

# Stability In A Flexible Manipulator Using Optimal Nonlinear Controller

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Received: June 10 2013

Accepted: July 10 2013

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## ABSTRACT

Flexible manipulators are extensively used in industries. In this paper, Generalized Backstepping Method (GBM) is used to control flexible manipulator. This method is called GBM because of its similarity to Backstepping and more applications in systems than it. Generalized backstepping method approach consists of parameters which accept positive values. The system responses differently for each value. It is necessary to select proper parameters to obtain a good response because the improper selection of the parameters lead to inappropriate responses or even may lead to instability of the system. Genetic Algorithm (GA), Cuckoo Optimization Algorithm (COA) and Particle Swarm Optimization Algorithm (PSO) are used to compute the optimal parameters for the generalized backstepping controller of flexible joint manipulator system. These algorithms can select appropriate and optimal values for the parameters. These minimize the Cost Function, so the optimal values for the parameters will be founded. Selected cost function is defined to minimizing the least square error. Cost function enforce the system error to decay to zero rapidly.

**KEYWORDS:** Flexible Manipulator, Generalized Backstepping Method, Genetic Algorithm, Cuckoo Optimization Algorithm, Particle Swarm Optimization Algorithm, Cost Function.

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## 1. INTRODUCTION

Industrial manipulator robots play an important role in the field of flexible automation. A flexible manipulator is the most basic one which is operated to perform tasks such as moving payloads or painting objects. To obtain a high performance flexible link manipulator, position controllers are necessary in order that the manipulator follows a preselected positional trajectory specified either as point-to-point or continuous path tracking motion with minimal deviation.

During recent years of considering flexible manipulator several papers were published. To reduce the end-point vibration of a single-link flexible manipulator without sacrificing its speed of response is a very challenging problem since the faster the motion, the larger the level of vibration. [1] presents a genetic algorithm (GA)-based hybrid fuzzy logic control strategy to achieve that goal. A proportional-derivative (PD) type fuzzy logic controller utilising hub-angle error and hub-velocity feedback is designed for input tracking of the system. A novel distributed sensing and actuation approach for actively suppressing vibrations within flexible link manipulators is presented in [2]. [3] proposes a method to modify the dynamics of single-link flexible arms in order to allow the design of a control system which is more robust to changes in the payload value. This is achieved by attaching some small lumped masses at some points of the link. [4] presents investigations into the design of a command-shaping technique using multi-objective genetic optimisation process for vibration control of a single-link flexible manipulator. Conventional design of a command shaper requires a priori knowledge of natural frequencies and associated damping ratios of the system, which may not be available for complex flexible systems. A feedback control law is proposed for regulating the contact force exerted by a very lightweight single-link flexible manipulator when it comes into contact with a motionless object. This control law is based on a lumped-parameter model [5]. The development of PID controller with PID and ILC feedback control strategy for vibration reduction based on genetic algorithms for flexible manipulators in vertical plane motion has been presented in [6]. [7] presents development of an approach with recursive least squares (RLS) and genetic algorithms (GAs) for modelling of single-link flexible manipulators. The backstepping design scheme is developed for the tip-position trajectory tracking control of single-link flexible robotic manipulator systems. An infinite dimensional dynamic model of a single-link flexible manipulator is derived through the assumed modes method associated with Lagrange approach [8]. A new approach for the control of flexible manipulators based on the Integral Resonant Control (IRC) scheme has been proposed in [9]. The non-linear behavior of a single-link flexible visco-elastic Cartesian manipulator is studied in [10]. A new methodology for modeling of single-link flexible manipulator with an arbitrarily large (infinite) number of deflection modes based on singular perturbation method was presented in [11]. Output feedback control of the tip position was pursued by utilizing the  $\varepsilon$ -dependent  $H_\infty$  technique on the rest part of the certain dynamics of the manipulator. The bond graph modeling of space robot with flexible link is presented. Links are modeled as Rayleigh beam. A new feedback joint trajectory control law has been presented based on distributed parameter dynamic model of a single flexible link robot manipulator [12].

The main contribution of this paper is to design a generalized backstepping controller using evolutionary algorithms optimization to control of a flexible manipulator.

The paper is organised as follows: Section 2. Describes models of a flexible joint manipulator. Section 3. Presents the Generalized Backstepping Method. Section 4. Describes controlling manipulator system. Section 5. proposes Genetic

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Algorithm, Cuckoo Optimization Algorithm and Particle Swarm Optimization Algorithm for optimizing the controller. In section 6. Represents simulation results of Manipulator system. Section 7. Provides conclusion of this work.

### 2. Modeling and Problem Formulation

The dynamics of a flexible manipulator, in state-space representation, can be described by[13].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{J_l} \sin x_1 - \frac{k_e}{J_l} (x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k_e}{J_r} (x_1 - x_3) - \frac{\mu}{J_r} x_4 + \frac{1}{J_r} u(t) \end{aligned} \tag{1}$$

Where  $x_1$  is the link position,  $x_2$  is the link angular velocity,  $x_3$  is the motor rotor position,  $x_4$  is the motor rotor angular velocity,  $J_l$  is the link inertia,  $J_r$  is the motor rotor inertia,  $k_e$  is the joint elastic constant,  $m$  is the link mass,  $l$  is the link length,  $g$  is the gravity constant,  $\mu$  is the viscosity, and  $u(t)$  is the control input. The system parameters are chosen from [13] that list in table 1.

**Table 1.** Parameters of System

Symbols	Values	Units
$mgl$	5	[N m]
$J_l$	1	[Kg m <sup>2</sup> ]
$J_r$	0.3	[Kg m <sup>2</sup> ]
$\mu$	0.1	[Kg m <sup>2</sup> /sec]
$k_e$	100	[N m]

### 3. The Generalized Backstepping Method

Generalized Backstepping Method [14-16] will be applied to a certain class of autonomous nonlinear systems which are expressed as follow

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X, \eta) + g_0(X, \eta)u \end{cases} \tag{2}$$

In which  $\eta \in \mathfrak{R}$  and  $x = [x_1, x_2, \dots, x_n] \in \mathfrak{R}$ . In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

**Theorem :** suppose equation (2) is available, then suppose the scalar function  $\varphi_i(x)$  for the  $i_{th}$  state could be determined in a manner which by inserting the  $i_{th}$  term for  $\eta$ , the function  $v(x)$  would be a positive definite equation (4) with negative definite derivative.

$$V(X) = \frac{1}{2} \sum_{i=1}^n x_i^2 \tag{3}$$

Therefore, the control signal and also the general control lyapunov function of this system can be obtained by equation 4,5.

$$u = \frac{1}{g_0(x, \eta)} \left\{ \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \varphi_i}{\partial x_j} [f_i(X) + g_i(X)\eta] - \sum_{i=1}^n x_i g_i(X) - \sum_{i=1}^n k_i [\eta - \varphi_i(X)] - f_0(X, \eta) \right\}, k_i > 0, i = 1, 2, \dots, n \tag{4}$$

$$V_t(X, \eta) = \frac{1}{2} \sum_{i=1}^n x_i^2 + \frac{1}{2} \sum_{i=1}^n [\eta - \varphi_i(X)]^2 \tag{5}$$

**Proof:** The equation (2) can be represented as the extended form of equation 6.

$$\begin{cases} \dot{x}_i = f_i(X) + g_i(X)\eta ; i = 1, 2, \dots, n \\ \dot{\eta} = f_0(X, \eta) + g_0(X, \eta)u \end{cases} \tag{6}$$

$V(X)$  is always positive definite and therefore the negative definite of its derivative should be examined; it means  $W(X)$  in equation (7) should always be positive definite, so that  $\dot{V}(X)$  would be negative definite.

$$\dot{V}(X) = \sum_{i=1}^n x_i \dot{x}_i = \sum_{i=1}^n x_i [f_i(X) + g_i(X)\eta] \leq -W(X) \tag{7}$$

By  $u_0 = f_0(X, \eta) + g_0(X, \eta)u$  and adding and subtracting  $g_i(X)\varphi_i(X)$  to the  $i_{th}$  term of equation (7) and (8) be obtained.

$$\begin{cases} \dot{x}_i = [f_i(X) + g_i(X)\varphi_i(X)] + g_i(X)[\eta - \varphi_i(X)] \\ \dot{\eta} = u_n \end{cases} \quad i = 1, 2, \dots, n \tag{8}$$

Now we use the following change of variable.

$$z_i = \eta - \varphi_i(X) \Rightarrow \dot{z}_i = u_0 - \dot{\varphi}_i(X) \tag{9}$$

$$\varphi_i(X) = \sum_{j=1}^n \frac{\partial \varphi_i}{\partial x_j} [f_j(X) + g_j(X)\eta] \tag{10}$$

Therefore, the equation (8) would be obtained as follows:

$$\begin{cases} \dot{x}_i = [f_i(X) + g_i(X)\varphi_i(X)] + g_i(X)[\eta - \varphi_i(X)] \\ \dot{z}_i = u_0 - \dot{\varphi}_i & ; i = 1, 2, \dots, n \end{cases} \tag{11}$$

Regarding that  $z_i$  has  $n$  states, the  $u_0$  can be considered with  $n$  terms, provided that the equation (12) would be established as follows.

$$u_0 = \sum_{i=1}^n u_i \tag{12}$$

Therefore, the last term of equation (11) would be converted to equation 13.

$$\dot{z}_i = u_i - \dot{\varphi}_i(X) = \lambda_i \tag{13}$$

At this Stage, the control Lyapanov function would be considered as equation 14.

$$V_t(X, \eta) = \frac{1}{2} \sum_{i=1}^n x_i^2 + \frac{1}{2} \sum_{i=1}^n z_i^2 \tag{14}$$

Which is a positive definite function. Now it is sufficient to examine negative definitly of its derivative.

$$\dot{V}_t(X, \eta) = \sum_{i=1}^n \frac{\partial V(X)}{\partial x_i} [f_i(X) + g_i(X)\varphi_i(X)] + \sum_{i=1}^n \frac{\partial V(X)}{\partial x_i} g_i(X) + \sum_{i=1}^n z_i \lambda_i \tag{15}$$

In order that the function  $\dot{V}_t(X, \eta)$  would be negative definite, it is sufficient that the value of  $\lambda_i$  would be selected as the equation 16.

$$\lambda_i = -\frac{\partial V(X)}{\partial x_i} g_i(X) - k_i z_i \quad ; k_i > 0 \tag{16}$$

Therefore, the value of would be obtained from following equation.

$$\dot{V}_t(X, \eta) = \sum_{i=1}^n x_i [f_i(X) + g_i(X)\varphi_i(X)] - \sum_{i=1}^n k_i z_i^2 \leq -W(X) - \sum_{i=1}^n k_i z_i^2 \tag{17}$$

Which indicates that the negative definitly status of the function  $\dot{V}_t(X, \eta)$ .

Consequently, the control signal function, using the equations (8,9) and (11) would be converted to 18.

$$u_0 = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial \varphi_i}{\partial x_j} [f_j(X) + g_j(X)\eta] - \sum_{i=1}^n x_i g_i(X) - \sum_{i=1}^n k_i [\eta - \varphi_i(X)] \tag{18}$$

Therefore, using the variations of the variables which we carried out, the equations (4,5) can be obtained. Now, considering the unlimited region of positive definitly of  $V_t(X, \eta)$  and negative definitly of  $\dot{V}_t(X, \eta)$  and the radially unbounded space of its states, global stability gives the proof.

#### 4. Controlling Manipulator System

The generalized backstepping method is used to control the states  $x_1, x_2, x_3, x_4$  to the origin point(0,0,0,0), via the torque  $u$ . In order to convert manipulator system equation (1) to the general state of equation (2), the change of variables  $X_1 = x_1 + x_3, X_2 = x_2 + x_3$  should be carried out. Therefore, the equation (1) would be converted to equation (19), as follows.

$$\begin{aligned} \dot{X}_1 &= X_2 - x_3 + x_4 \\ \dot{X}_2 &= -\frac{mgl}{J_l} \sin(X_1 - x_3) - \frac{k_e}{J_l} (X_1 - 2x_3) + x_4 \end{aligned} \tag{19}$$

$$\begin{aligned} \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k_e}{J_r} (X_1 - 2x_3) - \frac{\mu}{J_r} x_4 + \frac{1}{J_r} u(t) \end{aligned}$$

Stabilization of the state: In order to use the theorem, it is sufficient to establish equations 20,21 and22.

$$\varphi_1(X_1, X_2, x_3) = x_3 - X_2 - k_1 X_1 \tag{20}$$

$$\varphi_2(X_1, X_2, x_3) = \frac{mgl}{J_l} \sin(X_1 - x_3) + \frac{k_e}{J_l} (X_1 - 2x_3) - k_2 X_2 \tag{21}$$

$$\varphi_3(X_1, X_2, x_3) = -k_3 x_3 \tag{22}$$

According to the theorem, the control signal will be obtained from the equation 23.

$$u = J_r \left[ \left( \frac{\partial \varphi_1}{\partial x_1} + \frac{\partial \varphi_2}{\partial x_1} + \frac{\partial \varphi_3}{\partial x_1} \right) \dot{X}_1 + \left( \frac{\partial \varphi_1}{\partial x_2} + \frac{\partial \varphi_2}{\partial x_2} + \frac{\partial \varphi_3}{\partial x_2} \right) \dot{X}_2 + \left( \frac{\partial \varphi_1}{\partial x_3} + \frac{\partial \varphi_2}{\partial x_3} + \frac{\partial \varphi_3}{\partial x_3} \right) \dot{x}_3 - (X_1 + X_2 + x_3) - k_4(x_4 - \varphi_1) - k_5(x_4 - \varphi_2) - k_6(x_4 - \varphi_3) - \frac{k_e}{J_r}(X_1 - 2x_3) + \frac{\mu}{J_r}x_4 \right] \tag{23}$$

And Lyapunov function as.

$$V(X_1, X_2, x_3, x_4) = \frac{1}{2}X_1^2 + \frac{1}{2}X_2^2 + \frac{1}{2}x_3^2 + \frac{1}{2}x_4^2 + \frac{1}{2}(x_4 - \varphi_1)^2 + \frac{1}{2}(x_4 - \varphi_2)^2 + \frac{1}{2}(x_4 - \varphi_3)^2 \tag{24}$$

**5. Manuscript Structure**

The Genetic Algorithm [17,18], Cuckoo Optimization Algorithm [19] and Particle Swarm Optimization Algorithm [20] are used to search the optimal parameter (*k*) in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response.

The controller in the equation (23) is optimized by the Cost Function in the equation 25.

$$f(x_1, x_2, \dots, x_n) = \frac{1}{n} \sqrt{\sum_{i=1}^n \int (x_i(t) - x_{di})^2 dt} \tag{25}$$

*x<sub>i</sub>* is the system state and *x<sub>di</sub>* is the favorit mood for *x<sub>i</sub>*.

**Table 2.** Genetic Algorithm Parameters

Parameters	Values
Size population	100
Maximum of generation	100
Prob.crossover	0.75
Prob.mutation	0.001
<i>k</i> Search interval	[0.1 30]

**Table 3.** Cuckoo Optimization Algorithm Parameters

Parameters	Values
Size clusters	2
Maximum number of cuckoo	200
Size initial population	5
Maximum iterations of cuckoo	200
<i>k</i> Search interval	[0.1 30]

**Table 4.** Particle Swarm Optimization Algorithm Parameters

Parameters	Values
Size population	100
Size initial population	100
Initial and Final value of the global best acceleration factor	2 and 2
Initial and Final value of the inertia factor	1 and 0.99
<i>k</i> Search interval	[0.1 30]

**6. Numerical Simulations**

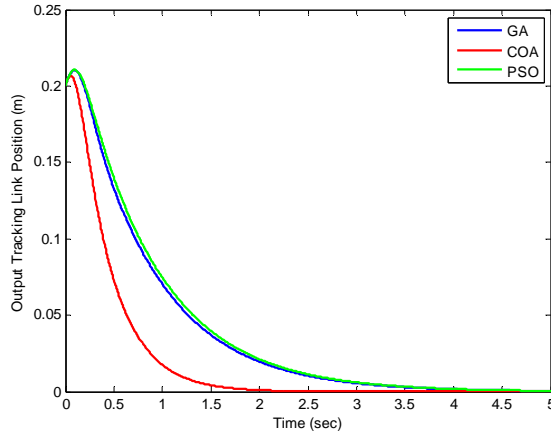
This section presents numerical simulations flexible link manipulator. The Optimal Generalized Backstepping Method (OGBM) is used as an approach to control manipulator system. The optimal Parameters of generalized backstepping controller using genetic algorithm, cuckoo optimization algorithm and particle swarm optimization algorithm are listed in tables 5.

**Table 5.** optimal parameters of generalized backstepping controller

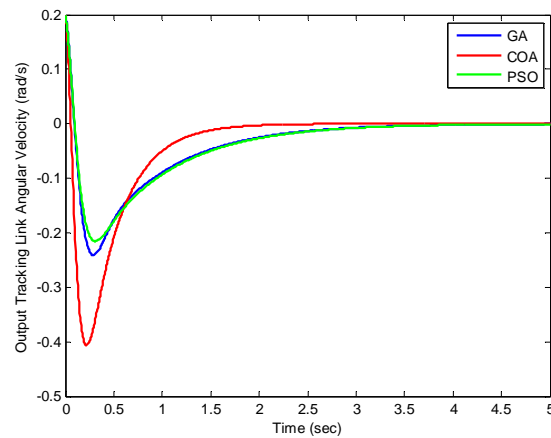
	<i>k</i> <sub>1</sub>	<i>k</i> <sub>2</sub>	<i>k</i> <sub>3</sub>	<i>k</i> <sub>4</sub>	<i>k</i> <sub>5</sub>	<i>k</i> <sub>6</sub>
GA	1.69	20.01	29.8	29.98	8.75	1.43
COA	17.49	18.43	30	0.1	28.4	28.4
PSO	30	30	30	0.1	7.09	4.53

Figure 1 shows that *x*<sub>1</sub> state of manipulator system can be stabilized with the control law *u*(23) to the origin point(0,0,0,0). Figure 2 shows that *x*<sub>2</sub> state of manipulator system can be stabilized with the control law *u*(23) to the origin point(0,0,0,0). Figure 3 shows that *x*<sub>3</sub> state of manipulator system can be stabilized with the control law *u*(23) to

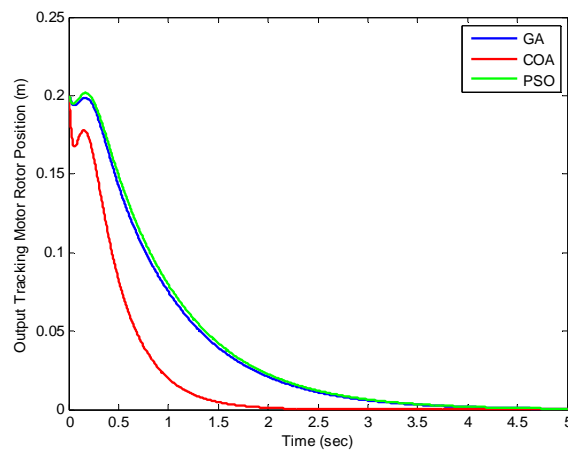
the origin point(0,0,0,0). Figure 4 shows that  $x_4$  state of manipulator system can be stabilized with the control law  $u(23)$  to the origin point(0,0,0,0). Figure 5 shows that the scalar control signal  $u(23)$  when manipulator stabilizes to the origin point(0,0,0,0).



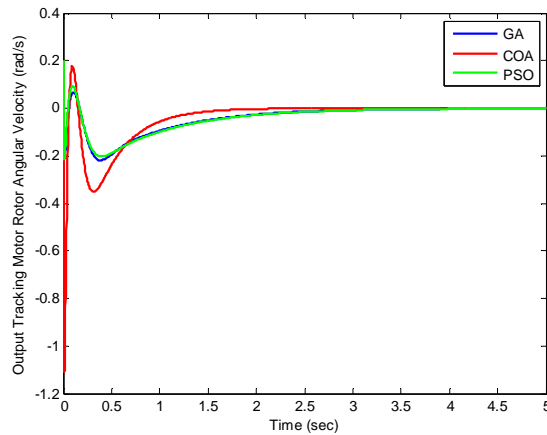
**Fig. 1.** The time response of the Link Position for the controlled Flexible Manipulator where the control input is (23).



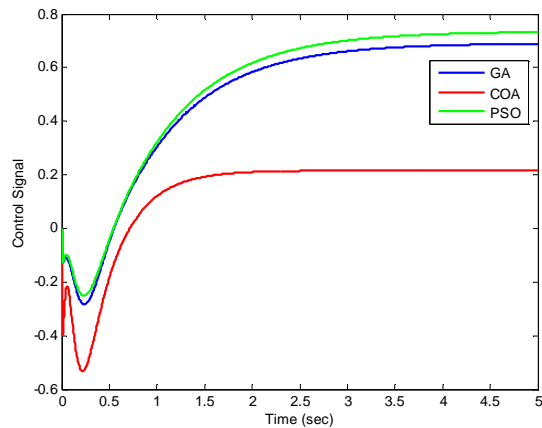
**Fig. 2.** The time response of the Link Angular Velocity for the controlled Flexible Manipulator where the control input is (23).



**Fig. 3.** The time response of the Motor Rotor Position for the controlled Flexible Manipulator where the control input is (23).



**Fig. 4.** The time response of the Motor Rotor Angular Velocity for the controlled Flexible Manipulator where the control input is (23).



**Fig. 5.** The time response of control signal (23) where a Flexible Manipulator stabilizes to the origin point(0,0,0,0).

## CONCLUSION

A flexible link manipulator is the most manipulators are extensively used in industries. In this paper, a flexible joint manipulator is controlled with generalized backstepping method. The designed controller consists of parameters which accept positive values. The controlled system presents different behavior for different values of these parameters. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system. It is needed to optimize these parameters. Evolutionary Algorithms are well known optimization method. Genetic Algorithm, Cuckoo Optimization Algorithm and Particle Swarm Optimization Algorithm optimize the controller to gain optimal and proper values for the parameters. For this reason this algorithms minimize the cost function to find minimum current value for it. On the other hand cost function finds minimum value to minimize least square errors. As the simulation results show, this approach, the errors, reach to their minimum values that is demonstrated to have more optimal values when compared with previous methods.

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