J. Basic. Appl. Sci. Res., 3(3)255-258, 2013
© 2013, TextRoad Publication
ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Adomian Decomposition Method with Hermitepolynomials for Solving Ordinary Differential Equations

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ABSTRACT

This paper illustrates the using of Hermite orthogonal polynomials to modify the Adomian decomposition method. This method can be successfully used for different types of ordinary and partial differential equations. The scheme is tested for some examples and the obtained results are compared with usual Adomian Decomposition Method. The results demonstrate that the Hermite polynomials provide the better estimation than usual Adomian Decomposition Method.
KEYWORDS: Adomian decomposition Method, Hermite polynomials, Ordinary differential equations.

1. INTRODUCTION

In the 1980s, George Adomian (1923-1996) introduced a powerful method for solving linear and nonlinear differential equations. Since then, this method is known as the Adomian decomposition method (ADM) [1,2]. The technique is based on a decomposition of the solution of a nonlinear differential equation in a series of functions. Each term of the series is obtained from a polynomial generated by a power series expansion of an analytic function.

A large variety of methods have been proposed for solving ordinary differential equations see for example [12]. Yucheng Liu employed Legendre polynomials to improve the Adomian decomposition method [3]. Hosseini [4] proposed the method of implementing ADM with Chebyshev polynomials, where the reliability and efficiency of the proposed scheme was verified to be applicable for both linear and nonlinear models. Yahya Qaid Hassan applied modified ADM to solve singular boundary value problems of higher-order ordinary differential equations [5]. D. J. Evans applied ADM for the approximate solution of delay differential equation [6]. M. Alabdullatif applied the ADM to find an analytic approximate solution for nonlinear reaction diffusion system of Lotka-Volterra type [7]. Awatif Hendi applied ADM to solve the linear and nonlinear differential equations to find the neutron energy density and flux, which can be used to calculate the neutron angular intensity through the Pomraning-Eddington approximation [8].

The goal of this paper is to introduce a new reliable modification of Adomian decomposition method with Hermit polynomials. There is a basic qualitative difference between ADM with Hermite polynomials and other methods that this method decreases the order of errors, especially when the computations volume arises.

This paper focuses on the ADM using Hermite polynomials. The Hermite polynomials are a sequence of orthogonal polynomials considered by Askey and Wimp in[9], who analytically derived a number of results about these polynomials. Hermite differential equation is defined as

$$y'' - 2xy' + 2ny = 0, \qquad (1)$$

wherenis a real number. The Hermite polynomials $H_n(x)$ can be expressed by Rodrigues' formula

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right), \qquad n = 0,1,2,\ldots \qquad (2)$$

They also defined recursively by using the following recurrence relation

$$H_0(x) = 1,$$
$$H_1(x) = 2x,$$
$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x), \qquad n \geq 1. \qquad (3)$$

Hermitepolynomials $H_n(x)$, form a complete orthogonal set on the interval $-\infty < x < \infty$ with respect to the weight function e^{-x^2}. This paper applies Hermite polynomials [9-11] to modify the ADM and compares with ADM on the basis of Taylor series expansion.

The remaining structure of this article is organized as follows: a brief introduction to the ADM and modified ADM is presented in section 2 and section 3, respectively. One example is documented in section 4. The last section includes our conclusion.

2. Adomian Decomposition Method

The Adomian decomposition method has been used in [1,2] to solve effectively, easily and accurately a large class of linear and nonlinear, ordinary, partial, deterministic or stochastic differential equations with

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approximate solutions which converge rapidly to accurate solutions. In this part, the concept of Adomian decomposition method is introduced. Consider the differential equation

$Lu + Ru + Nu = g(x)$, (4)

$Lu = g(x) - Ru - Nu$, (5)

where N is a nonlinear operator, L is the highest-order derivative which is assumed to be invertible, R is a linear operator of lower order than L and g is a given function. Applying the inverse operator to both sides of the (5), we get

$u = \varphi(x) + L^{-1}[g(x) - Ru - Nu]$, (6)

$u = \varphi(x) + L^{-1}g - L^{-1}(Ru) - L^{-1}(Nu)$, (7)

where $\varphi(x)$ represents the given conditions and L^{-1} is the inversed operator of L.

According to the decomposition method, we assume that a series solution of the unknown functions u are given by

$u = \sum_{n=0}^{\infty} u_n$. (8)

The nonlinear terms Nu can be decomposed into the in finite series of polynomials given as

$Nu = \sum_{n=0}^{\infty} A_n$, (9)

where the components u_n will b determined recursively, and the A_n's are the so called Adomain Polynomials. Specific algorithms were set in for calculating Adomian's polynomials for nonlinear term.

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{i=0}^{n} \lambda^i u_i)]_{\lambda=0}$, $\quad n \geq 0$. (10)

The components u_n for $n \geq 0$ are given by the following recursive relationships

$u_0 = L^{-1}(g) + \varphi(x)$,

$u_i = -L^{-1}(Ru_{i-1}) - L^{-1}(A_{i-1})$, $\quad i \geq 1$. (11)

Using the above recursive relationships, we construct the solution u as

$u = \lim_{n\to\infty} \vartheta_n$, (12)

where

$\vartheta_n = \sum_{i=0}^{n-1} u_i \quad n \geq 1$. (13)

It is interesting to note that, we obtain the series solution by using the initial condition only.

3. Modified Adomian decomposition method

To solve differential equation by the Adomian decomposition method, for an arbitrary natural number m (expand at $x = 0$), $g(x)$ can be expressed in the Taylor series or Hermite series, that is pointed by $g^{T,m}(x)$ and $g^{H,m}(x)$ respectively, where

$g(x) \approx g^{T,m}(x) = \sum_{n=0}^{m} \frac{g^{(n)}(0)}{n!} x^n$, (14)

$g(x) \approx g^{H,m}(x) = \sum_{n=0}^{m} c_n H_n(x)$, (15)

where

$c_n = \frac{1}{2^n n! \sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-x^2} g(x) dx$, $\quad n = 0, 1, \ldots$. (16)

where $H_n(x)$ is the orthogonal Hermite polynomial and from (2) we can deduce that

n	$H_n(x)$
0	1
1	$2x$
2	$4x^2 - 2$
3	$8x^3 - 12x$
4	$16x^4 - 48x^2 + 12$
…	…

Substitute (15) into (11) yields

$$
\begin{aligned}
u_0 &= L^{-1}(c_0H_0(x) + c_1H_1(x) + \cdots + c_mH_m(x)) + \varphi(x),\\
u_1 &= -L^{-1}(Ru_0) - L^{-1}(Nu_0),\\
u_2 &= -L^{-1}(Ru_1) - L^{-1}(Nu_1), \qquad (17)\\
&\vdots
\end{aligned}
$$

Alternatively, Wazwaz [10] had written (17) as

$$
\begin{aligned}
u_0 &= L^{-1}(c_0H_0(x)) + \varphi(x), \qquad (18)\\
u_1 &= L^{-1}(c_1H_1(x)) - L^{-1}(Ru_0) - L^{-1}(Nu_0),\\
u_2 &= L^{-1}(c_2H_2(x)) - L^{-1}(Ru_1) - L^{-1}(Nu_1),\\
&\vdots
\end{aligned}
$$

Where g(x) (Eq.(15)) can be represented as a standard polynomial form as

$$g(x) \approx \sum_{n=0}^{m} d_n x^n$$

And

$$\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 0 & 12 & . \\ 0 & 2 & 0 & -12 & 0 & . \\ 0 & 0 & 4 & 0 & -48 & . \\ 0 & 0 & 0 & 8 & 0 & . \\ 0 & 0 & 0 & 0 & 16 & . \\ . & . & . & . & . & . \end{bmatrix}_{(m+1)\times(m+1)} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}$$

Therefore Eq. (18) can be converted to

$$\begin{aligned} u_0 &= L^{-1}(d_0) + \varphi(x), \\ u_1 &= L^{-1}(d_1) - L^{-1}(Ru_0) - L^{-1}(Nu_0), \\ u_2 &= L^{-1}(d_2) - L^{-1}(Ru_1) - L^{-1}(Nu_1), \\ &\vdots \end{aligned} \tag{19}$$

Eqs.(17)-(19) are governing equations of modified ADM using Hermite polynomials. The approximate $u(x)$ is obtained from these equations as $u(x) = \sum_{n=0}^{m} u_n$, which can be very close to the Hermite expansion of the exact solution $u(x)$ for appropriate m.

4. NUMERICAL RESULTS

In this section, we solve a differential equation of second order by ADM based on Hermite polynomials. In order to compare the precision of ADM on the basis of Taylor and Hermite, their absolute errors are drown in figure 1.

We consider the following differential equation with the exact solution $u(x) = e^{-x^2}$.

$$u'' + u' - uu' = (-2 + 4x^2 - 2x)e^{-x^2} + 2xe^{-2x^2}, \qquad u(0) = 1, \quad u'(0) = 0 \tag{20}$$

The operator from (20) respect to $Lu = g(t) - R(u) - F(u)$, $0 \le x \le 2$,is

$$L(u) = \frac{d^2u}{dx^2} = u'', F(u) = Nu = -uu', R(u) = u', g(x) = (-2 + 4x^2 - 2x)e^{-x^2} + 2xe^{-2x^2}.$$

Then the inverse operator L^{-1} can be regarded as the definite integral in the following form

$L^{-1} = \int_0^x \int_0^x (.)dxdx.$

The Adomianpolynomials are

$$\begin{aligned} A_0 &= -u_0u_0', \\ A_1 &= -u_0u_1' - u_1u_0', \\ A_2 &= -u_2u_0' - u_0u_2' - 2u_1u_1', \\ &\vdots \end{aligned} \tag{21}$$

In this work we expand $g(x)$ with taylor series and Hermite polynomials (14), (15), then we obtain u_i for $i = 0,1,2,\ldots$ by using (17)and $u(x) = \sum_{n=0}^{m} u_n$.

By Eq. (21),$u_T(x)$, $u_l(x)$ can be evaluated based on $g(x)$, u_i and Adomian polynomials A_n as

$u_0 = L^{-1}(g(x)) + \emptyset(x)$, (22)

$u_k = L^{-1}\left(\frac{du_{k-1}}{dx}\right) - L^{-1}(A_{k-1}), \;\; k \ge 1.$

Case (**A**): Let m=10, we first expand $g(x)$ with Taylor series

$g^{T,10}(x) \approx -2 + 6x^2 - 2x^3 - 5x^4 + 3x^5 + \frac{7}{3}x^6 - \frac{7}{3}x^7 - \frac{3}{4}x^8 + \frac{5}{4}x^9 + \frac{11}{60}x^{10} + O(x^{11})$,(23)

By using Eq. (22) we obtained$u_T(x)$ based on Eq. (23) as

$$u^{T,10}(x) = \sum_{m=0}^{10} u_m = 1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6 + \frac{1}{24}x^8 - \frac{1}{120}x^{10} + \frac{1}{720}x^{12} - \frac{31}{9360}x^{13} + \cdots$$

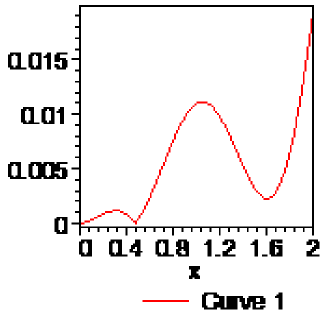
Case (**B**): By setting $m = 10$ and from recurrence relation (14) and Eqs. (15), (16) we can have

$$\begin{aligned} g^{H,10} \approx\; & -1.9308192 - 0.3438746x + 5.0148302x^2 - 0.2099379x^3 - 2.8829793x^4 + 0.1673955x^5 \\ & + 0.6541658x^6 - 0.0281526x^7 - 0.0621480x^8 + 0.0013264x^9 + 0.0020255x^{10}. \end{aligned} \tag{24}$$

Similarly, placing (24) in $g(x)$ at (22), the approximate solution based on Hermite polynomials is

$$u^{H,10}(x) = \sum_{m=0}^{10} u_m = 1 - 0.9654096x^2 - 0.0573124x^3 + 0.41794025x^4 + 0.0827046x^5$$

$$-0.08688776x^6 - 0.0534150x^7 - 0.0012928x^8 + \cdots.$$



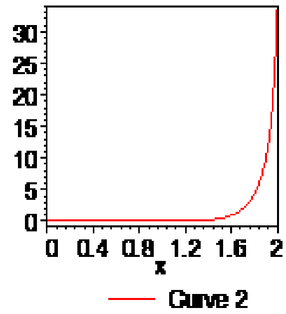


Figure-1: Absolute errors of ADM by Taylor and Hermite polynomials.

5. Conclusion

In this paper, Hermite polynomials is used to improve the Adomian decomposition method. Considering presented example and their figures, we conclude that solutions of Adomian decomposition method on the basis of orthogonal polynomials expansion (Hermite polynomials) is better than Taylor expansion, especially when the approximate interpolation is wider than [0,1] (we have considered [0,2]).

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