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The ($\frac{G'}{G}$)-Expansion Method for the (2+1)-Dimensional Boiti-Leon-Manna-Pempinelli Equation

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ABSTRACT

In this work, we construct new and more general exact traveling wave solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation with the aid of the ($\frac{G'}{G}$)-expansion method. As a result, the traveling wave solutions with three arbitrary functions are obtained including hyperbolic function solutions, trigonometric function solutions and rational solutions. The method appears to be easier and faster by means of some mathematical software.

KEYWORDS: The ($\frac{G'}{G}$)-expansion method, The (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation, Traveling wave solutions.

1. INTRODUCTION

In the nonlinear sciences, it is well-known that many nonlinear evolution equations (NLEEs) are widely used to describe the complex phenomena. Phenomena in physics, fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and other fields are often described by nonlinear evolution equations. The analytical solutions of such equations are of fundamental importance since a lot of mathematical–physical models are described by NLEEs.

In the recent years, many effective methods for obtaining exact solutions of NLEEs have been presented, such as the inverse scattering transform method [1-2], the standard tanh and extended tanh methods [3-6], the Exp-function method [7-8], the complex hyperbolic function method [9], the F-expansion method [10], homogeneous balance method [11-12], the Hirota's bilinear method [13], Jacobi elliptic function expansion method [14], the Sub-ODE method[15-16] and so on.

Recently, Wang et al. [17] introduced a new method called the ($\frac{G'}{G}$)-expansion method to look for traveling wave solutions of nonlinear evolution equations. Next, Bekir applied the method to some nonlinear evolution equations gaining traveling wave solutions. [18] Later, Zhang et al. [19] have generalized the method to obtain non-traveling wave solutions and coefficient function solutions. The method was soon been applied to other non-linear problems by several authors[20-26].

The ($\frac{G'}{G}$)-expansion method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation.

In this paper, we use the ($\frac{G'}{G}$)-expansion method to find the exact solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation

$$u_{yt} - 3u_{xy}u_x - 3u_{xx}u_y + u_{xxxy} = 0, \tag{1}$$

which was derived by Gilson et al [27].

The rest of this work is organized as follows. In Section 2, we describe the ($\frac{G'}{G}$)-expansion method. In Section 3, we apply this method to Eq. (1). In Section 4, some conclusions are given.

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2. Description of The ($\frac{G'}{G}$)-expansion method

In this section we describe the ($\frac{G'}{G}$)-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in three independent variables x, y and t, is given by

$$P(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{tt}, u_{xy}, u_{xt}, u_{yt}, ...) = 0 \tag{2}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the ($\frac{G'}{G}$)-expansion method.

The transformation $u(x, y, t) = U(\xi)$, $\xi = x + y - \omega t$ reduces Eq. (2) to the ordinary differential equation(ODE)

$$Q(U, U', U'', ...) = 0 \tag{3}$$

where $U = U(\xi)$, and prime denotes the derivative with respect to ξ, and ω are constants. Eq. (3) is then integrated as long as all terms contain derivatives, where integration constants are considered zeros.

We assume that the solution of Eq. (3) can be expressed by a polynomial in ($\frac{G'}{G}$) as follows:

$$U = \sum_{i=1}^{m} \alpha_i (\frac{G'}{G})^i + \alpha_0, \qquad \alpha_m \neq 0, \tag{4}$$

where $G = G(\xi)$, is the solution of the auxiliary linear second-order ordinary differential equation

$$G'' + \lambda G' + \mu G = 0 \tag{5}$$

where $G' = \frac{dG}{d\xi}, G'' = \frac{d^2G}{d^2\xi}, \alpha_m \neq 0, ..., \alpha_1, \alpha_0, \lambda$ and μ are real constants to be determined later.

So, a direct computation with use from Eqs. (4) and (5) gives

$$U' = -\sum_{i=1}^{m} i\alpha_i [(\frac{G'}{G})^{i+1} + \lambda (\frac{G'}{G})^i + \mu (\frac{G'}{G})^{i-1}], \tag{6}$$

$$\begin{aligned} U'' = \sum_{i=1}^{n} i\alpha_i [(i+1)(\frac{G'}{G})^{i+2} + (2i+1)\lambda (\frac{G'}{G})^{i+1} + i(\lambda^2 + 2\mu)(\frac{G'}{G})^i \\ + (2i-1)\lambda\mu (\frac{G'}{G})^{i-1} + (i-1)\mu^2 (\frac{G'}{G})^{i-2}], \end{aligned} \tag{7}$$

and so on, in other hands with using the general solutions of Eq. (5) we have

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \frac{\sqrt{\lambda^2 - 4\mu}}{2} (\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}) - \frac{\lambda}{2}, & \lambda^2 - 4\mu > 0 \\ \frac{\sqrt{4\mu - \lambda^2}}{2} (\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}) - \frac{\lambda}{2}, & \lambda^2 - 4\mu < 0 \\ \frac{C_1}{C_1\xi + C_2} - \frac{\lambda}{2}, & \lambda^2 - 4\mu = 0. \end{cases} \tag{8}$$

To determine U explicitly, we take the following four steps:

Step 1. Determine the integer m by substituting Eq. (4) along with Eq. (5) into Eq. (3), and balancing the highest order nonlinear term(s) and the highest order partial derivative.

Step 2. By substituting Eqs. (4) and (5) into Eq. (3) with the value of m obtained in Step 1, and collecting all term(s) with the same order of $(\frac{G'}{G})$ together, the left-hand side of Eq. (3) converted into polynomial in $(\frac{G'}{G})$. Then setting each coefficient to zero, we obtained a set of algebraic equations for λ, μ, ω, α_0 and α_i.

Step 3. Solve the system of algebraic equations obtained in step 2 for ω, α_0 and α_i by use of Maple.

Step 4. By substituting the results obtained in the above steps, we can obtain a series of fundamental solutions of Eq.(3).

3. Applications of the method

In this section, we would like to use the $(\frac{G'}{G})$-expansion method to obtain new and more general exact solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation.

Let us consider the following the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation in the form:

$$u_{yt} - 3u_{xy}u_x - 3u_{xx}u_y + u_{xxxy} = 0, \tag{9}$$

Now, to seek for the traveling wave solutions of Eq. (9) we make the transformation $u(x, y, t) = U(\xi)$, $\xi = x + y - \omega t$, where $U(\xi)$ is polynomial in the $(\frac{G'(\xi)}{G(\xi)})$ of degree m, and ω is constant to be determined. Then, we get

$$-\omega U'' - 6U'U'' + U^{(4)} = 0, \tag{10}$$

where the primes denotes the derivation with respect to ξ.

Integrating the above equation and neglecting the constant of integration, we find

$$-\omega U' - 3U'^2 + U''' = 0. \tag{11}$$

Balancing the terms U'^2 and U''' in Eq.(11), we get 2(m+1) = m+3, which yields the leading order m=1. we suppose that the solution $U(\xi)$ of Eq. (11) is of the form:

$$U = \alpha_1(\frac{G'}{G}) + \alpha_0, \qquad \alpha_1 \neq 0, \tag{12}$$

where α_0, α_1, are unknown constants that to be determined later.

Now substituting Eq. (12) along with Eq. (5) into Eq. (11) and equating all terms with the same power of $(\frac{G'}{G})$ to zero, we obtain set of algebraic equations as:

$$(\frac{G'}{G})^0 : \omega\alpha_1\mu - 3\alpha_1^2\mu^2 - 2\alpha_1\mu^2 - \alpha_1\lambda^2\mu\ ,$$

$$(\frac{G'}{G})^1 : \omega\alpha_1\lambda - 6\alpha_1^2\lambda\mu - 8\alpha_1\lambda\mu - \alpha_1\lambda^3\ ,$$

$$(\frac{G'}{G})^2 : \omega\alpha_1 - 6\alpha_1^2\mu - 3\alpha_1^2\lambda^2 - 8\alpha_1\mu - 7\alpha_1\lambda^2\ ,$$

$$(\frac{G'}{G})^3 : -6\alpha_1^2\lambda - 12\alpha_1\lambda\ ,$$

$$(\frac{G'}{G})^4 : -3\alpha_1^2 - 6\alpha_1.$$

On solving the above algebraic equations by use of Maple, we have

$$\alpha_1 = -2, \quad \alpha_0 = \alpha_0, \quad \omega = \lambda^2 - 4\mu. \tag{13}$$

where α_0, λ, and μ are arbitrary constants.

Hence in view of the solutions (13), Eq (12) become

$$U(\xi) = -2(\frac{G'}{G}) + \alpha_0, \tag{14}$$

where $\xi = x + y - (\lambda^2 - 4\mu)t$.

Inserting the general solutions (8) into Eq. (14), we have three types of traveling wave solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation.

When $\lambda^2 - 4\mu > 0$, we obtain hyperbolic function solutions,

$$U(\xi) = -\sqrt{\lambda^2 - 4\mu}\left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right) + \lambda + \alpha_0, \tag{15}$$

where $\xi = x + y - (\lambda^2 - 4\mu)t$, and C_1, C_2 are are arbitrary constants.

When $\lambda^2 - 4\mu < 0$, we obtain trigonometric function solutions,

$$U(\xi) = -\sqrt{4\mu - \lambda^2}\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right) + \lambda + \alpha_0, \tag{16}$$

where $\xi = x + y - (\lambda^2 - 4\mu)t$, and C_1, C_2 are are arbitrary constants.

When $\lambda^2 - 4\mu = 0$, we obtain rational function solutions,

$$U(\xi) = \frac{-2C_1}{C_1\xi + C_2} + \lambda + \alpha_0, \tag{17}$$

where $\xi = x + y - (\lambda^2 - 4\mu)t$, and C_1, C_2 are are arbitrary constants.

If we set $C_1 \neq 0, C_2 = 0$, and $C_1 = 0, C_2 \neq 0$, in Eqs. (15), respectively, and $\lambda = 2, \mu = 0, \alpha_0 = -2,$ then we have two solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation

$$u_1 = -2\tanh(x + y - 4t), \qquad u_2 = -2\coth(x + y - 4t). \tag{18}$$

So, If we set $C_1 \neq 0, C_2 = 0$, and $C_1 = 0, C_2 \neq 0$, in Eq (16), respectively, and $\lambda = 0,\ \mu = 1,\ \alpha_0 = 0,$ then we have two solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation

$$u_3 = 2\tan(x + y + 4t), \qquad u_4 = -2\cot(x + y + 4t). \tag{19}$$

Another solution of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation was obtained with setting $C_1 \neq 0, C_2 = 0$, and $\lambda = 2,\ \mu = 1,\ \alpha_0 = -2,$ in Eq.(17),

$$u_5 = -\frac{2}{x + y}. \tag{20}$$

The diagrams of these solutions are given in Figs. 1-4 for some particular value of t variable.

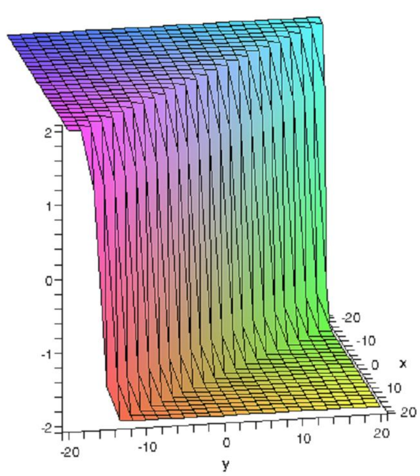

Figure 1: 3D Plots of $u_1(x, y, t)$, for $t = 1$

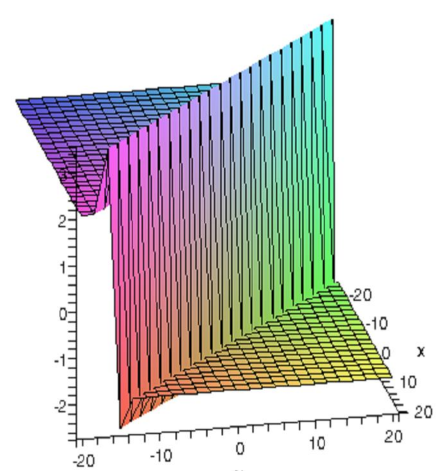

Figure 2: 3D Plots of $u_2(x, y, t)$, for $t = 1$

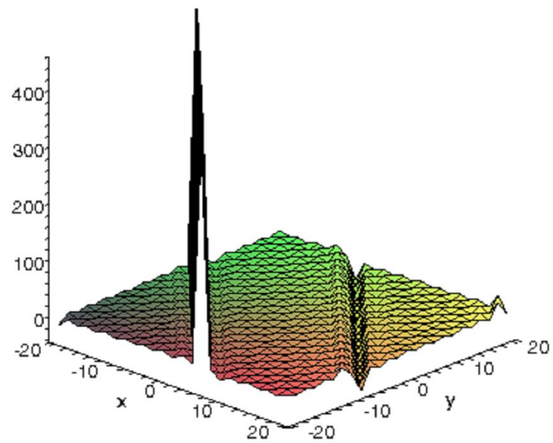

Figure 1: 3D Plots of $u_3(x, y, t)$, for $t = 1$

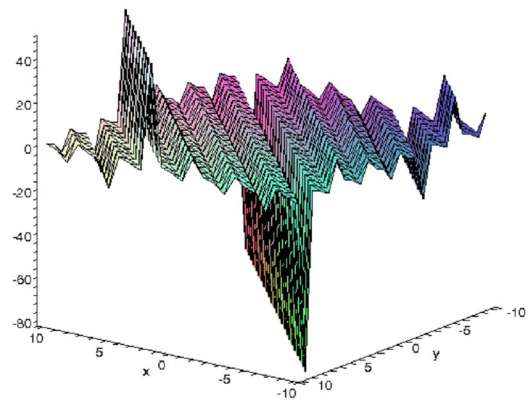

Figure 2: 3D Plots of $u_4(x, y, t)$, for $t = 1$

On comparing our results (18-20) and results in [28] by using the Hirota bilinear method and Riemann theta function and results in [29-30] by using the Backlund transformation, then it can be seen that the results are the same and some of our results are new exact traveling wave solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation.

4. Conclusions

In this work, we have applied the ($\frac{G'}{G}$)-expansion method for the solving the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation. The results show that the ($\frac{G'}{G}$)-expansion method is a powerful mathematical tool for solving systems of nonlinear partial differential equations having wide applications in sciences and engineering. we have seen that three types of traveling wave solutions in terms of hyperbolic, trigonometric and rational functions for the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation are successfully found out by using the ($\frac{G'}{G}$)-expansion method. Some of these results are in agreement with the results reported by others in the literature, and new results are formally developed in this work. In our work, we use the maple software to carry the computations.

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