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Relative Efficiency in Two-Stage Dea and Its Application to Bank Branches

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ABSTRACT

In the recent decade, data envelopment analysis (DEA) has been extended to evaluate the performance of two-stage processes. Two-stage process known as two-stage DEA is divided into two stages. The outputs produced from the inputs of the first stage are considered as inputs to produce the outputs of the second stage. These data are called intermediate data. In the recent works on two-stage DEA, the efficiency of the total process is defined by a combination of the efficiencies of the two stages. However, the presented methods may not measure the relative efficiencies of two-stage processes. Therefore, present paper proposes a new two-stage DEA model which can obtain the relative efficiencies of two-stage processes. The method also preserves relation between the total efficiency and the efficiencies of the stages. Some theorems related to the proposed approach are provided. Moreover, the proposed two-stage DEA model is compared to other approaches by using a case study in the literature. Furthermore, the utilization of the proposed approach is demonstrated to evaluate 23 bank branches in Iran.

KEYWORDS: Data envelopment analysis, Two-stage process, Total efficiency, Relative efficiency.

1. INTRODUCTION

Performance evaluation is one of the most serious concerns for managers, since it can be utilized as a reference in decision making with regard to performance improvement. Performance is conventionally defined either as organizational outputs or inputs, or as a relationship between them. Because the evaluation characteristics are generally multi-dimensional, there is no appropriate aggregation schema for them and the basic problem of performance measurement is how to evaluate the relative performance of units. To overcome this difficulty, data envelopment analysis (DEA) is a widely employed technique for efficiency evaluation within a group of decision making units (DMUs).

DEA is a mathematical programming technique, which is used to evaluate relative efficiency of homogeneous DMUs on the basic of multiple inputs and outputs and has been suggested by Charnes et al. (1978) (CCR model). The concept of the relative efficiency in the CCR model (Charnes et al. 1978) was indicated by Thompson et al. (1993) (Maximin DEA model). The technique used to construct CCR model (Charnes et al. 1978) has been expanded by Banker et al. (1984) (BCC model). DEA is an important analysis tool and research way in management science, operational research, system engineers, decision analysis and so on. A thorough review on DEA up 2009 can be found in Cook and Seiford (2009).

Systems with more than one production process are called multi-stage processes. The simplest kind of theses processes is two-stage process. Analysis two-stage processes in DEA is called two-stage DEA. In two-stage DEA, the first stage utilizes inputs to generate outputs which become the inputs to the second stage. The first stage outputs are called intermediate data. The second stage then utilizes these intermediate data to produce outputs. Bank systems are good examples of two- stage processes. In the first stage, labor product deposits and the second stage uses the deposits to generate profit. Two-stage DEA was also applied to evaluate Fortune Global companies (Zhu 2000), life and health insurance companies (Yang 2006), and printed circuit board industry (Liu and Wang 2009), and non-life insurance companies (Kao and Hwang 2008).

Several attempts have been made to evaluate the performances of the two-stage processes. For example, Seiford and Zhu (1999) suggested three independent models based on classic DEA to measure the efficiency of the total process and the two stages. In the model proposed to evaluate the total process, intermediate data are not used. Also, there is no relation between those three efficiencies. To overcome these difficulties, Chen and Zhu (2004) compounded the feasible regions of the input oriented model for evaluation the first stage and the output oriented model for evaluation the second stage and then

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minimized the efficiency of the first stage minus the efficiency of the second stage. To evaluate the performance of the total process, Kao and Hwang (2008) considered the product of the efficiencies of the two stages as efficiency of the total process. When the multipliers of the outputs of the first stage are equal to the inputs of the second stage, their model is transformed to linear program. Chen et al. (2009) proved that Chen and Zue's model (2004) under constant return to scale (CRS) is equivalent to the model presented by Kao and Hwang (2008). Chen et al. (2009) changed the efficiency defined by Koa and Hwang (2008) with considering the weighted sum of the efficiencies of the two stages as the efficiency of the total process. Using specific forms of the weights, their model was also converted to a linear fractional program. There are other methods to analyze two-stage DEA. Castelli et al. (2010) presented a full review of DEA approaches with intermediate structures.

To evaluate the efficiencies of the two-stage processes, two kinds of constraints are added to models. These constraints are the efficiencies of the two stages that must be less than or equal to one. The constraints are incorporated to models to preserve a relation between the efficiency of the total process and the efficiencies of the two stages. Adding the constraints into two-stage DEA models is similar to considering weight restrictions in DEA models. DEA models with weight restrictions maximize the absolute efficiency of a unit which may not equal to the relative efficiency of the unit. Also, in this case, distinguishing DEA frontier and determining target points for inefficient DMUs may not easy. Moreover, wrong reference set may be obtained for inefficient DMUs. Full details about the DEA models with weight restrictions can be found in the works of Podinovski and athanassopoulos (1998) and Podinovski (1999, 2001, 2004). To avoid the aforementioned problems, in this paper, the Maximin DEA model (Thompson et al. 1993) is used for measuring the relative efficiencies of the two-stage processes. Empirical examples are also used to compare our proposed method to previous methods in the literature.

The rest of the paper is organized as follows. In section 2, two-stage DEA is introduced, briefly. A new DEA model to assess the two-stage processes is suggested in section 3 which explicitly measures the relative efficiency of a two-stage process. Some facts related to the proposed models are also provided in this section. In section 4, the proposed two-stage DEA method is compared to other approaches in the literature. The last section comprises our conclusions.

Background of two-stage DEA

Consider n DMUs. Each DMU_j (j = 1,...,n) has m inputs to the first stage, x_{ij}, (i = 1,...,m), and d outputs from this stage, z_{lj}, (l = 1,...,d). These d outputs are the inputs to the second stage, and known as intermediate data. The outputs from the second stage are denoted y_{rj}, (r = 1,...,s).

Chen et al. (2009) proposed that the weighted sum of the efficiencies of two stages is considered as the total efficiency. Then, they were presented below model to measure the total efficiency of DMU_p as:

$$
\begin{aligned}
&\max w_1\left(\sum_{r=1}^{s}u_r y_{rp} \Big/ \sum_{l=1}^{d}\mu_l z_{lp}\right) + w_2\left(\sum_{l=1}^{d}\mu_l z_{lp} \Big/ \sum_{i=1}^{m}v_i x_{ip}\right),\\
&s.t.\ \left(\sum_{l=1}^{d}\mu_l z_{lj} \Big/ \sum_{i=1}^{m}v_i x_{ij}\right) \le 1,\ j=1,\dots,n,\\
&\left(\sum_{r=1}^{s}u_r y_{rj} \Big/ \sum_{l=1}^{d}\mu_l z_{lj}\right) \le 1,\ j=1,\dots,n,\\
&v_i \ge 0,\ i=1,\dots,m,\\
&\mu_l \ge 0,\ l=1,\dots,d,\\
&u_r \ge 0,\ r=1,\dots,s, \quad (1)
\end{aligned}
$$

where w_1 and w_2 are such that $w_1 + w_2 = 1$. In particular case, when $w_1 = \frac{\sum_{l=1}^{d}\mu_l z_{lp}}{\sum_{l=1}^{d}\mu_l z_{lp} + \sum_{i=1}^{m}v_i x_{ip}}$ and $w_2 = \frac{\sum_{i=1}^{m}v_i x_{ip}}{\sum_{l=1}^{d}\mu_l z_{lp} + \sum_{i=1}^{m}v_i x_{ip}}$, the model is converted to a linear fractional program, which using a simple variable transformation is change to a linear program, as:

$$\max\left(\sum_{r=1}^{s}u_r y_{rp}+\sum_{l=1}^{d}\mu_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m}v_i x_{ip}+\sum_{l=1}^{d}\mu_l z_{lp}\right),$$
$$s.t.\ \left(\sum_{l=1}^{d}\mu_l z_{lj}\Big/\sum_{i=1}^{m}v_i x_{ij}\right)\le 1,\ j=1,\dots,n,$$
$$\left(\sum_{r=1}^{s}u_r y_{rj}\Big/\sum_{l=1}^{d}\mu_l z_{lj}\right)\le 1,\ j=1,\dots,n,$$
$$v_i\ge 0,\ i=1,\dots,m,$$
$$\mu_l\ge 0,\ l=1,\dots,d,$$
$$u_r\ge 0,\ r=1,\dots,s,\quad (2)$$

Chen et al. (2009) defined $\left(\sum_{r=1}^{s}u_r y_{rp}+\sum_{l=1}^{d}\mu_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m}v_i x_{ip}+\sum_{l=1}^{d}\mu_l z_{lp}\right)$, as the absolute efficiency of DMU_p. In this model, to preserve relation between the total efficiency and the efficiencies of the stages, constraints $\left(\sum_{r=1}^{s}u_r y_{rj}\Big/\sum_{l=1}^{d}\mu_l z_{lj}\right)\le 1$, $\left(\sum_{r=1}^{s}u_r y_{rj}\Big/\sum_{l=1}^{d}\mu_l z_{lj}\right)\le 1\ (j=1,\dots,n)$ were incorporated to the model. Adding these constraints to the model is similar to consider weight restrictions in the CCR model (Charnes et al. 1978). However, incorporating weight restrictions in the CCR model (Charnes et al. 1978) leads to the absolute efficiency of the evaluating DMU which is not always equal to its relative efficiency. In other words, the primary goal in DEA is obtaining the relative efficiency for evaluating DMU, while the value of the optimal objective function of the model (2) is the absolute efficiency. In fact, all DMUs with the model (2) may have the scores strictly less than one. As an example, consider two DMUs A and B with one input, one output and one intermediate data which are provided in Table 1. Using the models in the literature, the efficiency scores for DMUs A and B are 0.766 and 0.8, respectively. Note that none of the scores is equal to one. Thus, the model cannot measure the relative efficiencies of the units. In what follows, to overcome this difficulty, based on the definition of the relative efficiency (Cooper et al. 2007) and using the Maximin DEA model (Thompson et al. 1993), we provide an approach to measure the relative efficiency in two-stage DEA.

***Table 1**. The data set*

DMU	x	z	y	Total efficiency
A	2	3	4	0.766
B	2	2	5	0.800

METHODOLOGY

In classic DEA, for evaluating $DMU_p\ (p=1,\dots,n)$ with m inputs and s outputs, is formed a virtual input by weights $v_i\ (i=1,\dots,m)$ as $\sum_{i=1}^{m}v_i x_{ip}$ and a virtual output by weights $u_r\ (r=1,\dots,s)$ as $\sum_{r=1}^{s}u_r y_{rp}$. The absolute efficiency of DMU_p based on Charnes et al. (1978) is defined by the ratio of the virtual output to the virtual input, which is shown by E_p, as (Cooper et al. 2007):

$$E_p=\sum_{r=1}^{s}u_r y_{rp}\Big/\sum_{i=1}^{m}v_i x_{ip},\ (3)$$

To measure the relative efficiency of DMU_p in comparison to the other DMUs is used from the ratio of the absolute efficiency of DMU_p to the maximum absolute efficiencies all DMUs, which is shown by RE_p, as:

$$RE_p=\left(\sum_{r=1}^{s}u_r y_{rp}\Big/\sum_{i=1}^{m}v_i x_{ip}\right)\Big/\max_{j=1,\dots,n}\left\{\sum_{r=1}^{s}u_r y_{rj}\Big/\sum_{i=1}^{m}v_i x_{ij},\right\},(4)$$

In two-stage DEA, in addition to inputs and outputs of DMU_p, there are intermediate data as $z_{lp}\ (l=1,...,d)$ that are the outputs of the first stage and the inputs of the second stage. We form a virtual intermediate measure by weights $\mu_l\ (l=1,...,d)$ as $\sum_{l=1}^{d}\mu_l z_{lp}$. Because intermediate data are both inputs and outputs, we therefore, in two-stage DEA, define the absolute efficiency of DMU_p as:

$$E_p = \left(\sum_{r=1}^{s} u_r y_{rp} + \sum_{l=1}^{d}\mu_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ip} + \sum_{l=1}^{d}\mu_l z_{lp}\right), (5)$$

According to above assumptions, the relative efficiency of $DMU_p\ (p=1,...,n)$ in two-stage DEA is defined as:

$$RE_p = \left(\left(\sum_{r=1}^{s} u_r y_{rp} + \sum_{l=1}^{d}\mu_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ip} + \sum_{l=1}^{d}\mu_l z_{lp}\right)\right)\Big/\max_{j=1,...,n}\left\{\left(\sum_{r=1}^{s} u_r y_{rj} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\right\}, (6)$$

We must determine the multipliers to measure the relative efficiency of DMU_p. To determine the multiplier, according to Thompson et al. (1993), the relative efficiency of DMU_p must be maximized. To preserve relation between efficiencies of two stages and the efficiency of total process, the relative efficiency must be maximized under the assumption that the efficiency of each stage must be less than or equal to one. So, to measure the relative efficiency of DMU_p, we have a fractional program as:

$$\begin{aligned}
RE_p^* = \max & \left(\left(\sum_{r=1}^{s} u_r y_{rp} + \sum_{l=1}^{d}\mu_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ip} + \sum_{l=1}^{d}\mu_l z_{lp}\right)\right)\Big/\max_{j=1,...,n}\left\{\left(\sum_{r=1}^{s} u_r y_{rj} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\right\},\\
s.t.\ & \left(\sum_{l=1}^{d}\mu_l z_{lj}\Big/\sum_{i=1}^{m} v_i x_{ij}\right) \le 1,\ j=1,...,n,\ (7-1)\\
& \left(\sum_{r=1}^{s} u_r y_{rj}\Big/\sum_{l=1}^{d}\mu_l z_{lj}\right) \le 1,\ j=1,...,n,\ (7-2)\\
& v_i \ge 0, i=1,...,m,\\
& \mu_l \ge 0, l=1,...,d,\\
& u_r \ge 0, r=1,...,s,\quad (7)
\end{aligned}$$

Using substitution $\frac{1}{t} = \max_{j=1,...,n}\left\{\left(\sum_{r=1}^{s} u_r y_{rj} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{l=1}^{d}\mu_l z_{lj}\right)\right\}$ and variable transformations $\bar{u}_r = t\, u_r, \bar{\mu}_l = t\, \mu_l$, the above problem is converted to a linear fractional program as:

$$\begin{aligned}
E_p^* = \max & \left(\sum_{r=1}^{s}\bar{u}_r y_{rp} + \sum_{l=1}^{d}\bar{\mu}_l z_{lp}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ip} + \sum_{l=1}^{d}\mu_l z_{lp}\right),\\
s.t.\ & \left(\sum_{r=1}^{s}\bar{u}_r y_{rj} + \sum_{l=1}^{d}\bar{\mu}_l z_{lj}\right)\Big/\left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{l=1}^{d}\mu_l z_{lj}\right) \le 1,\ j=1,...,n,\ (8-1)\\
& \left(\sum_{r=1}^{s}\bar{u}_r y_{rj}\Big/\sum_{l=1}^{d}\bar{\mu}_l z_{lj}\right) \le 1,\ j=1,...,n,\ (8-2)\\
& \left(\sum_{l=1}^{d}\mu_l z_{lj}\Big/\sum_{i=1}^{m} v_i x_{ij}\right) \le 1,\ j=1,...,n,\ (8-3)\\
& v_i \ge 0, i=1,...,m,\\
& \mu_l \ge 0, l=1,...,d,\\
& \bar{\mu}_l \ge 0, l=1,...,d,\\
& \bar{u}_r \ge 0, r=1,...,s,\quad (8)
\end{aligned}$$

Then, the linear fractional program is converted to a linear program with defining new variable $\frac{1}{t'} = \sum_{i=1}^{m} v_i x_{ip} + \sum_{l=1}^{d} \mu_l z_{lp}$ and variable transformations

$\tilde{u}_r = t' \, \overline{u}_r, \tilde{\mu}_l = t' \, \overline{\mu}_l, \hat{v}_i = t' \, v_i, \hat{\mu}_l = t' \, \mu_l$. The linear program is as:

$$\begin{aligned}
&\overline{E}_p^* = \max \sum_{r=1}^{s} \tilde{u}_r y_{rp} + \sum_{l=1}^{d} \tilde{\mu}_l z_{lp} \\
&s.t. \ \sum_{i=1}^{m} \hat{v}_i x_{ip} + \sum_{l=1}^{d} \hat{\mu}_l z_{lp} = 1, \ (9-1) \\
&\sum_{r=1}^{s} \tilde{u}_r y_{rj} + \sum_{l=1}^{d} \tilde{\mu}_l z_{lj} - \sum_{i=1}^{m} \hat{v}_i x_{ij} - \sum_{l=1}^{d} \hat{\mu}_l z_{lj} \le 0, \ j = 1,...,n, \ (9-2) \\
&\sum_{r=1}^{s} \tilde{u}_r y_{rj} - \sum_{l=1}^{d} \tilde{\mu}_l z_{lj} \le 0, \ j = 1,...,n, \ (9-3) \\
&\sum_{l=1}^{d} \hat{\mu}_l z_{lj} - \sum_{i=1}^{m} \hat{v}_i x_{ij} \le 0, \ j = 1,...,n, \ (9-4) \\
&\hat{v}_i \ge 0, i = 1,...,m, \\
&\hat{\mu}_l \ge 0, l = 1,...,d, \\
&\tilde{\mu}_l \ge 0, l = 1,...,d, \\
&\tilde{u}_r \ge 0, r = 1,...,s, \quad (9)
\end{aligned}$$

Theorem1. The model (7) has non-zero feasible solution.

Proof. For predetermined multipliers v_k, μ_t, let $v_k = \max_{j=1,...,n}\{z_{tj}\}$ and $\mu_t = \min_{j=1,...,n}\{x_{kj}\}$. Also, suppose $v_i = 0 \, (i = 1,...,m), i \ne k$ and $\mu_l = 0 \, (l = 1,...,d), l \ne t$ and $u_r = 0 \, (r = 1,...,s)$. So, we have $\left(\sum_{r=1}^{s} u_r y_{rj} \big/ \sum_{l=1}^{d} \mu_l z_{lj}\right) = \left(0 \big/ \mu_t z_{tj}\right) = 0 \le 1$. Besides, $\frac{\sum_{l=1}^{d} \mu_l z_{lj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{\mu_t z_{tj}}{v_k x_{kj}} = \frac{\min_{j=1,...,n}\{x_{kj}\} z_{tj}}{\max_{j=1,...,n}\{z_{tj}\} x_{kj}} = \left(\min_{j=1,...,n}\{x_{kj}\} / x_{kj}\right)\left(z_{tj} / \max_{j=1,...,n}\{z_{tj}\}\right) \le 1$, because $\left(\min_{j=1,...,n}\{x_{kj}\} / x_{kj}\right) \le 1$ and $\left(z_{tj} / \max_{j=1,...,n}\{z_{tj}\}\right) \le 1$. Also, all variables are non-negative. Thus, the problem (7) has non-zero feasible solution.

Theorem2. The model (8) measures the relative efficiency of DMU_p in two-stage DEA.

Proof. At first, we prove that, for each optimal solution of the problem (8), at least one of the constraints (8-1) is binding. Consider, $\overline{u}_r^* \, (r = 1,...,s), \overline{\mu}_l^* \, (l = 1,...,d), \mu_l^* \, (l = 1,...,d), v_i^* \, (i = 1,...,m)$ be the optimal solution of the problem (8). By contrapositive assumption, we have

$$\forall j, \left(\left(\sum_{r=1}^{s} \overline{u}_r^* y_{rj} + \sum_{l=1}^{d} \overline{\mu}_l^* z_{lj} \right) \Big/ \left(\sum_{i=1}^{m} v_i^* x_{ij} + \sum_{l=1}^{d} \mu_l^* z_{lj} \right) \right) < 1$$

$$\Rightarrow \forall j, \exists \Delta_j > 0, \left(\left(\sum_{r=1}^{s} \overline{u}_r^* y_{rj} + \sum_{l=1}^{d} \overline{\mu}_l^* z_{lj} + \Delta_j (\alpha \sum_{r=1}^{s} y_{rj} + \sum_{l=1}^{d} z_{lj}) \right) \Big/ \left(\sum_{i=1}^{m} v_i^* x_{ij} + \sum_{l=1}^{d} \mu_l^* z_{lj} \right) \right) = 1$$

where $0 < \alpha \le \min_{j=1,...,n}\{\sum_{l=1}^{d} z_{lj} \big/ \sum_{r=1}^{s} y_{rj}\}$. Let $\Delta = \min_{j=1,...,n}\{\Delta_j\}$, we have

$$\forall j, \left(\left(\sum_{r=1}^{s} \overline{u}_r^* y_{rj} + \sum_{l=1}^{d} \overline{\mu}_l^* z_{lj} + \Delta (\alpha \sum_{r=1}^{s} y_{rj} + \sum_{l=1}^{d} z_{lj}) \right) \Big/ \left(\sum_{i=1}^{m} v_i^* x_{ij} + \sum_{l=1}^{d} \mu_l^* z_{lj} \right) \right) \le 1$$

$$\Rightarrow \forall j, \left(\left(\sum_{r=1}^{s} (\overline{u}_r^* + \Delta\alpha) y_{rj} + \sum_{l=1}^{d} (\overline{\mu}_l^* + \Delta) z_{lj} \right) \Big/ \left(\sum_{i=1}^{m} v_i^* x_{ij} + \sum_{l=1}^{d} \mu_l^* z_{lj} \right) \right) \le 1 \ (10)$$

Regarding definition α, we have $\alpha \sum_{r=1}^{s} y_{rj} - \sum_{l=1}^{d} z_{lj} \le 0$. From constraints (8-2), we have $\sum_{r=1}^{s} \overline{u}_r^* y_{rj} - \sum_{l=1}^{d} \overline{\mu}_l^* z_{lj} \le 0$. Therefore,

$$\sum_{r=1}^{s}\bar{u}_r^* y_{rj} - \sum_{l=1}^{d}\bar{\mu}_l^* z_{lj} + \Delta(\alpha\sum_{r=1}^{s}y_{rj} - \sum_{l=1}^{d}z_{lj}) \le 0$$

$$\Rightarrow \sum_{r=1}^{s}(\bar{u}_r^* + \Delta\alpha)y_{rj} - \sum_{l=1}^{d}(\bar{\mu}_l^* + \Delta)z_{lj} \le 0$$

$$\Rightarrow \left(\sum_{r=1}^{s}(\bar{u}_r^* + \Delta\alpha)y_{rj} \middle/ \sum_{l=1}^{d}(\bar{\mu}_l^* + \Delta)z_{lj}\right) \le 1 \quad (11)$$

From the constraints (8-3) and (10) and (11), $\bar{u}_r^* + \alpha\Delta\ (r=1,\ldots,s),\ \bar{\mu}_l^* + \Delta\ (l=1,\ldots,d),\ \mu_l^*\ (l=1,\ldots,d),\ v_i^*\ (i=1,\ldots,m)$ is a feasible solution of the problem (8). Bu

$$\left(\left(\sum_{r=1}^{s}\bar{u}_r^* y_{rp} + \sum_{l=1}^{d}\bar{\mu}_l^* z_{lp}\right)\middle/\left(\sum_{i=1}^{m}v_i^* x_{ip} + \sum_{l=1}^{d}\mu_l^* z_{lp}\right)\right) < \left(\left(\sum_{r=1}^{s}(\bar{u}_r^* + \Delta\alpha)y_{rp} + \sum_{l=1}^{d}(\bar{\mu}_l^* + \Delta)z_{lp}\right)\middle/\sum_{i=1}^{m}v_i^* x_{ip} + \sum_{l=1}^{d}\mu_l^* z_{lp}\right)$$

This contradicts the fact that $\bar{u}_r^*\ (r=1,\ldots,s),\ \bar{\mu}_l^*\ (l=1,\ldots,d),\ \mu_l^*\ (l=1,\ldots,d),\ v_i^*\ (i=1,\ldots,m)$ is the optimal solution of the problem (8). Thus, at least one of the constraints (8-1)is binding. As a result, $\max_{j=1,\ldots,n}\{(\sum_{r=1}^{s}\bar{u}_r^* y_{rj} + \sum_{l=1}^{d}\bar{\mu}_l^* z_{lj})/(\sum_{i=1}^{m}v_i^* x_{ij} + \sum_{l=1}^{d}\mu_l^* z_{lj})\} = 1$. So, we have

$$E_p^* = \left(\sum_{r=1}^{s}\bar{u}_r^* y_{rp} + \sum_{l=1}^{d}\bar{\mu}_l^* z_{lp}\right)\middle/\left(\sum_{i=1}^{m}v_i^* x_{ip} + \sum_{l=1}^{d}\mu_l^* z_{lp}\right)$$

$$= \left(\left(\sum_{r=1}^{s}\bar{u}_r^* y_{rp} + \sum_{l=1}^{d}\bar{\mu}_l^* z_{lp}\right)\middle/\left(\sum_{i=1}^{m}v_i^* x_{ip} + \sum_{l=1}^{d}\mu_l^* z_{lp}\right)\right)\middle/\max_{j=1,\ldots,n}\left\{\left(\sum_{r=1}^{s}\bar{u}_r^* y_{rj} + \sum_{l=1}^{d}\bar{\mu}_l^* z_{lj}\right)\middle/\left(\sum_{i=1}^{m}v_i^* x_{ij} + \sum_{l=1}^{d}\mu_l^* z_{lj}\right)\right\}$$

Hence, E_p^* is the relative efficiency of DMU_p.

Theorem3. The model (9) measures the relative efficiency of DMU_p in two-stage DEA.

Proof. Proo is similar to Teorem 2.

Example

Commercial banks as financial institutions receive the payments of consumers and use them in kinds of projects to generate profit. Therefore, the first stage in banking is acquiring pecuniary resources and the second stage is investment the resources to earn profit. Thus, commercial banks can be considered as two-stage processes for evaluation in DEA.

Here, 23 supervision branches of Iranian banks are studied. The inputs of the process, which are the inputs of the first stage, are payable interest (X_1), staff (X_2), and non-performing loans (X_3). The outputs of the process, which are the outputs of the second stage, are loan granted (Y_1), received interest (Y_2), and fee (Y_3). The intermediate data, which are the outputs of the first stage and the inputs of the second stage, are the total sum of four main deposits (Z_1) and other deposits (Z_5). The data are presented in Table 2.

Table 2. *Data of the bank branches in Iran*

Branch	X_1	X_2	X_3	Z_1	Z_2	Y_1	Y_2	Y_3
B_1	1639.17	14.21	16675.00	250952.00	14924.00	203538.00	354.98	138.41
B_2	1307.91	12.15	7438.00	182427.10	12188.23	157796.70	728.12	244.29
B_3	981.03	13.11	10386.79	149269.00	28110.70	202161.10	932.39	145.42
B_4	659.82	15.99	9270.50	150047.00	18359.00	651237.00	1832.74	336.52
B_5	2372.7	11.13	19420	251208.35	9649.83	570838.3	581.23	225.13
B_6	1187.25	11.76	5276.28	167316.90	5410.85	73978.42	999.96	97.46

B_7	679.26	11.70	6978.00	154246.50	4442.00	83922.00	294.46	50.10
B_8	393.78	24.48	89426.00	145740.00	164665.00	494529.00	646.66	85.48
B_9	554.70	11.37	3964.00	73316.00	1141.00	14227.00	109.04	16.16
B_{10}	1510.91	20.30	2888.00	171590.00	22340.00	93208.00	568.77	96.85
B_{11}	563.41	17.17	9941.50	80458.00	5285.00	208500.00	135.79	14.22
B_{12}	372.96	17.47	22019.00	77512.00	5656.00	24035.00	1534.69	19.82
B_{13}	817.74	11.61	8749.71	103557.40	15572.71	99227.71	326.75	48.79
B_{14}	651.69	9.71	20447.00	88949.00	3022.00	75052.00	284.04	26.47
B_{15}	437.36	16.57	12440.00	362278.00	5550.00	182611.00	391.84	163.96
B_{16}	493.44	14.19	3995.00	87227.00	6757.00	93974.00	476.60	39.86
B_{17}	505.24	16.23	9308.33	112742.00	49604.66	105776.66	950.00	358.90
B_{18}	173.51	16.32	7507.00	73176.00	4410.00	30231.00	351.65	24.02
B_{19}	26.75	5.68	2705.00	155061.00	3725.00	32691.00	3.84	26.43
B_{20}	570.79	17.70	1649.00	154306.00	3583.00	48989.00	242.37	145.38
B_{21}	680.43	17.74	42614.33	115517.30	10586.66	68866.00	1427.05	310.97
B_{22}	569.04	20.76	6837.00	213858.00	246918.00	638265.00	2171.42	458.80
B_{23}	381.02	19.99	474.66	68773.65	5177.00	99312.33	544.35	39.82

Table 3 represents the numerical results of applying three methods on this data. The second and third columns of Table 5 consists of the efficiencies obtained from Kao and Hwang's method (2008) and Chen et al.'s method (2009). The last column of Table 3 shows the efficiencies of the branches by applying our proposed method. Using our approach, $B_4, B_5, B_{19}, B_{20}, B_{22}, B_{23}$ have efficiency scores unity. Therefore, these DMUs are efficient. The rest of the DMUs are inefficient. As shown in Table 5, the efficiencies of branches by other methods are less than one and therefore the obtained efficiencies are not relative efficiencies of Branches.

Table 3. *Efficiencies of the bank branches using different methods*

DMU	Kao and Hwang's method	Chen et al.'s method	Our method
B_1	0.172	0.507	0.845
B_2	0.319	0.570	0.997
B_3	0.219	0.485	0.823
B_4	0.393	0.564	1.000
B_5	0.494	0.726	1.000
B_6	0.220	0.497	0.946
B_7	0.082	0.382	0.718
B_8	0.194	0.548	0.996
B_9	0.034	0.259	0.495
B_{10}	0.205	0.565	0.952
B_{11}	0.117	0.253	0.442

B_{12}	0.174	0.296	0.578
B_{13}	0.090	0.344	0.604
B_{14}	0.088	0.320	0.585
B_{15}	0.167	0.534	0.968
B_{16}	0.135	0.364	0.666
B_{17}	0.334	0.527	0.870
B_{18}	0.051	0.192	0.362
B_{19}	0.149	0.574	1.000
B_{20}	0.388	0.694	1.000
B_{21}	0.262	0.415	0.755
B_{22}	0.497	0.748	1.000
B_{23}	0.570	0.785	1.000

CONCLUSION

In this paper, the evaluation of DMUs in two-stage DEA was studied. Measuring the relative efficiencies of decision making units is the primary aim in DEA literature. The recently suggested models to acquire the efficiencies of DMUs in two-stage DEA are based on the CCR model. But, the CCR model obtain the absolute efficiency of assessed DMU and this efficiency in classic DEA is coincide to the relative efficiency of the DMU. In the present paper, a numerical example was used to show that previous two-stage DEA methods may not measure the relative efficiency of evaluating unit. To overcome this problem, in this paper, a fractional model for obtaining the relative efficiencies of DMUs is proposed in two-stage DEA using the definition of the relative efficiency. The model was transformed to a linear fractional program and the program was converted to an equivalent linear program by simple variable transformations. It is proved that the linear fractional program and its equivalent linear programming problem measure the relative efficiency of assessed DMU in two-stage DEA. Furthermore, to indicate the ability of the proposed method, the method was applied to evaluate the performance of the supervision branches of Iranian banks. Finally, considering the method evaluated in this paper to study more than two-stage processes in DEA can be suggested for further research.

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