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A Method to Recognize Congestion in FDH Production Possibility Set

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ABSTRACT

In a business market or Decision Making Unit (DMU) congestion occurs when reducing some inputs causes an increase outputs. Recently economical implications of congestion in data envelopment analysis (DEA) have been studied. All of models that have investigated congestion are in Production Possibility Set (PPS) with constant or variable technology. so, DMUs might be compared with unreal DMUs (virtual DMUs) that sometimes are meaningless in real life and some of them have computational complexity. Therefore, we are going to introduce a new simple approach to recognize congestion by Free Disposal Hull (FDH) PPS. We will show that the proposed approach realizes congestion only with paired comparisons. Numerical examples are also prepared for illustration.
KEYWORDS: Free Disposal Hull; Congestion; Data Envelopment Analysis; Production Possibility Set.

1. INTRODUCTION

Most of DMUs or business markets desire to use resources (inputs) as much as possible to have more supply (outputs) but sometimes increasing inputs will cause decreasing outputs. This condition referred to as congestion. So, recognition it helps managements with decreasing resources (inputs) to get more supply (outputs).

First of all, congestion was introduced by Fare and Svensson (1980) [9] and subsequently a DEA model was proposed to recognize congestion by Fare et al. (1983) [10]. Cooper et al. (2001) [2] introduced an alternative approach for evaluating congestion. Cooper et al. (2002) [7] proposed a one model approach to congestion by DEA. Tone and Sahoo (2004) [18] proposed a method to recognize congestion with return to scale (RTS) concept. But, when the model has alternative optimal solutions it cannot give correct results. So a modified approach that measures the degree of congestion under the occurrence of multiply solution was proposed by Suyoshi and Sekitani in 2009[17].This model is comprehensive but its computations is complex. All of these models deal with constant or variable technology, so DMUs might be compared with unreal DMUs (virtual DMUs) that sometimes are meaningless in real life. For the first time in 1984 another PPS, the Free Disposal Hull, to which most of research considerations were taken, was formulated by Deprins, Simar et al. and Tulkens et al. This PPS is based on the observed activities, possibility and smallest set principles. The models in this PPS don't need to solve a linear programming and only with paired comparisons we can achieve optimum solution. Since reference set is included real DMUs, thus is More matches with real life. So this paper introduces a new method for identifying and recognizing congestion in FDH. The following sections, we will define some concepts and review some models (section 2), and a new method for recognizing congestion in FDH will be introduced in section 3. We have also empirical examples in section 4, and conclusion is given in section 5.

2. BACKGROUND

Suppose that we have n DMUs that each one has m inputs to produce s outputs. let $x_{ij} \geq 0$ be the level of ith input(i=1,…,m) and $y_{rj} \geq 0$ the level of rth output(r=1,…,s) of DMU_j (j=1,…,n). (Input-Output vector $(X_O, Y_O) = (x_{1o}, \dots, x_{mo}, y_{1o}, \dots, y_{so})$)

2.1 Definition 1

DMU_o is *technical efficient* if the evidence shows that, it is not possible to improve some of its inputs or outputs without worsening any of other inputs or outputs.

2.2 Definition 2

DMU_o is *technical inefficient* if the evidence shows that, it is possible to improve some of its inputs or outputs without worsening any of other inputs or outputs.

A PPS based on; 1: Observed activities 2: Possibility and 3: Smallest set principles was made like this

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$T_{FDH_v} = \cup_{j=1}^{n} \{(X,Y) | X \geq X_j, Y \leq Y_j\}$

So evaluating DMU_o in this PPS is as follows

$$\begin{aligned} &\varphi_o = \max_{1\leq j\leq n} \max \varphi_j^o \\ &s.t \quad x_{ij} + s_i^- = x_{io} \quad i = 1 \dots m \quad (a) \qquad (1) \\ &\quad y_{rj} - s_r^+ = \varphi_j^o y_{ro} \quad r = 1 \dots s \quad (b) \end{aligned}$$

According to (b) we have

$$\varphi_j^o \leq \frac{y_{rj}}{y_{ro}}, \quad r = 1 \dots s \; (y_{ro} > 0)$$

So

$$\varphi_j^o = \min \left\{ \frac{y_{rj}}{y_{ro}} \middle| r = 1 \dots s \;\; y_{ro} > 0 \right\} \& (x_{ij} \leq x_{io} \; i = 1 \dots m)$$

In T_{FDH_v} we accept possibility in inputs, hence in each DMU with increasing input (any amount) output is at least available with initial amount (output will not decrease) but sometimes it is not. In some cases increasing input causes decreasing output. This condition is called as congestion.

2.3 Definition 3

DMU_o has congestion if increasing one or more inputs cause decreasing one or more outputs without improving any of its other outputs, and conversely, reductions one or more inputs can be associated with increasing one or more outputs.

For example, if a large number of miners work in a mine, it may lead to reduction output, because they don't have enough space to work.

Congestion is a kind of inefficiency but not technical inefficiency. It is an inefficiency that is due to the accumulation of inputs.

3. THE PROPOSED MODEL

This section presents a model for identifying congestion in FDH technology

For this purpose we introduce a PPS with the observed activities, possibility in outputs and smallest set principles(T'_{FDH_v}).

$T'_{FDH_v} = \cup_{j=1}^{n} \{(X,Y) | X = X_j, Y \leq Y_j\}$

So evaluating DMU_o in this PPS is as follows

$$\begin{aligned} &\varphi'_o = \max_{1\leq j\leq n} \max \varphi'^o_j \\ &s.t \; x_{ij} = x_{io} \quad i = 1 \dots m \qquad (2) \\ &y_{rj} - s_r^+ = \varphi'^o_j y_{ro} \quad r = 1 \dots s \end{aligned}$$

3.1Theorem1

Model (2) is feasible.

Proof:

$(x_{i0}, y_{ro}), s_r^+ = 0 (i = 1 \dots m, r = 1 \dots s)$ Is a feasible solution for (2).■

To identify congestion, the following method is introduced.

First we evaluate DMU_o with model (1).if $\varphi_o^* = 1$ and $s_r^{+*} = 0 (r = 1 \dots s)$ then DMU_o is on T_{FDH_v} frontier and it is efficient (strong or weak) so congestion is not exist in DMU_o.

Afterward model (2) is applied for the rest of DMUs

Suppose that $\varphi'^*_o = 1$ namely DMU_o is on T'_{FDH_v} frontier. Otherwise we project DMU_o on T'_{FDH_v} frontier as follow:

$$\begin{cases} \hat{X}_o = X_0 \\ \hat{Y}_o = \varphi'^*_o Y_o + S^{+*} \end{cases}$$

So after this we suppose DMU_o is on T'_{FDH_v} frontier. Then reemploy model (1) for these DMUs. Now if φ_o^*=1 then DMU_o is inefficient but doesn't have any congestion.

Otherwise set $K_o = \{ j \epsilon J \,| X_j \leq \hat{X}_o , Y_j \geq \varphi_o^* \hat{Y}_o , j \neq o \}$ that $J = \{1 \dots n\}$

3.2 Theorem 2

The K_o is not empty.

Proof: because the reference set belongs to it. ■

If $card(K_o) = 1$ then consider DMU_t that $t \epsilon K_o$ else we use the following model

$$\begin{aligned} &\min_{j \epsilon K_o} s_o = \sum_{i=1}^{m} s_{ij} \\ &x_{ij} + s_{ij} = x_{io} \quad j \epsilon K_o \\ &\quad s_{ij} \geq 0 \end{aligned}$$

This model finds the nearest DMU to DMU_o (from K_o set)
Suppose that DMU_t is the optimal solution.
Now we define T set as
$T = \{j\epsilon J | X_j \geq X_t , X_j \neq X_t\}$
Namely the set of DMUs that have more inputs than DMU_t

3.3 Theorem 3

If $T = \emptyset$ then DMU_o is without congestion.
Proof:
because there is not DMU that can produce more outputs with more inputs.
If $T \neq \emptyset$ we use the following model
$\beta = \max_{j\in T} \max \beta_j$
$y_{rj} \geq \beta_j y_{rt} \quad r = 1 \dots s \quad j\epsilon T \quad (4)$

3.4 Theorem 4

If $\beta^* \geq 1$ then DMU_o is without congestion.
Proof:
Because there are some DMUs that can produce more outputs with more inputs.

3.5Theorem`5

If $\beta^* < 1$ then DMU_o has congestion.
Proof:
Because all of DMUs that have more inputs than DMU_t cannot produce more output than DMU_t.

4. Numerical examples

4.1Example 1

Consider the nine DMUs of Table 1 with single input and single output. We illustrate the method by themes

Table1. The nine DMUs

DMU	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Input	1	2	3	3	4	4	5	5	6
Output	2	6	1	4	3	6	3	5	3

This data is plotted in Figure 1

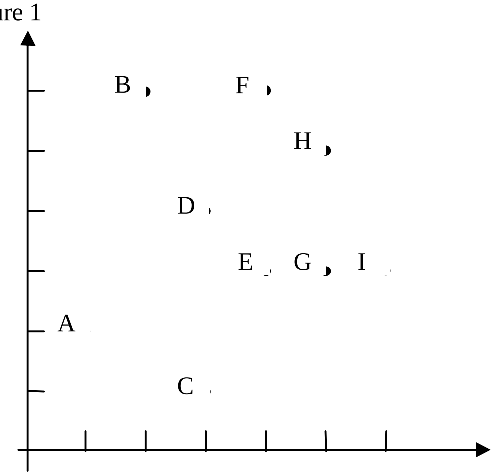

Figure1. The nine DMUs

It is illustrated T_{FDH_v} and T'_{FDH_v} in figure 2, 3

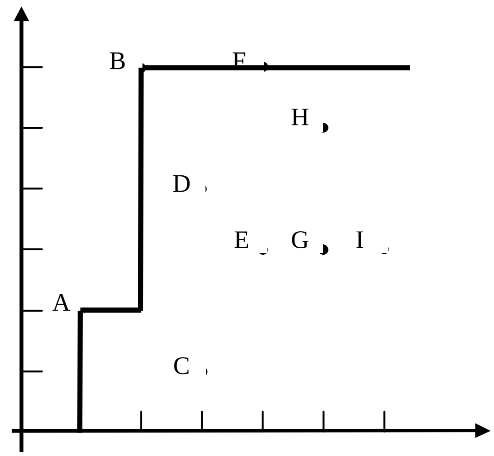

Figure2. T_{FDH_v}

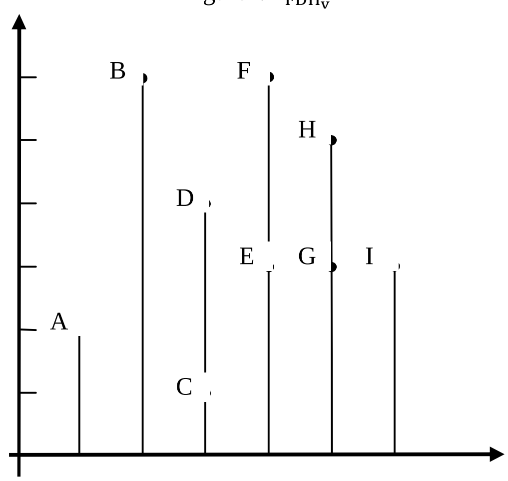

Figure 3. T'_{FDH_v}

By model (1) DMUs, A, B and F are efficient, so they don't have congestion
Now we employ model (2) for other DMUs

Table 2.the $\varphi_o'^*$, $\hat{X}_o$, $\hat{Y}_o$ for these DMUs

DMU	C	D	E	G	H	I
$\boldsymbol{\varphi_o'^*}$	3	1	2	5/3	1	1
$\hat{\mathbf{X}}_o$	3	3	4	5	5	6
$\hat{\mathbf{Y}}_o$	4	4	6	5	5	3

So after this we use Table 2. And reemploy model (1) for these DMUs

Table3. The φ_o^* for these DMUs

DMU	$\hat{C}$	$\hat{D}$	$\hat{E}$	$\hat{G}$	$\hat{H}$	$\hat{I}$
$\boldsymbol{\varphi_o^*}$	3/2	3/2	1	6/5	6/5	2

Hence according to Table 3 DMU_E is inefficient and doesn't have congestion.
Now sets K_o and T, DMU_t for the rest DMU's are
$K_C = \{B\} \rightarrow DMU_t = B \rightarrow T = \{C, D, E, H, G, I, F\}$
$K_D = \{B\} \rightarrow DMU_t = B \rightarrow T = \{C, D, E, H, G, I, F\}$
$K_G = \{B, F\} \rightarrow DMU_t = F \rightarrow T = \{H, G, I\}$
$K_H = \{B, F\} \rightarrow DMU_t = F \rightarrow T = \{H, G, I\}$
$K_I = \{B, F\} \rightarrow DMU_t = F \rightarrow T = \{H, G, I\}$

Table 4.the β_O^* for these DMUs

DMU	C	D	G	H	I
β_0^*	1	1	5/6	5/6	5/6

By Theorems 4, 5 and Table 4 we conclude DMUs C, D don't have congestion and G, H, I have congestion.

4.2 Example 2

In this example, the proposed approach is compared with Tone and Sahoo(2004)[18] and Sueyoshi and Sekitani 's approaches[17].

For this purpose let us use four DMUs with two inputs and two outputs of Table 4(data is listed in [18]). The congestion identification of this study is compared with their result on congestion

Table5. An illustrative data

DMU	Input 1	Input 2	Output 1	Output 2
1	1	1	1	1
2	2	2	2	2
3	2	3	2	1
4	3	3	1	1

These data is listed in [18]

Table 6 summarizes the congestion identification results measured by these three different approaches.

Table6. Congestion identification by three approaches.

DMU	Identification of[18]	Identification of[17]	Proposed approach				
1	Not congestion	Not congestion	$\varphi_1^* = 1$	$S^{+*} = 0$	Not congestion		
2	Not congestion	Not congestion	$\varphi_2^* = 1$	$S^{+*} = 0$	Not congestion		
3	Weak congestion	Wide congestion	$\varphi_3^* = 1$	$S^{+*} = 1$	$K_3 = \{2\}$	$\beta_3^* = 1/2$	congestion
4	Strong congestion	Wide congestion	$\varphi_4^* = 1/2$	$S^{+*} = 0$	$K_4 = \{2\}$	$\beta_4^* = 1/2$	congestion

The approach of Tone and Sahoo determines that DMU {3} has weak congestion and DMU {4} has strong congestion. The Sueyoshi and Sekitani's approach indicates that two DMUs {3, 4} are widely congested. As it can be seen the proposed approach also indicates that two DMUs {3, 4} are congested. The proposed approach in this study needs to solve maximum five Paired comparison problems in order to identify congestion.

5. Conclusion

All of models that have investigated congestion were in Production Possibility Set with constant or variable technology, so DMUs might compare with unreal DMUs (virtual DMUs) that sometimes are meaningless in real life further some of them have computational complexity. Because of this we introduced a new simple approach to recognize congestion by FDH technology, This method showed that congestion can be found only by paired comparisons and does not require solve linear programming models. The proposed approach used in our paper identifies congestion and reduces the computational effort required for congestion identification. It can be seen that, in example 2, the proposed approach also indicates congestion identification as Tone and Sahoo(2004)[18] and Sueyoshi and Sekitani 's approaches[17] is, but by simpler method.

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