J. Basic. Appl. Sci. Res., 3(2s)682-688, 2013
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ISSN 2090-4304
Journal of Basic and Applied
Scientific Research
www.textroad.com

Determination of Skm Mathematical Model for Estimation of Transverse Velocity Distribution in Compound Channels

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Received: June 10 2013
Accepted: July 10 2013

ABSTRACT

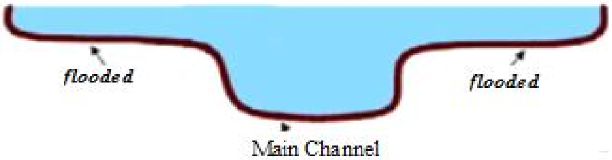
In compound channels with bed roughness, the existing methods for estimation of velocity distribution suffer from excess of errors which may be accounted for by considering strong secondary flows caused by varied bed roughness distribution. The present paper was formulated to determine a general model of velocity distribution where two novel and common analytic solutions were adopted for simple and trapezoidal compound channels in order to depict their differences as well as importance of secondary flow. The 2-D numerical method, SKM, was proposed for obtaining velocity distribution in compound channels; the method is based upon Navier-Stokes equations. Comparing analytical results with experimental data showed that SKM method is capable of predicting transverse velocity distributions in compound channels along with mutual effect of secondary flow.

KEYWORDS: velocity distribution, SKM 2-D method, secondary flows, eddy flows.

1. INTRODUCTION

Transverse velocity distribution and boundary-level shear stress are considered as important aspects of river hydraulics. Rivers usually have compound cross-sections in their ends so that the main channel of rivers gets over-flooded and water enters floodplains. Due to fast variations of flow depth and bed roughness coefficient, transverse velocity distribution and boundary-level shear stress will be considerably uneven. In this situation, the main channel has a high velocity owing to high flow depth and low roughness coefficient while flow velocity in floodplains is rather lower owing to lower depth and higher roughness. The channels with compound transverse cross-section are composed of one main channel and one or two flood bed; therefore, such channels are termed as compound or two-stage channels [1].

If compound cross-section is composed of deep cross-section (main channel) and shallow cross-section (floodplains), an abrupt change will occur in depth causing momentum exchange between main channel and floodplains. Combination of higher effect of flow depth and lower influence of roughness in main channel and floodplains brings about a considerable velocity difference in the main channel and floodplains. Such a difference causes transverse mass and momentum exchange which reduces cross-section carrying capacity considerably while increases flow in floodplain. Velocity witnesses a sudden reduction in the area between the main channel and floodplains which can be attributed to apparent shear and momentum exchange. Compound cross-sections are a combination of a deep main channel and wide floodplains; floodplains are usualy dry and have higher roughness coefficient than the main channel (Fig. 1) [2,3].



(a)Transverse cross-section of river

(b)Regular compound cross-section

Figure 1. a sample of flooded rivers with compound cross-sections (along with main channel and floodplains) [2]

2. SECONDARY FLOWS

Centrifugal force is exerted as the flow bends; such a force is varied in terms of bend radius and direction due to velocity fluctuations. Centrifugal force in the bend forms a transverse slope on water surface; consequently, water surface rises in external bend while it falls in internal bend. Such a phenomenon results in lateral pressure gradient inside the

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cross-section. Now, if the pressure gradient overcomes centrifugal force, a flow (known as secondary flow) is formed in transverse direction inside the cross-section.

Secondary flow makes the particles on the surface water and in the bottom move towards external and internal walls, respectively.Aspecial flow model, termed as helical flow, is formed as a result of interaction of secondary flow and irregular profile of longitudinal velocity; helical flow exerts several changes in flow pattern of the bend compared with the flow in direct channel. Furthermore, flow velocity inside the channels encounters considerable variations with formation of the flow caused by transverse rotational cell in channel cross-section. Before facing to a bend, flow moves through direct channel with the maximum longitudinal velocity being virtually in the center of channel cross-section; as it enters the bend, the flow changes and its maximum moves toward internal wall. Secondary flow as well as pressure longitudinal gradient should, thus, come along in order to determine flow pattern in the bend [2]. Determination of secondary flows is important due to their influence on flow hydraulics. Such flows originate from shear stress differences between liquid layers. Owing to velocity difference between main channel and floodplains in compound cross-sections, shear stress is formed inside liquid layers forming secondary flows in this area.

3. VELOCITY TRANSVERSE DISTRIBUTION

Velocity transverse distribution in compound cross-sections is much more different and complex than in simple cross-sections. Flow cross-section can be regarded as the main channel as long as there is water in the main channel; however, as soon as water enters floodplain, flow hydraulics cannot be considered as it is in simple cross-section anymore. Since such phenomena as eddy flows, momentum exchange, secondary flows, etc. increase in the cross-sections, conventional relations are no longer responsive to the situations in compound cross-sections; therefore, it is necessary to make use of the relations capable of modeling such flow phenomena with more complex cross-sections considering physical status of flow hydraulics. Accuracy of determining capacity of compound cross-sections in design process is questionable due to huge eddy flows in interface of main channel and floodplains.

Predicting transfer capacity of velocity distribution is very difficult. Variety of prediction models have been recently proposed for simulation of complex behavior of irregular flows in channels according to mean depth momentum equation. The proposed method, SKM (Shiono and Knight Method), has been unanimously accepted because it presents proper analytic solutions for distribution of lateral velocities in mean depth on the basis of hydraulic parameters. Such eddy results in transfer of momentum of main channel to floodplains, energy loss, and consequently, reduction of speed and flow rate of the river [4]. Flow rotational cells in floodplains have a direction against direction of main channel eddies. So, simulation of velocity transverse distribution and obtaining rating curve are pivotal in floods. Such eddy flows are mainly due to shear stress between fast and slow flows in insignificant cross-sections which causes momentum exchange from main channel to floodplain; as a result, velocity decreases in main channel while increases in floodplain [3]. The considerable difference in flow velocities between floodplains and main channel accounts for formation of an interaction area in the interface between them; this brings about a significant mass exchange and movement. Complex and 3-D architecture of flow in this area and development in cross-section width toward neighboring areas invalidates 1-D flow hypothesis and therefore, considerable errors occur in estimation of Maning 1-D relation from transfer capacity [6]. Influence of secondary flow on latitudinal velocity due to mutual effect of flow is evident in Figure 2 [2].

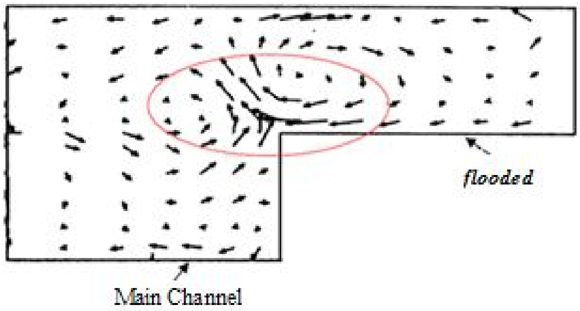


Figure 2. secondary flow between channel main channel and floodplain [2]

4. FLOW BEHAVIOR IN DIRECT COMPOUND CHANNEL

Flow behavior in direct channels with floodplains along the channel has been evaluated for 50 years. In 1985, Sciences and Engineering Research Council and Hydraulics Research Institute in Wallingford decided to make a series of equipped facilities. These facilities involved a 56m long and 10m wide channel with a capacity of 1.1 m3/s built for research about mutual effect of flow between main channel and floodplains. The dimensions of FCF-SERC channel enabled it to form such a flow as the one in natural rivers where the channel was absolutely 3-D along with high momentum change between different cross-sections [Zahiri, 1999]. The most important flow behavior along regular flow is disturbed shear reaction between fast flow in main channel and low-velocity flow in floodplain (Fig. 3).

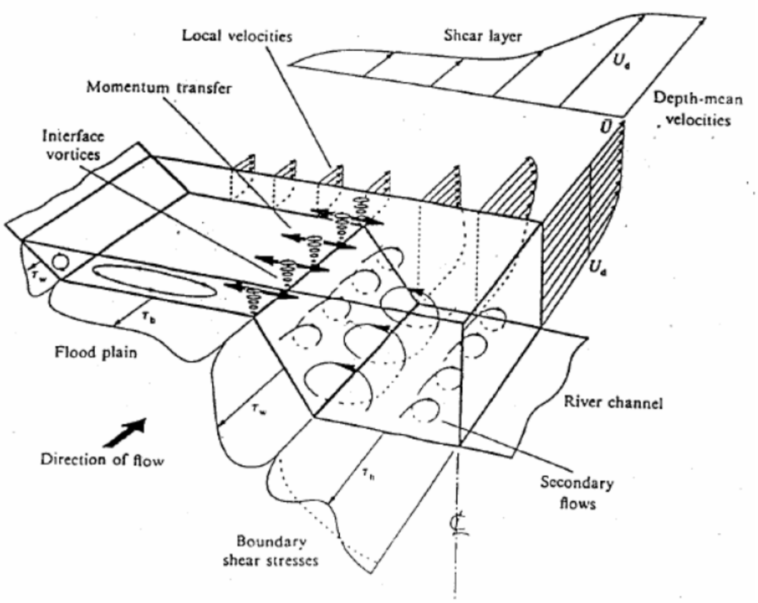
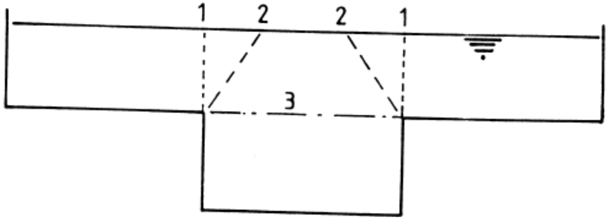


Figure 3. mechanism of flow architecture in a channel with compound cross-section [SKM, 1991]

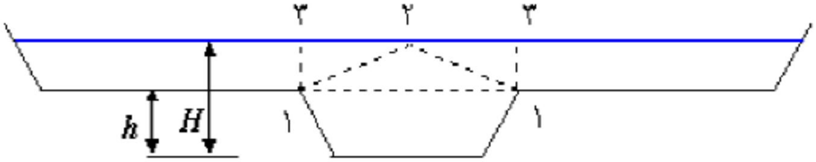
Moreover, rotational flow with vertical axes can be seen; this flow results in a significant reduction of energy and lateral momentum transfer which occurs from main channel to floodplain.

5. RATING CURVE

Manning equation is useful for rating in compound cross-sections. Hydraulic radius occurs haphazardly in shallow floodplains and therefore, if flow rate is estimated for an individual channel, it cannot be free from errors. The conventional method for overcoming this problem is considering vertical walls with friction and calculation of flow rate for each sub cross-section (Fig. 4).



(a) Rectangular cross-section



(b) Trapezoidal cross-section

Figure 4. various models of analysis in compound cross-sections (1:1 horizontal pattern; 1:2 inclined pattern; 1:3 vertical pattern)

This method was validated by Chow (1956) as follows:

$$Q = \frac{1}{n} A_1 R_1^{\frac{2}{3}} S^{\frac{1}{2}} + \frac{1}{n} A_2 R_2^{\frac{2}{3}} S^{\frac{1}{2}} + \frac{1}{n} A_3 R_3^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (1)$$

If relative depth is very low in the above equation, $\frac{Y_f}{Y_C}$ (Yf and Yc stand for floodplain and main channel depths, respectively) causes disturbance reaction leading to faulty estimation of Q.

$$Q = K_c Q_c + K_f Q_f \qquad (2)$$

Q_C ,Q_f Occur when there is no reaction; k_c ,k_f stand for coefficients of floodplain and main channel, respectively where k_c is 0.6-1.5 and k_f 1-1.2. These methods were further developed by Wormleaton et al. (1988). These experiments were performed on a fixed bed with varying transvers cross-section and two different amounts of roughness in floodplain.

6. FLOW HYDRAULICS IN FLOODED RIVERS (COMPOUND CROSS-SECTIONS)

As discussed before, velocity transverse distribution in compound cross-sections is much more different and complicated than that in simple cross-sections. As water flows in main channel, flow cross-section may be considered a simple cross-section; however, once entered floodplain, flow hydraulics cannot be considered as that in simple cross-sections anymore because such phenomena as eddy flows, momentum flows, and secondary flows aggravate. An analytic-2-D model for solving velocity transverse distribution in compound cross-sections was proposed in 1988 based upon Navier-Stokes equations. Effect of secondary flows is neglected in this mathematical model. The results obtained from this mathematical model in experimental and river compound cross-sections showed that secondary flows play important role in determination of velocity transverse distribution. Numerical solution for this 2-D model by use of limited subtractions [Ayubzadeh and Zahiri, 2002] and limited components is also proposed. Application of these two numerical solutions in flood trend finding in regular compound cross-sections has resulted absolutely identical results [1].

In 2000, a 2-D method according to Navier-Stokes equations was proposed for direct compound cross-sections [4] where effect of secondary flow in compound cross-section with bends was shown to be 10 times as much as its counterpart in compound cross-sections with direct route. In 2004, a method was recommended for calculation of rating curve in regular and river heterogeneous compound cross-sections. In this method, secondary flows coefficient of main channel in heterogeneous compound cross-sections was corrected as a third-order function of relative roughness.

7. EFFECTIVE FACTORS ON MUTUAL EFFECT OF FLOW

When compound cross-section involves deep (main channel) and shallow (floodplains) cross-sections, an abrupt change will occur in depth leading to momentum exchange phenomenon between main channel and floodplains. Combination of big effect of flow depth and low effects of roughness in main channel and flood-plains results in considerably different velocities in main channel and floodplains. Such a velocity difference leads to momentum exchange which in turn, reduces carrying capacity of the cross-section. Recent investigations by Sellin (1964) and Zheleznyakov (1971) showed rotational movements and their effect on velocity and flow rate in overflows.

8. HELICAL SECONDARY FLOWS

Helical flows are formed in main channel on the walls. These longitudinal rotations are seen all along disturbed flows leading to disturbance in points pattern [Perkins, 1979; Knight & Patel, 1985]. Helical secondary flows are formed in the bottom of cross-section and on walls in a clockwise manner. Nevertheless, directions of these flows on corners of the main channel are more intensive and formed in anticlockwise. Individual rotational flows exist on the corners of cross-section of floodplains. These flows and helical secondary flow in the bottom of the cross-section are unable to expand to free water surface due to low energy (Figs. 5 and 6).

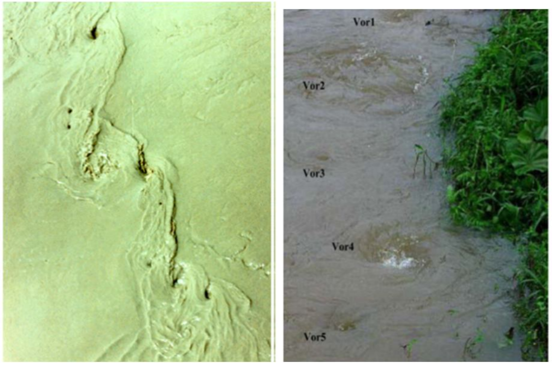


Figure 5. Figure 5: rotational flows in the interface of main channel and floodplain

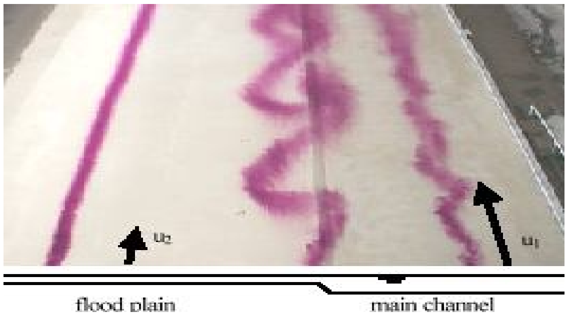


Figure 6. disturbance of flow in the interface of main channel and floodplain

9. MAIN EFFECTIVE PARAMETER ON FLOW MUTUA EFFECT PHENOMENON IN COMPOUND CROSS-SECTION

A compound cross-section can be generally imagined as follows:

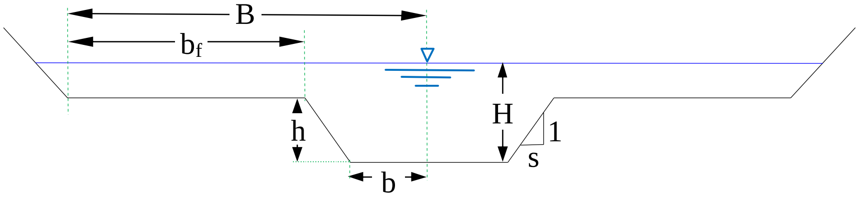


Figure 7. compound cross-section (trapezoidal cross-section) with the parameters

In figure 7, H, B, h, bf, bc, Sc, and Sf stand for total height of the cross-section (water height in the main channel), total width of water level, height difference between bottom of floodplain and main channel (depth of main channel), width of floodplain, width of main channel width, lateral slope of main channel, and lateral slope of floodplain cross-section, respectively. Main parameters in determination of flow in compound cross-sections are ordered in terms of importance as follows:

- Relative depth (D_r) (ratio of flow depth in floodplain to flow height in main channel)
- Relative width (W_r) (ratio of half width of cross-section in floodplain to half width of main channel)
- Relative roughness (n_r) (ratio of roughness in floodplain to that in main channel)
- Shape ratio (ASP) (the ratio of half width of main channel to depth of main overflown cross-section
- Number of floodplains in the considered compound cross-section (N_f)
- Lateral slope of main channel (S_c)

Parametric description of effects of above-mentioned factors may vividly present variations of mutual effect.

10. 2-D METHODS IN COMPOUND CROSS-SECTIONS

1-D methods for determination of insignificant flow rates, velocity distribution, etc., as conventional methods in the investigations on sediment transfer, walls erosion, and sustainable channels design, lack efficiency. 2-D models are able to calculate velocity distribution, insignificant flow rates and consequently, total flow rate. Variety of 2-D models have been proposed such as Wark method (LDM), Shiono-Knight [8], Lambert-Selin [2], Sponer-Shiono [9], Projen [7].

11. 2-D MODEL FOR VELOCITY TRANSVERSE DISTRIBUTION (SKM)

Several 2-D hydraulic methods have been proposed by Shiono-Knight [12], Lambert-Salin [2], Arvin et al. [11], and Projen et al. [7].

$$\rho\left[v\frac{\partial u}{\partial y}+\omega\frac{\partial \omega}{\partial z}\right]=\rho g sin\theta+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z} \qquad (3)$$

2-D equations of SKM method have been put forth for solving transverse distribution of flow velocity in compound cross-sections; the following equation is the equation presented in [10] without considering secondary flow.

$$\rho g H S_0-\frac{1}{8}\rho f U_d{}^2\left(1+\frac{1}{s^2}\right)^{1/2}+\frac{\partial}{\partial y}\left\{\rho\lambda H^2\left(\frac{f}{8}\right)^{1/2}U_d\frac{\partial U_d}{\partial y}\right\}=0 \qquad (4)$$

where Ud, …, g, and λ stand for mean velocity in depth, volume mass of water, gravitational acceleration, and coefficient without viscosity (disturbed flow), respectively. Furthermore, H, S□, and y stand for flow depth in each point, slope of channel bottom, and transverse direction of channel, respectively [10]. Analytic solution of equation [4] is presented below for the areas with fixed depth in main channel and floodplain (Fig. 8):

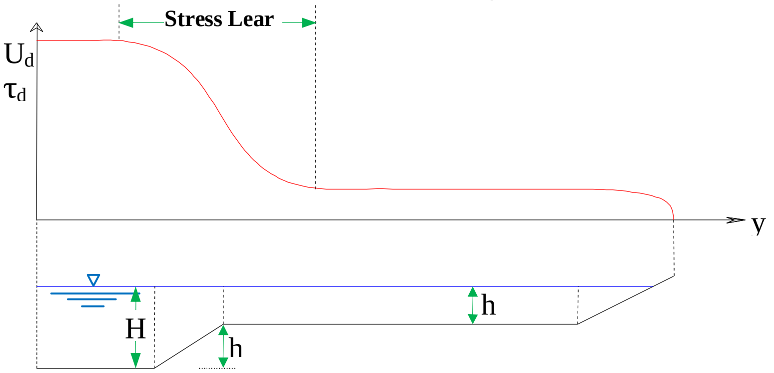


Figure 8. shear layer in the interface of main channel and floodplain

$$U_d = \sqrt{\frac{8}{\rho f}\left[\beta + C_1 \exp\left(\frac{-2}{e+\sqrt{4a+e^2}}y\right) + C_2\exp(\frac{-2}{e-\sqrt{4a+e^2}}y)\right]} \tag{5}$$

where $a = \frac{\lambda_{H^2}}{2}$, $\beta = \rho g H S_\circ$, $e = \frac{8KH}{f}$, C1, and C2 are unknown constants of the equation. The function depth for linear slope range shown in Fig. 7 is as follows:

$$H(y) = H - \frac{y-b}{s} \tag{6}$$

where S stands for lateral slope of main channel. Substituting equation (6) in (4), the analytic solution for mean depth velocity in linear slope amplitude can be obtained. Equation (3) id used only if secondary flows are not considered; otherwise, (4), which was proposed by SKM in 1991, is adopted instead of the differential equation (3) [10].

$$\rho g H S_0 - \frac{1}{8}\rho f\, U_d{}^2\left(1+\frac{1}{s^2}\right)^{1/2} + \frac{\partial}{\partial y}\left\{\rho\lambda H^2\left(\frac{f}{8}\right)^{1/2} U_d\frac{\partial u_d}{\partial y}\right\} = \frac{\partial}{\partial y}\left[H(\rho\, UV)_d\right] = \Gamma \tag{7}$$

where s stands for transverse slope of the channel. SKM method is related to effect of secondary flows through the right phrase of (7) and is mean velocities over time in longitudinal and latitudinal directions, respectively. In general, whenever vertical and horizontal components of flow velocity become important, secondary flow is formed and it affect hydraulics of compound cross-sections considerably. Like SKM method [12], the method proposed by Arvin et al. [11] is applicable for heterogeneous compound cross-sections. By solving Navier-Stokes equations in regular compound cross-sections with direct route (4), Arvin et al. [11] indicated importance of secondary flows phrase to be 7-10% of that in rotational compound cross-sections. Through experimental data for homogeneous compound cross-section with direct route and big dimensions (FCF), SKM could propose the effect of secondary flows as a function of bed shear stress:

$$\frac{\partial H(\rho \overline{U}\overline{V})_d}{\partial y} = \beta_s\, \rho g S_0 H \tag{8}$$

In this equation, the coefficient for homogeneous compound cross-sections in normal and flood flows are 0.05 and 0.15 in the main channel and -0.25 in floodplains. Definite coefficients have not been proposed for heterogeneous compound cross-sections where roughness coefficient of floodplain is higher than the main channel (as it is in majority of rivers).

$$V = kU \tag{9}$$

where k is proportion coefficient. This coefficient is a function of depth and roughness of floodplain and rotation degree of river. After analysis of the proposed 2-D mathematical model, proportion coefficient in the main channel and floodplains are 0.25 and 0.00 for experimental compound channels with direct direction, respectively. This coefficient in wide and homogenous compound cross-section of rivers with direct route is negligible.

13. PROPOSED MODELS OF SKM

13.1. SKM model A

2-D analytic solutions for velocities distribution has been examined in open channels. Secondary flow is a very important parameter because it is directly engaged in force balance of momentum equation. Therefore, it is of high importance for a proper determination of velocity distribution. It can be seen in the recommended model A through SKM method that the distributions [ρUV]d are somewhat identical to different depths Dr. The distributions [H ρUV]d, however, are largely depth-related. As a result, secondary flow can be presented in a more efficient manner by [ρUV]d than [H ρUV]d distributions.

13.2. SKM model B

This model is used for simulation of secondary flow in sloped lateral wall. Analytic solution of this model is performed for velocity lateral distribution by considering Γ=HΓ^*.

13.3. SKM model C

This model is adopted for simulation of secondary flow in sloped lateral wall. This model is solved for velocity lateral distributions by considering Γ=(∂(HΨ))/∂y. the most common model for determination of secondary flow has been recommended on the basis of Ψ considering Ψ=[ρUV]d to have linear distribution.

14. CONCLUSION

The present paper was formulated in order to determine 2-D flow model and effect of secondary flow on flow velocity distribution in compound channels by use of the mathematical method SKM. The results obtained from the present study about variation pattern of secondary flow coefficient, it can be concluded that the proposed models in compound channels have a good performance and negligence of secondary flows in compound channels leads to more errors in calculations of flow rate. Therefore, abrupt change in profile of shear stress may be prevented by considering linear distribution from secondary flow coefficient and draw acceptable results. SKM can make proper results by considering parameter estimation of secondary flow. Considering the proposed models and identical status for the models, the following conclusions can be drawn:

- The models *B* and *C* set no-slip consition during analutic solution without any numerical problem.
- All three models have the same solutions for amplitude of channels with fixed depth; however, there are differences for varied linear flow. There are also differences in the results of U_d predicted for simple channels with the three models where the higher U_d, the higher secondary flow. Such increase usually occurs in lateral walls where there is a strong shear layer.
- The differences between the proposed models are higher in the channels with rather same secondary flows. If Γ is not that big in the models (especially in the channel with rather small apparent ratio), the difference is very insignificant.
- No-slip boundary condition has a tiny effect in the model *A* while boundary condition is always set in the models *B* and *C.* also, the model *C* is a suitable general model for wide variety of applications.

Comparison of analytic results shows that all the proposed models by SKM are capable of forming the velocity distributions conforming to experimental data. It is also noteworthy that the mentioned data are validated by comparing with experimental data.

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