimized solution for the problem by using branch and bound (B & B) method. He also plotted a try-and-error diagram based on the diagrams of operational costs and expected failure costs and calculated expected failure costs for different quantities of operational costs. He ultimately came to the conclusion that the system’s reliability can be considerably boosted if operational costs increase slightly. Snyder et al

*Corresponding Author: M.Ghomi-Avili, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran. E-mail address: ghoni@iust.ac.ir. Phone: +989121694204
addressed a min-max reliable facility location problem (MMRFLP) to minimize the maximum cost and proposed a mixed-integer programming model for the problem.

Berman et al. [3] studied a p-median problem considering the reliability issue and concluded that the stochastic p-median problem, in which the probability of failure or breakdown is non-zero, converges to the deterministic P-median problem if the asymptotic property is taken into account. They also studied the stochastic P-median problem in the most pessimistic circumstances and developed innovative methods with different upper and lower bounds for solving the problem.

Zhan [4] studied un-capacitated fixed charge location problem (UFLP) in single-level and multi-levels by considering unstable (unreliable) facilities and developed a mixed-integer mathematical model. He also proposed useful algorithms based on a genetic algorithm (GA) to solve the problem. Besides, he studied the ways of enhancing the reliability of a designed and stable system from a mathematical viewpoint. Zhan et al [5], introducing a GA, and Cui et al [6], addressing the discrete UFLP, theorized that considering different failure probabilities for facilities in different places may have a considerable effect on the selection of facility location and site selection. The objective function is to minimize the initial setup and expected transportation costs. A mixed-integer programming model is presented for the problem. Moreover, they improved the solution for small-sized and medium-sized problems based on Continuum Approximation (CA).

Lim et al [7] proposed an integer programming (IP) model for FLP with constant costs addressing the issue of reliability when facility hardening is a matter of concern and solved the mathematical model with the LR method. Gade and Pohl [8] studied the UFLP by assuming a limited capacity for each facility and considering the issue of facility instability. They put forward a discrete mixed-integer mathematical model for the problem and introduced a scenario in which failure probability distribution is regarded as Bernoulli. They then developed a sample average approximation (SAA) procedure for solving the problem.

Pourali khani et al [9] proposed a multi-period multi-stage integrated forward/reverse supply chain network design under risk with consideration of the strategic network design decisions along with tactical material flow to avoid sub-optimality led from separated design in both forward and backward flows. The demands of first market customer zones are assumed stochastic. The problem is formulated in a mixed integer non-linear programming (SMINLP) decision making form as a multi-stage stochastic problem with objective function maximizing the total expected profit.

Xin Miao et al [10] proposed an uncertainty evaluation (UE) method for supply chain reliability. Their objective was to scientifically evaluate the reliability issue in a supply chain. They maintain that the proper understanding of supply chain reliability depends on a few factors. They addressed the problem in a fuzzy environment and set dynamic variables for the problem. They finally proposed the UE method along with a numerical example.

Aryanezhad et al. [11] presented an integrated model for location-inventory problem with disruption and studied the problem of designing a supply chain when distribution centers are liable to accidental breakdown. Because of this vulnerability, it is possible that one or more distribution center fails to serve the customers. They assumed that customer demands are stochastic; as a result, each distribution center keeps a certain amount of safety stock to meet the customer demands at an appropriate service level. They modeled the problem using non-linear IP model, the objective function of which is to minimize the costs of location, inventory, transportation, and lost sale by considering the accidental breakdowns. The model determined the location of each distribution center and simultaneously assigned the customer to each. They used GA to solve the problem.

Jabbarzadeh et al. [12] formulated a supply chain design problem with the risk of disruptions to facilities to maximize the total profit for the entire system. Two solution methods based on LR and GA were developed to achieve near-optimal solutions for large-scale problem instances.

Nader Azad et al. [13] proposed a capacitated supply chain network design model with random disruptions to facility and transportation targeting at determining the optimal location, types of distribution centers, and the best design to allocate customers to each opened distribution center. Peng et al. [14] developed a capacitated supply chain network design model with random disruptions, stochastic p-robustness criteria, and site-dependent disruption probabilities. Qi Chen et al. [15] studied a reliable inventory-location problem optimizing facility locations, customer assignments, and inventory management decisions when facilities are subjected to disruption risks. They proposed an IP model minimizing the sum of costs under normal and failure circumstances and developed a solution based on LR method for the problem.

The aim of this paper is to propose a model for a multi-level supply chain considering strategic and operational levels simultaneously. Furthermore, when one or more distribution center incurs breakdown, the ways of meeting a customer demands are expressed through reliability. Regarding the importance of the subject in designing a supply chain, six general objectives are discussed. First, locating the site of factories and distribution centers; second, determining the procedure of assigning customers to distribution centers in the primary and backup assignment; third, choosing the suppliers and selecting the amount of raw material supply from each supplier to each factory for manufacturing products; fourth, calculating the amount of each manufactured product in each factory; fifth, calculating the amount of products shipped from each facility to another one in the network; sixth, introducing a solution method based on Simulated Annealing to obtain optimal solution for large-scale problem instances.
Reviewing the above-mentioned literature on supply chain network design problems and facility failure and breakdown issues as well as SA algorithms, it is noted that most of the existing models lack of the relations between the reliability issue with supply chain network design problems. Therefore, considering facility failure and breakdown in the proposed supply chain network model is the aim of this paper. In addition, the logistics network design problem is an NP-hard problem. Thus an efficient algorithm is essential for enterprises. From the literatures, we can note that SA algorithms have been successfully applied to a wide range of domains including facility location problems. Therefore, to tackle such an NP-hard problem we adopt SA to solve our model in large-size problems.

The rest of the paper is organized as the following. In section 3, the proposed model is discussed and its components are explained in details. In section 4, the solving method for the proposed model is elaborated and experimental results and detailed analysis are discussed. Finally, in section 5, conclusion and suggestions for further research are presented.

3. Problem Definition and Mathematical Modeling

In this section, the definition of the problem and the characteristics of parameters, decision variables, and the mathematical modeling are addressed in details.

3.1. Problem Definition

The purpose of modeling the problem is to locate the site of factories and distribution centers and specify the flow volume among the facilities. Customer demands are fulfilled from one distribution center. Since the distribution centers may incur failures, a backup assignment must be determined beforehand. It must be noted that if each customer in primary assignment is allocated to a reliable distribution center, it will not need backup assignment, and if the customer is allocated to an unreliable distribution center in primary assignment, it will be assigned inevitably to a reliable distribution center in the next assignment. The reliability concept used in this model is the one studied by Snyder et al. [2] and Lim et al. [8]. A schematic general supply chain network of the model is depicted in Fig 1. Note that complete arrows denote the customer primary assignment and dashed-arrows indicate customer backup assignment.

3.1.1. Characteristics and Assumptions

The problem is formulated as a single-product model. The supplier and customer sites are constant and definite. The potential sites for establishing factory and distribution centers are definite and discrete. Materials flow can only exist between two sequential network levels. There are not any connections among facilities being at the same level. Customer demands are constant and definite. Failure probabilities of distribution centers (the percentage of time when the distribution center fails or is out of service) are independent of each other. It must be noted that when one distribution center fails, it does not have any destructive influence on other distribution centers. Each customer can only be responded by one distribution center.

3.1.2. The most Remarkable Decisions

- Locating the site of factories and distribution centers.
- Determining the procedure of assigning customers to distribution centers in their primary and backup assignments.
• Choosing the suppliers and the amount of raw material supply from each supplier to each factory for manufacturing products.
• Calculating the amount of shipped products from each facility to another one.

3.1.3. Defined Sets
- I: The set of suppliers constant sites \( \forall i \in I \)
- J: The set of candidate sites for establishing factories \( \forall j \in J \)
- K: The set of candidate sites for establishing distribution centers \( \forall k \in K \)
- E: The set of constant customer sites \( \forall e \in E \)

3.1.4. Defined Parameters
- \( c_j \): Fixed cost of establishing factory \( j \)
- \( f_k^a \): Fixed cost of establishing distribution center \( k \) if distribution center \( k \) is uncertain and unreliable
- \( f_k^r \): Fixed cost of establishing distribution center \( k \) if distribution center \( k \) is certain and reliable
- \( c_{ij} \): Transportation cost from supplier \( i \) to factory \( j \) per raw material unit
- \( h_{jk} \): Transportation cost from factory \( j \) to distribution center \( k \)
- \( h_{ke}^p \): Transportation cost from distribution center \( k \) to customer \( e \) if distribution center located at \( k \) is primary assignment center for customer \( e \)
- \( h_{ke}^b \): Transportation cost from distribution center \( k \) to customer \( e \) if distribution center located at \( k \) is backup assignment center for customer \( e \)
- \( p_j \): Cost of manufacturing a product per unit in factory \( j \)
- \( r_{ij} \): Cost of purchasing raw material from supplier for manufacturing product
- \( q_k \): Failure probability of distribution center \( k \)
- \( d_e \): Demand of customer \( e \)
- \( \text{hold}_k \): Holding cost per product unit in distribution center \( k \)

3.1.5. Decision variables

- Continuous variable:
  - \( v_{ij} \): Amount of raw materials sent from supplier \( i \) to factory \( j \) for manufacturing
  - \( B_{jk} \): Amount of product sent from factory \( j \) to distribution center \( k \)

- Binary variables:
  - \( x_j = \begin{cases} 1 & \text{If a factory is established at site } j \\ 0 & \text{Otherwise} \end{cases} \)
  - \( y_k^u = \begin{cases} 1 & \text{If an unreliable distribution center is established at site } k \\ 0 & \text{Otherwise} \end{cases} \)
  - \( y_k^r = \begin{cases} 1 & \text{If a reliable distribution center is established at site } k \\ 0 & \text{Otherwise} \end{cases} \)
  - \( T_{ke}^p = \begin{cases} 1 & \text{If the demand of customer } e \text{ is assigned to distribution center } k \text{ as the primary assignment} \\ 0 & \text{Otherwise} \end{cases} \)
  - \( T_{ke}^b = \begin{cases} 1 & \text{If the demand of customer } e \text{ is assigned to distribution center } k \text{ as the backup assignment} \\ 0 & \text{Otherwise} \end{cases} \)
  - \( T_{ke}^s = \begin{cases} 1 & \text{If the demand of customer } e \text{ is assigned to distribution center } k \text{ both as the primary and backup assignment} \\ 0 & \text{Otherwise} \end{cases} \)

3.1.6. Objective Function
The problem’s objective function is to minimize the costs of providing raw material, establishing factories and distribution centers, and transporting raw material and product through the network:
\[
\text{Min} \sum_{j} c_j x_j + \sum_{k} f^u_k y^u_k + \sum_{k} f^R_k y^R_k + \sum_{i} \sum_{j} (r_{ij} + c_{ij}) v_{ij} + \sum_{j} \sum_{k} p_{jk} B_{jk} + \sum_{j} \sum_{k} h_{jk} B_{jk} \\
+ \sum_{e} \sum_{k} (1-q_k) d_{e} h_{e} T^p_{ke} + \sum_{e} \sum_{k} q_k d_{e} h_{e} T^B_{ke} - \sum_{e} \sum_{k} q_k d_{e} (h_{ke}^B - h_{ke}^R) T^S_{ke} + \sum_{k} \sum_{e} h_{ke}^B d_{e} (T^B_{ke} - T^S_{ke})
\] (1)

The first three terms calculate the fixed costs of establishing factories, unreliable distribution center, and reliable distribution center, respectively. The fourth term calculates the cost of purchasing raw material being transported from supplier i to factory j. The fifth term denotes the total production cost in all factories. The sixth term computes the product transportation cost from factory j to distribution center k (reliable or unreliable). The seventh term calculates the transportation cost from distribution center j to the customer e. In this case, the distribution center assigned to customer e is a primary assignment; as a result, this center may incur failure. The eighth term calculates the transportation cost from distribution center j to the customer e. In this case, the distribution center assigned to the customer e is a backup assignment; subsequently, this distribution center would not incur failure and always works properly. The ninth term calculates the additional transportation cost from distribution center k to the customer e. Since one distribution center may be assigned to a customer both as backup and primary assignment, this additional cost should be subtracted from the objective function. Finally, the tenth term denotes the holding cost of product in distribution centers; this cost must only be calculated for distribution centers assigned to the customers as backup assignment.

3.1.7. Constraints

\[
\sum_{i} v_{ij} = \sum_{k} B_{jk} \quad \forall j \tag{2}
\]

\[
\sum_{j} B_{jk} = \sum_{e} d_{e} (T^p_{ke} + T^B_{ke} - T^S_{ke}) \quad \forall k \tag{3}
\]

\[
\sum_{i} v_{ij} \leq M x_{j} \quad \forall j \tag{4}
\]

\[
\sum_{k} B_{jk} \leq M x_{j} \quad \forall j \tag{5}
\]

\[
\sum_{j} B_{jk} \leq M(y^u_k + y^R_k) \quad \forall k \tag{6}
\]

\[
\sum_{k} T^p_{ke} = 1 \quad ; \quad \sum_{k} T^B_{ke} = 1 \quad \forall e \tag{7}
\]

\[
T^S_{ke} \leq T^B_{ke} \quad ; \quad T^S_{ke} \leq T^p_{ke} \quad \forall k, e \tag{8}
\]

\[
T^p_{ke} \leq y^R_k + y^u_k \quad ; \quad T^B_{ke} \leq y^R_k \quad \forall k, e \tag{9}
\]

\[
y^R_k + y^u_k \leq 1 \quad \forall k \tag{10}
\]

\[
\sum_{k} y^R_k \geq 1 \tag{11}
\]

\[
T^B_{ke}, T^p_{ke}, x_j, y^u_k, y^R_k \in \{0, 1\} \quad \forall k, e, j \tag{12}
\]

\[
v_{ij}, B_{jk} \geq 0 \quad \forall j, k \tag{13}
\]

It should be noted that M is a large-enough positive number in all above terms.

Constraints (2) ensure that the sum of factory inflows for manufacturing the product must be equal to the sum of factory outflows to the distribution centers. Constraints (3) ensure that the sum of inflow to distribution center k must be equal to the sum of outflow from that distribution center. Constraints (4) ensure that flow can be initiated from suppliers to factory j if and only if factory j is established. Constraints (5) denote that flow can be initiated from factory j to distribution centers if and only if factory j is established. Constraints (6) ensure that flow can be initiated from factory j to distribution center k if and only if distribution center k is established. Constraints (7) indicate that each customer in primary assignment is assigned only to one distribution center and is assigned only to one backup centering backup assignment. Constraints (8) ensure that if customer e is assigned to distribution center k in both primary and backup distribution levels, the coefficient \(T^S_{ke}\) in the objective function will be equal to 1 to subtract the additional cost from it. Constraints (9) ensure that each customer in
primary assignment can only be allocated to a distribution center that one certain or uncertain had been previously established. Moreover, each customer in backup assignment can only be allocated to the distribution center that one certain and reliable distribution center had been previously established. Constraints (10) denote that one reliable distribution center or one unreliable distribution center will be located at site k. Constraints (11) ensure that since all the customers should be assigned to a reliable distribution center in backup assignment level or should be assigned to a reliable distribution center in the primary level, we will have at least one reliable distribution center. Constraints (12) are the definitions of binary variables. Constraints (13) indicate that product-flow transition variables among facilities are non-negative.

The NP-hardness of the supply chain network design problem is proved in many studies [16]. The proposed model consists of two problems, capacitated facility location problem and allocation optimization. It is worth to note that the capacitated facility location problem is NP-hard [17], then the reliable facility location problem presented in this paper is also NP-hardness.

4. Solution Method and Numerical Example
Since the network design problem considering reliability issues is a capacitated location-allocation problem; and also can be viewed as a multiple-choice Knapsack problem, it is known to be NP-hard, therefore, an efficient algorithm should be developed to solve such an NP-hard problem, which is the aim of this section.

A standard optimization solver such as CPLEX can be used for solving the proposed linear mixed-integer model; however, the solution time increases as the problem size becomes larger and larger. Therefore, for large-scale problems, we propose a meta-heuristic algorithm based on simulated annealing (SA). The proposed models are introduced in this section and the experimental results of solving the problem with SA and CPLEX methods will be discussed in section 5.

4.1. Simulated Annealing (SA) Algorithm
Simulated Annealing (SA) is a common heuristic optimization technique for solving large-scale supply chain design problems and has been proven to be a very effective heuristic procedure [18]. This technique is used for accidental local searches. The search starts from an initial feasible solution and each solution has a specific cost value. The neighboring solution is randomly generated. If the cost value of the candidate solution is lower than that of the prior solution, a transition to the candidate solution is made. However, if the candidate solution does not improve the prior solution, there is still a chance of transition according to a special probability function. Detailed discussions of SA can be found in [19, 20]. Fig. 2 represents the basic structure of SA.
Before discussing the numerical example, the definition procedure of initial solution is introduced.

4.2. Definition Procedure of Initial Solution

1) Assuming that the number of suppliers is a, each cell represents the amounts of product sent by the supplier i.

\[ \text{Number of cells} = a \]

\[
\begin{array}{cccc}
1 & 2 & \cdots & a \\
\end{array}
\]

2) The number of candidate factories is b.

\[ \text{Number of cells} = 2b \]

\[
\begin{array}{cc}
1 & 2 \\
\end{array}
\]

This matrix with zero-and-one elements denotes that factory j is established or not.

\[
\begin{array}{cc}
1 & 2 \\
\end{array}
\]

This matrix indicates the amount of production in factory j.

3) The number of candidate sites for distribution centers is c.
4) Sending procedure from suppliers to the factories. The number of sent items is shown in the cells.

\[ \text{Number of cells} = a \times b \]

5) Sending procedure from factories to the distribution centers (DC). The number of sent items is shown in the cells.

\[ \text{Number of cells} = a \times 2c \]

6) Sending procedure from distribution center to the customers. The number of sent items is shown in the cells.

\[ \text{Number of cells} = c \times d \]

7) Procedure of assigning distribution centers to customers.

\[ \text{Number of cells} = 2d \]

The dimension of solution matrix is \( 1 \times N \), where \( N = a + 2b + 2c + a \times b + b \times 2c + c \times d + 2d \).
5. NUMERICAL RESULTS

In this section, the numerical examples are solved by CPLEX and the results are compared with SA outputs.

5.1. Getting the Exact Solution

In order to prove that the proposed model is practical, a case example is solved by CPLEX solver of GAMS software and experimental results are then analyzed. The results are shown in Table 4. We employ data sets of demand parameters and their geographical coordinates (length and width of candidate and demand points) from Snyder [1] for this model. Other parameters added to the proposed model are taken from Snyder et al. [2] and Pishvaee et al. [21]. The disruption probability and failure occurrence data are assumed independent of each other and are randomly and uniformly generated on (0.025, 0.075). The cost of stochastic and unreliable centers is calculated by adding up fixed and variable costs, which are a function of demand at each candidate point:

\[ f_k^u = 500000 + 1.7 \, d_k \]  \hspace{1cm} (14)

Unreliable center hardening cost is presented by a linear function based on failure probability:

\[ d = (f_k^R - f_k^u) = 5000000 \, q_k \]  \hspace{1cm} (15)

Hence, for fixed cost of deterministic and reliable distribution centers establishment, the following is the case:

\[ f_k^R = f_k^u + 5000000 \, q_k \]  \hspace{1cm} (16)

Consequently, if a site incurs more failure probability, the costs of establishment would augment to compensate for more reliability. We use Euclidean distance for calculating distances between two sites. For computing the transportation cost, the distance is multiplied by 1000. To consider additional transportation cost for backup assignment, the following equation is of used:

\[ h_{k_e}^B = 1.25 \, h_{k_e}^R \]  \hspace{1cm} (17)

Other parameters are shown in Table 1. Five case problems with different dimension are defined to analyze the model. The fixed number of suppliers, the number of potential sites for establishing factories and distribution centers, and the fixed number of customers in each example are shown in Table 2.

**Table 1: Values of the parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factory establishment cost ( c_f )</td>
<td>( 800000 + 1.9(d_f) )</td>
</tr>
<tr>
<td>Raw material purchase cost ( c_{r_i} )</td>
<td>Uniform(10,20)</td>
</tr>
<tr>
<td>Production cost in factory ( p_f )</td>
<td>Uniform(15,30)</td>
</tr>
<tr>
<td>Product holding cost in distribution center ( h_k )</td>
<td>Uniform(10,15)</td>
</tr>
</tbody>
</table>

**Table 2: Number of facilities in different examples for analyzing the model**

<table>
<thead>
<tr>
<th>Example No.</th>
<th>suppliers</th>
<th>Potential sites for establishing factory</th>
<th>Potential sites for establishing distribution centers</th>
<th>Number of customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>10</td>
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<tr>
<td>4</td>
<td>5</td>
<td>7</td>
<td>20</td>
<td>20</td>
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<tr>
<td>5</td>
<td>20</td>
<td>20</td>
<td>49</td>
<td>49</td>
</tr>
</tbody>
</table>

Information pertinent to numerical example is available in Table 3. It should be noted that computations are done by CPLEX solver of GAMS software on a personal computer provided with a Quad Core Intel core i7 CPU and 4.00 GB of RAM.

To analyze the structure of the model, all the parameters are set fixed, and the features of the solution are analyzed by the variations of demand and distribution centers failure probabilities. For this purpose, example 2 is used. The results are indicated in Figures 3 to 5. As illustrated in Fig 3, the objective function value increase significantly with increasing percentage of demand.
Fig 3: Objective Function variation in terms of the percentage of demand changes

Fig 4 can be interpreted as first-increase and then-fixed value of system costs when the distribution center failure probability increases. Fig 5 demonstrates that although the establishment of reliable distribution center imposes a significant cost on the company, its merits outweigh the additional transportation costs spent for customer assignment to the backup distribution centers.

Fig 4: Objective Function variation in terms of different distribution centers failure probability values
5.2. Getting the results by Simulated Annealing Algorithm

To analyze the proposed meta-heuristic algorithm (SA), sixteen samples in terms of three different sizes are considered, which are shown in Table 3. Comparing CPLEX and proposed SA algorithm, the results of objective function value and CPU time for test problems of Table 3 indicated briefly in Table 4.

**Table 3:** Sample example in terms of the number of facilities in different sizes

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Test problems</th>
<th>Suppliers</th>
<th>Potential sites for establishing factory</th>
<th>Potential sites for establishing distribution centers</th>
<th>Number of customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>2</td>
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<td>6</td>
<td>5</td>
<td>7</td>
<td>20</td>
<td>20</td>
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<tr>
<td>Medium sizes</td>
<td>7</td>
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<td>25</td>
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<td></td>
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**Table 4:** Summary of algorithm comparison between CPLEX and SA

<table>
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<th>Test problems</th>
<th>Objective Function</th>
<th>Cpu Time (s)</th>
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According to the results of the above table and Fig 6, changing in the objective function can be observed for different test problems. Furthermore, the low difference between objective function value obtained from CPLEX and SA algorithm indicate that the proposed SA algorithm has desirable performance. The CPU time for both algorithms increased exponentially with the size of the problem. However, note that the CPLEX time grew more rapidly.

**Fig 6:** Objective value comparison between CPLEX and SA algorithm

**Fig 7:** CPLEX and SA algorithm comparison on CPU time (s)

6. CONCLUSIONS AND FUTURE RESEARCH

In the classical models of facility location in distribution chains, less attention is paid to the failure of distribution centers. It is generally assumed that the facilities never incur failure and break down; however, in reality, facilities are constantly inclined to failure. This fact resulted in developing more practical models in distribution chain design considering the failure probability for each of the distribution centers. In this paper, a mathematical programming model was introduced by considering different features and conditions of real world such as paying attention to facility failure and breakdown. After introducing the proposed simulated annealing algorithm for this NP-hard model, Numerical experiments were presented, and the results showed that the proposed model and algorithms were able to support the logistics decisions in the proposed supply chain network model efficiently and accurately.

The following are some recommendations for future research:

- In most cases, determining the sequence of product delivery to the customer is only according to transportation costs; customer desired time for delivery and customer priority among other customers during delivery prioritizing are not considered.
- In all previously proposed models in the field of reliability, only P-median model is regarded as the basic model and minimizing the average transportation costs is not desirable.

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For simplification, it is assumed that the single type of product or service is presented to the customer; however, in practice, the customer may be served multiple types of services concurrently.

REFERENCES