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# Correction in Linear Programming Method for Generating the Most Favorable Weights from a Pairwise Comparison Matrix

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## ABSTRACT

Data envelopment analysis (DEA) is a useful tool to measure and recognize the effectiveness and performance of decision-making units. On the other hand, analytical hierarchy process (AHP) is a useful tool in the field of multi-criteria decision-making problems (MCDA) and it can be helpful to rank the set of options with distinct criteria and choose the best or most appropriate option among them. In 2008, Wang et al. proposed a linear programming model to determine most desirable weight at the hierarchy process and preserve the ranking model. However, with a violation example we showed that the proposed model does not preserve the ranking, because of substantial problem in their model relevant to ignoring the fourth principle of hierarchy analysis process. Therefore, in the current research a new evaluation process including a new option has been done to save the rank in Wang et al.'s linear programming model.

**KEY WORDS:** data envelopment analysis, analytical hierarchy process, multi-criteria decision-making analysis, linear programming approach to determine the most desirable weight.

## 1. INTRODUCTION

In the recent years, different applications of the data envelopment analysis have been used to evaluate the performance of different entities in many organizations. The major reason for using this method is that it is non-parametric.

Data envelopment analysis has been used to determine the best weight through measuring the effectiveness and its aim is to recognize the performance of a system or decision-making unit. The analytical hierarchy process (AHP) is a useful tool in the field of multi-criteria decision-making problems (MCDA) and it can be helpful to rank the set of options with distinct criteria and choose the best option from the set of options having distinct quantitative and qualitative criteria. In 1987, Tomas Saati [1] (the founder of this method) introduced the following four principles as the principles of analytical hierarchy process and he has established all calculations, rules and regulations on the base of these principles. They are:

**Principle1:** if the preference of element A to element B is equal to n, then the preference of element B to element A will be 1/n.

**Principle2:** (principle of homogeneity) element A must be homogenous and comparable with element B. in other words, the preference of element A to element B cannot be infinite or zero.

**Principle3:** (dependency) each hierarchy element can be related to its higher element and this dependency can be continued up to the highest level.

**Principle4:** (expectations) whenever any change occurs in the hierarchy structure, evaluation process should be done again.

Apart from Saaty's eigenvector method (EM) [2], which is the most widely used priority method, Chu et al. [3] proposed a weighted least-squares method (WLSM). Crawford [4] proposed a logarithmic least-squares method (LLSM). Saaty and Vargas [5] presented a least-squares method (LSM). Cogger and Yu [6] suggested a gradient Eigen weight method (GEM) and a least distance method (LDM). Islei and Lockett [7] developed a geometric least-squares method (GLSM). Bryson [8] put forward a goal programming method (GPM). Bryson and Joseph [9] also brought forward a logarithmic goal programming approach (LGPA).

The analytical hierarchy process is an easy method that all people including social and physics scientists, engineers, politicians and even public can apply this method without using the specialists.

Analytical hierarchy process as the most known tool of multi-criteria decision making has been considerably criticized because of the possibility of reverse rank that means the change or incompatibility in the relative ranking of options after adding or removing a new option.

DEAHP method [10] has some difficulties and the main problem of this method is that it allows decision making units to choose weights freely and gives the high weights to outputs having strengths, gives the zero weight to outputs that have weaknesses, evaluate the non-logical relative weights to the paired comparison matrixes and also does not maintain the ranking.

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Hence, due to fundamental difficulties by the DEAHP method, Wang et al. [11] in 2008 proposed a new method called linear programming approach to determine the most desirable weight from paired comparison matrixes in analytical hierarchy process (LP-GFW) that does not have disadvantages of DEAHP method and produces better weights and also claims that it maintains the ranking. Of course, we will show that in this proposed model, the ranking won't be kept with adding new option and the previous order will be destroyed with adding the new option.

In continue the paper will be as follow: in section 2, the proposed model by Wang et al. [11], is presented. In section 3, the violation example will be presented and in the final section, the conclusion of paper will be presented.

## 2. METHODS AND RESULTS

### 1. LP-GFW method

Suppose A= $[a_{ij}]_{n\times n}$ is pair comparison matrix. The LP model to estimate the most desirable weight of options and criteria is as follow:

$$\begin{aligned} &Max && w_i \\ &s.t. && (A-nI)W \geq 0, \\ & && e^T W = 1, \\ & && W \geq 0. \end{aligned} \tag{1}$$

Where, I is matrix n×n with the elements on the main diagonal 1 and other elements are zero, $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of matrix A and $e^T= (1,\ldots,1)$.

The above model seeks a maximum weight for each criterion or option. Thus, the defined maximum weights by this model are called the most desirable weights which the most desirable obtained weight through this model is unique and is one which does not have the DEAHP defects. Wang et al. [11] have proposed the following model to maintain the rank while adding new option and they claim that the ranking will be maintained with solving this model:

$$\begin{aligned} &Max && w_i \\ &s.t. && [A-(n+1)I]W \geq 0, \\ & && \sum_{j\neq j_0} w_j = 1, \\ & && w_j \geq 0, \qquad j=1,2,\ldots,n+1. \end{aligned} \tag{2}$$

By putting aside the new option from limit, the normalization of the most desirable final weights of all existing options would be maintained and their ranks also can be kept. But the above mentioned model has two substantial problems that are:

1- With solving the above model, the fourth principle of hierarchy analysis process will be ignored. According to principle 4, the evaluation process should be done again by adding the new option, while evaluating with the above model does not have the same results with re-evaluating.

2- Regardless of the first problem, the model does not save the rank.

In the next section, we showed with a simple example that two above mentioned problems are presented correctly.

### 2. Violation example

Consider the following paired comparison matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix}$$

By using of model 1, we can obtain weights. Therefore, that is sufficient to solve the following problem:

$$\begin{aligned}
&\text{Max} \quad w_i \qquad (i=1,2,3)\\
&\text{s.t.} \quad -2w_1+2w_2+3w_3 \geq 0,\\
&\qquad 1/2w_1-2w_2+2w_3 \geq 0,\\
&\qquad 1/3w_1+1/2w_2-2w_3 \geq 0,\\
&\qquad w_1+w_2+w_3=1,\\
&\qquad w_1, w_2, w_3 \geq 0.
\end{aligned}$$

Then, the obtained weights are $w_1^*=0.541, w_2^*=0.298, w_3^*=0.163$.

So, the ranking of weight are as follow: $w_1^* > w_2^* > w_3^*$

Now, we add another option. The paired comparison matrix would be as follow:

$$A=\begin{bmatrix} 1 & 2 & 3 & 1/2\\ 1/2 & 1 & 2 & 1\\ 1/3 & 1/2 & 1 & 4\\ 2 & 1 & 1/4 & 1 \end{bmatrix}$$

Now we bring up two above mentioned problems for this example: We will obtain the weights from the LP-GFW model using the model in which a new option is added.

$$\begin{aligned}
&\text{Max} \quad w_i \qquad (i=1,2,3,4)\\
&\text{s.t.} \quad -3w_1+2w_2+3w_3+1/2w_4 \geq 0,\\
&\qquad 1/2w_1-3w_2+2w_3+w_4 \geq 0,\\
&\qquad 1/3w_1+1/2w_2-3w_3+4w_4 \geq 0,\\
&\qquad 2w_1+w_2+1/4w_3-3w_4 \geq 0,\\
&\qquad w_1+w_2+w_3 \qquad =1,\\
&\qquad w_1, w_2, w_3, w_4 \geq 0.
\end{aligned}$$

By solving this model we would have:

$$w_1^*=0.533, w_2^*=0.411, w_3^*=0.529, w_4^*=0.450.$$

So, by using the obtained weights, the ranking of weights will be as follow:

$$w_1^* > w_3^* > w_4^* > w_2^*$$

It is seen that with adding the new option, the previous ranking would be destroyed and it demonstrates that the second problem is true. Additionally, if we do the hierarchical analysis of evaluation again according to principle 4, we will have:

$$
\begin{aligned}
\text{Max} \quad & w_i \qquad (i = 1,2,3,4) \\
\text{s.t.} \quad & -3\,w_1 + 2\,w_2 + 3\,w_3 + 1/2 w_4 \geq 0, \\
& 1/2 w_1 - 3\,w_2 + 2\,w_3 + w_4 \geq 0, \\
& 1/3 w_1 + 1/2 w_2 - 3\,w_3 + 4\,w_4 \geq 0, \\
& 2\,w_1 + w_2 + 1/4 w_3 - 3\,w_4 \geq 0, \\
& w_1 + w_2 + w_3 + w_4 = 1, \\
& w_1, \quad w_2, \quad w_3, \quad w_4 \geq 0.
\end{aligned}
$$

Then we will have: $w_1^* = 0.414, w_2^* = 0.323, w_3^* = 0.390, w_4^* = 0.310.$

So, the ranking is as follow: $w_1^* > w_3^* > w_2^* > w_4^*$

It is seen that the result of re-using the model differs from that of using the proposed model. Therefore, it is demonstrated that the model that the LP-GFW presents for adding the new option is in contrast with the principle 4 of AHP in which whenever a change occurs in a hierarchical structure, the evaluation process must be performed again.

## 3. CONCLUSIONS

Data envelopment analysis (DEA) is a nonparametric method in operations research and economics for the estimation of production frontiers. It is used to empirically measure productive efficiency of decision making units. On the other hand, analytical hierarchy process (AHP) is a useful tool in the field of multi-criteria decision-making problems and it can be helpful to rank the set of options with distinct criteria and choose the best or most appropriate option among them. Wang et al. [11] in 2008 proposed a new method called linear programming approach to determine the most desirable weight from paired comparison matrixes in analytical hierarchy process that does not have disadvantages of DEAHP method and produces better weights and also claims that it maintains the ranking. But their model has substantial problems.

The problem is that the fourth principle of hierarchy analysis process will be ignored. According to principle 4, the evaluation process should be done again by adding the new option, while evaluating with their model does not have the same results with re-evaluating. Also, Regardless of the first problem, the model does not save the rank.

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