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Viscous Damping of Surface Waves

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ABSTRACT

In this article an expression for the decay of horizontal velocity is obtained in a semi-infinite channel flow of a viscous liquid for weakly viscous fluids, by similarity solution. For viscous liquids when linear velocity profiles with z exist; the derivative $\frac{\partial^2 u}{\partial z^2}$vanish. Near the free surface; horizontal velocity can be determined from the obtained diffusion equation. The amplitude of the vertical wave on the free surface is also obtained as an integral. Horizontal velocity and amplitude of vertical displacement are attenuated since energy supplied by the source is dissipated by viscous forces.

KEYWORDS: Surface waves - viscous damping - similarity solution - continuity equation - momentum equation.

1. INTRODUCTION

Surface wave greatly affects marine navigation. Great risks are confronted by vessels and boats travelling across oceans and seas. These waves are strong and high enough to sink ships. Meteorological reports are provided to give warnings. Although seas and oceans are formed with water which is regarded invicid, viscous surface waves are encountered in oil tanks and in many other industries.

The subject of this article deals with two main topics; surface waves[1,2,3,4,5,6], and viscous dissipation[7,8,9]. Reference is made to some important articles in each topic.

Formulation and analysis

Consider a horizontal layer of viscous liquid of fixed depth extending horizontally along the x-axis from x=0 to x→ ∞. The z-axis extends vertically from the bottom of the layer (z= -H) to the free liquid surface (z= 0). At x = 0, a reciprocating source causes horizontal velocity $u(x = 0, z) = u_0 \cos \Omega t$; u_0, Ω are specified parameters. Due to viscous dissipation of the source energy; a damped horizontal velocity front will occur. We seek an expression of this damped velocity.

The horizontal motion of this viscous liquid will be subject to the Navier-Stokes equations given by

$$\frac{\partial u}{\partial t} + \left(\underline{u} \cdot \underline{\nabla}\right) u = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u \tag{1}$$

The following notation is adopted:

Horizontal velocity u,
vertical velocity w,
the velocity vector $\underline{u} = (u, w)$,$\nabla^2 u = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$,
the pressure p,
the density ρ,
and the kinematic velocity ν.

The time is t and the horizontal position is x. If the frequency Ω is relativelyhigh; and the amplitude u_0 is relatively low. The quadratic term $\left(\underline{u} \cdot \underline{\nabla}\right) u$ can be neglected and we have:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u \tag{2}$$

Although the flow is essentially viscous, we postulate that in wide passage velocity gradient $\frac{\partial u}{\partial z} = k$. i. e. the velocity gradient is linear. Furthermore; for small k we have $\nabla^2 u = \frac{\partial^2 u}{\partial x^2}$ since k is a constant or nearly a constant.The flow is approximately irrotational.

$$\left(\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} - \nu \nabla^2 u\right) = 0 \tag{3}$$

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We also add that in the neighborhood of the free surface; the pressure is uniform, andhasits atmospheric value with the gauge value zero ($p|_{z=0} = 0$). The pressuregradient $\frac{\partial p}{\partial x}$ vanishes, accordingly, we have :

$$\frac{\partial u}{\partial t} - \nu \nabla^2 u = 0 \tag{4}$$

Introducing $\frac{\partial^2 u}{\partial z^2} \cong 0$, we have:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} \tag{5}$$

Introducing the similarity variable[10] $\eta = \frac{x}{2\sqrt{\nu t}}$, the solution of (5) is

$$u(\eta) = c_1 + c_2 \int_0^{\frac{x}{2\sqrt{\nu t}}} e^{-\eta^2} \, d\eta \tag{6}$$

Together with the conditions

$$u(0,t) = u(\eta = 0) = u_0 \cos \Omega\, t, \tag{7a}$$

and

$$u(\infty, t) = u(x, 0) = u(\eta = \infty) = 0 \tag{7b}$$

It follows that $c_1 = u_0 \cos \Omega t$ and $c_2 = - u_0 \cos \Omega t$ giving

$$u(x,t) = u_0 \cos \Omega t \; erfc\left(\frac{x}{2\sqrt{\nu t}}\right) \tag{8}$$

Where $erfc\left(\frac{x}{2\sqrt{\nu t}}\right)$ is the complementary error function defined by $erfc(\eta) = \frac{2}{\sqrt{\pi}} \int_\eta^\infty e^{-\theta^2} \, d\theta$. Figure 1 shows schematic of the problem. Also, Figure 2 shows a plot of equation (8) for different values of η.

To obtain the vertical displacement of the free surface ξ; we first obtain the vertical velocity w from the continuity equation for an incompressible liquid

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

$\therefore \frac{\partial w}{\partial z} = -\frac{\partial u}{\partial x}, \; \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}\left(\frac{d\xi}{dt}\right) = -\frac{\partial u}{\partial x}$. Integrating with respect to z, we get
$w = \frac{d\xi}{dt} = c_3(x,t) - z\frac{\partial u(x,t)}{\partial x}$, c_3 is a constant of integration.

On the bottom z=-H, $\frac{\partial u}{\partial x} = 0 = \frac{d\xi}{dt}$, accordingly $c_3 = 0$ and

$$\frac{d\xi}{dt} = -\xi \frac{\partial u}{\partial x} \text{ i.e. } \xi = \xi_0 e^{-\int_0^t \frac{\partial u}{\partial x} dt} \tag{10}$$

The integral in (10) cannot be obtained in a closed form. Series solution can be obtained and truncation to any required degree of accuracy is suggested.

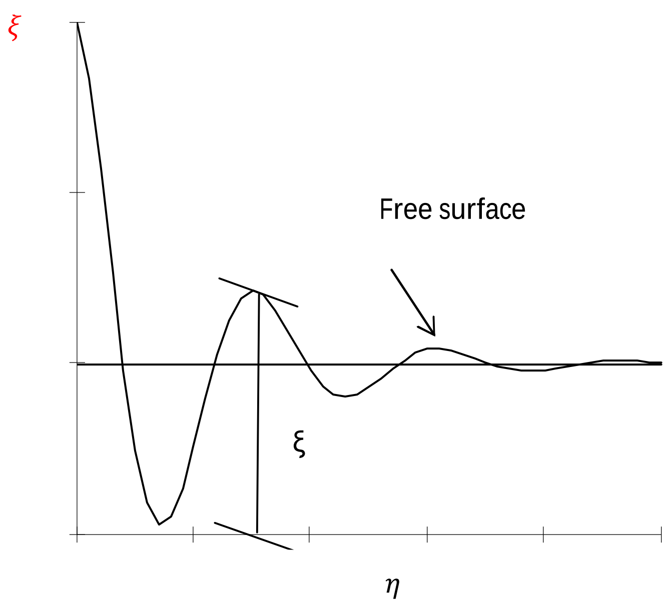

Figure 1: Schematic of the problem

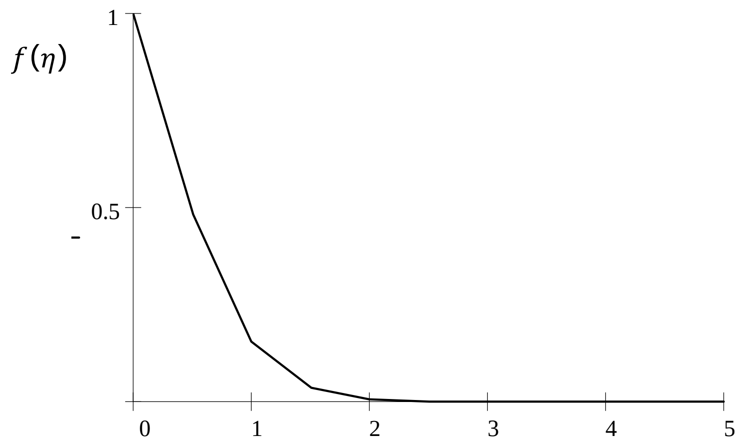

Figure 2: $f(\eta) = erfc(\eta)$ η

Conclusion

Similarity solution of the resulting diffusion equation for the horizontal velocity of the free surface is obtained. An integral for the vertical amplitude of the wave on the free surface is also provided. The velocity is attenuated since energy supplied by the source is dissipated by viscous forces.

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