Application of the FDTD Method to Identification of Detected Objects in Archaeology

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ABSTRACT

Geological radars or GPR (Ground Penetrating Radar), are electromagnetic systems used for the non-destructive study of subsoil. They are based on the transmission by an antenna coupled to the ground, of electromagnetic pulses with short duration or infrequent of harmonic waves sweeping a certain frequency band.

The goal of the paper is to study the temporal and spatial discretization of Maxwell’s equations and the formulation of iterative terms, which calculate numerically the electromagnetic fields. For this, we have written a code in FORTRAN 90 based on the FDTD numerical method that we have applied to some electromagnetic problems and also a model, which is interesting in archeology, the wall in this study.

KEYWORDS: GPR; FDTD; Archaeological Objects, Walls

1. INTRODUCTION

Ground Penetrating Radar (GPR) is used to image the subsurface. It is a geophysical method based on the propagation, reflection and scattering of high frequency (from 10 MHz to 2.5 GHz) electromagnetic (EM) waves in the subsurface. The investigation depth depends on the EM wave attenuation, which grows as the conductivity of the subsoil materials increases, and on the frequencies used. Generally, the penetration depth is higher at lower frequency and varies from about 1m to some tens of meters [1].

GPR is used for the exploration of the subsoil in several research fields such as the detection of landmines [2], geology [3], civil engineering [4], glaciology [5, 6] and archeology [7]. It was also successfully applied to detected objects in archaeology [8].

The Finite Difference Time Domain (FDTD) method has proven to be one of the most popular techniques reported in literature. Reasons for this are that the FDTD approach is conceptually simple in contrast to other methods, is accurate for arbitrarily complex models, and is capable of accommodating realistic antenna designs and important features such as dispersion in electrical properties [9][10]. Computational tools, such as FDTD has become a powerful tool for GPR users, since the time-domain nature of FDTD-based programs enables the visualization of the causal evolution of complex electromagnetic phenomena such as the propagation of electromagnetic pulses in GPR scenarios which involve layered media, dispersive media, objects of arbitrary shape, etc..

In this paper we present FORTRAN [11] and Matlab [12] Code for the FDTD modeling of GPR in 2D and 3D. Geometries are modeled using a Transverse Electric (TE) mode formulation. Differential notation is used in the codes wherever possible to optimize them for speed in the FORTRAN environment. We implement Perfectly Matched Layer (PML) absorbing boundaries. Although our codes are 2D and 3D. Three-Dimensional Finite-Difference Time-Domain is a powerful method for modelling the electromagnetic field. The 3D FDTD buried object detection model is emerging as a useful in archaeology to help the archaeologists for interpretation the results, such as the wall.

The remainder of the paper was organized as follows: In section II is for explaining the methodology, in section III is for presentation of numerical result, finally, in section VI is for presentation of conclusion.

1. METHODOLOGY

A. Theory

The propagation of electromagnetic fields is governed by the Maxwell equations,

\[ \frac{\partial D}{\partial t} = \nabla \times H - J \quad (1a) \]
\[ \frac{\partial B}{\partial t} = \nabla \times E \quad (1b) \]
\[ \nabla \cdot D = \rho \quad (1c) \]
\[ \nabla \cdot B = 0 \quad (1d) \]

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Where $B$ is the magnetic flux density in V/m$^2$, $D$ is the electric flux density in A/m$^2$, $H$ is the magnetic field in A/m, $E$ is the electric field in V/m, $J$ is the electric current density in A/m$^2$ and $\rho$ is the electric charge density in As/m$^2$.

The FDTD method directly solves a discrete difference form of Maxwell’s equations for closely spaced points on an orthogonal mesh. Since the FDTD method is performed by repeatedly applying the same equations over a mesh in such a way that information is only exchanged between nearest neighbors.

For the TE mode the FDTD equations becomes

$$
\begin{align*}
E_x^{n+1}_{i+\frac{1}{2},j} &= D_a|_{i+\frac{1}{2},j} E_x^n_{i+\frac{1}{2},j} + D_b|_{i+\frac{1}{2},j} \left[ \frac{H_x^{n+\frac{1}{2},j} - H_y^{n+\frac{1}{2},j}}{\Delta y} \right] \quad (2a) \\
E_y^{n+1}_{i,j+\frac{1}{2}} &= D_a|_{i,j+\frac{1}{2}} E_y^n_{i,j+\frac{1}{2}} - D_b|_{i,j+\frac{1}{2}} \left[ \frac{H_x^{n+\frac{1}{2},j+1} - H_y^{n+\frac{1}{2},j+1}}{\Delta x} \right] \quad (2b) \\
H_z^{n+\frac{1}{2},j+\frac{1}{2}} &= H_z^{n-\frac{1}{2},j+\frac{1}{2}} + C^{\frac{1}{2},j+\frac{1}{2}} \left[ \frac{E_x^{n+\frac{1}{2},j} - E_x^{n+\frac{1}{2},j}}{\Delta y} - \frac{E_y^{n+\frac{1}{2},j+1} - E_y^{n+\frac{1}{2},j+1}}{\Delta x} \right] \quad (2c)
\end{align*}
$$

Where,

$$
\begin{align*}
c &= \frac{\Delta t}{\mu} \\
D_a &= \frac{1 - \sigma \Delta t / 2 \varepsilon}{1 + \sigma \Delta t / 2 \varepsilon} \\
D_b &= \frac{\Delta t / \varepsilon}{1 + \sigma \Delta t / 2 \varepsilon}
\end{align*}
$$

In the perfectly matched layer, PML, [7] is used as an absorbing boundary condition (ABC) to simulate an open space [8,9]. Apart form its numerical efficiency, one of the major advantages of the PML over previously proposed ABCs when simulating dispersive media is that its absorption properties hold independently of the frequency, thus not requiring the knowledge of the dispersive nature of the propagating field.

**B. The FDTD Algorithm**

Figure 1 provides a flowchart of the algorithm that we used to implement the FDTD method. The algorithm starts by performing an initialization operation. Initialization is performed by first determining the mapping of data structures, then loading the standard mesh into the FDTD mesh. After initialization the algorithm enters the main loop of the flowchart. After each iteration of handling boundary conditions, updating fields by using the Finite Difference equations, and incrementing the time step counter. At appropriate times the loop takes a detour to write data to files. While the FDTD equations used to update electromagnetic field values can be considered to be the core of the algorithm, this aspect is actually one of the easier parts of the algorithm to implement.
2. Numerical results

We’ll choose the electric field $E_x$ of the source in Sinusoidal form:

$$S(n) = \sin 2\pi nf\Delta t$$  \hspace{1cm} (3)

Figure 2 show the simulation of the propagation for this wave, first in the vacuum and then in the dielectric medium $\varepsilon_r$ after crossing the interface (vacuum-dielectric) located at $k=100$. The frequency of the wave is fixed at $f=700$ MHz.

Steps are fixed at $\Delta x=0.01$ m and $\Delta t=\Delta x/c_0$.

Figure 2: Simulation of propagation of a sinusoidal wave of frequency $f = 700$ MHz incident on a medium of relative dielectric constant $\varepsilon_r=4$. 
When the sinusoidal wave encounters the dielectric medium, a part of it is transmitted and the other part is reflected. However notice that the wave length $\lambda_d$ in the dielectric medium is lower than $\lambda_0$ (Wave length of the vacuum), which is consistent with the theory that provides, in this case, wave length in the dielectric [2]:

$$\lambda_d = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = 0.214 \text{ m}$$

The one that’s been calculated by our code is:

$$\lambda_d = 0.2 \text{ m}$$

A. **Propagation in a medium with losses**

Studying now the case of a dielectric medium with losses characterized with a conductivity $\sigma$ and relative dielectric constant $\varepsilon_r$. The one that’s been calculated by our code is:

$$\lambda_d = 0.2 \text{ m}$$

B. **Applying the FDTD method to simulate a cement wall**

The aim is to study the structure of a cement wall using the FDTD method and obtain information about this structure. We can use this information in several fields such as archeology.

The simulation Results for this work is obtained by using a code in Fortran 90. The medium used in our simulation correspond to an homogeneous medium (dry sand with dielectric properties ($\varepsilon_r = 3$ et $\sigma = 0.0001 \text{ S/m}$) contain a simulated model of a cement wall with dielectric properties ($\varepsilon_r = 5.5$, $\sigma = 0.043 \text{ S/m}$). The dimension of this model is 10 m x 15 m simulated in a frequency $f = 700$MHz.

Isosurfaces are suggested as the best method for imaging small areas in high detail [13]. An example of an isosurface map produced for the simulated model in different iterations (Figure 4). This figure indicates the presence of walls (yellow patch).

**Figure 3:** Simulation of propagating a sinusoidal wave hitting a lossy dielectric material and a dielectric constant $\varepsilon_r = 4$, with a conductivity $\sigma = 0.04 \text{ (S / m)}$ and $f = 700$MHz

The curve represented in the figure 3 show clearly that the magnitude of the transmitted wave attenuate with distance $z$.

**Figure 4:** Isosurface 2D simulation of a wall at a frequency $f = 700$ MHz
The 3D Image presented in the figure 5 show the existence of defects. These defects could correspond to walls (red patch). This figure helps archaeologists to interpret the results collected by GPR systems.

Figure 5: 3D representation of a wall

3. Conclusion

In this work, we’ll study the simulation of GPR radar signals using numeric code (programming code). This code is based on Finite-difference time-domain method (FDTD). Indeed, the modeling is a scientific tool that allow to provide results of archeological objects (or their residues) using Simulation. The result has helped the archaeologists to plan their next excavation season, since they can indicate the built structures, like house foundations, such as the walls (Fig 4 and Fig 5).

4. BIBLIOGRAPHY