

Meta Heuristic for Multi Depot Inventory Routing Problem Backlogging

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ABSTRACT

Inventory routing problem is one of the major issues in supply chain operations planning. The aim is to Integrate activities associated with transportation and inventory management throughout the supply chain avoid inefficiencies caused by solving the underlying vehicle routing and inventory sub-problems separately. This paper considered to multi-depot inventory routing problem with backlog order. We develop a mixed integer mathematical model for multi-depot inventory routing problem with backlog order. The mixed integer problem is used to solve problem in small instances. A method based on parallel genetic algorithm is proposed to solve large instances. The result of proposed method is compared with result of Abdelmaguid et al (2009) saw [21] in case of single depot with backlog order instances, and it is compared with lower bound obtained Cplex solver in case of multi depot. Computational results show that the proposed method has good performance.

KEYWORDS: Inventory routing problem, mixed integer problem, meta heuristic method, backlogging, multi-depot

1.INTRODUCTION

In recent years, one of the most interesting issues in the field of supply chain management is to integrate and coordinate activities. Inventory routing problem is one of the famous issues in this area. (Golden et al, 1984 [1]) have introduced this problem as a combination of transport equipment and inventory control problems and showed that it is NP-hard. Another interpretation of the problem is presented by (Custodio and Oliveira, 2006 [2]). This is interpreted as a set of retailers with known demand in a supply network can be served daily. A more precise definition by (Campbell et al, 1998 [3]) have been proposed to explain that Inventory routing problem is the distribution of a single product from a single supplier to a set of retailers and customers who have placed in a scattered geographical area by a homogeneous flow of vehicles during the period. Retailers save defined amounts of inventories that can be consumed with a certain rate (Boudia et al, 2007 [4]).

The aim of this problem is finding a minimum cost of holding inventory and distribution, while the customer is assured that demand will be met on time. Other restrictions have been added to the problem over time, leading to a variety of inventory routing problem. Several applications of this type of problem have been reported in the literature, for example, we can mention the following: the distribution of liquid Propane in the petrochemical industry (Savelsbergh and Song, 2007 [5]), retailing in the Netherlands (Dror and Ball, 1987 [6]), Auto Parts (Alkayyal and Hwang, 2007 [7]) and frozen products (Cerny, 1985 [8]) and the Ammonia distribution through sea routes from a central repository to a group of consumers have been reported by (Andersson et al 2010 [9]). According to a new study by Andersson et al, there are few studies on inventory routing problems with backlog orders. (Chien et al, 1989 [10]) have suggested mixed integer mathematical model for an inventory routing problem by probability of orders and requests postponement that it was assumed the central warehouse is a set of customers serve using the same equipment. Later, a single depot inventory routing problem was introduced by (Chandra, 1993 [11]) as the demand rate is assumed to be deterministic. Method used to solve the problem is based on the analysis on the problem of ordering and distribution. (Barnes-Schuster and Bassok, 1997 [12]) and (Bard et al, 1998 [13]) have studied on inventory routing problem backlogging with stochastic demands and unlimited time horizon. They suggested low-cost transportation services to the customers by the limited number of direct transport proposals homogenized. (Reiman et al, 1999 [14]) studied on single depot inventory routing problem with backlog orders and stochastic demands. They assumed a single transportation serves to retailers in direct and multi-station structures and their problem-solving approach is using Monte Carlo simulation and heavy traffic analysis. (Schwartz et al, 2006 [15]) have mentioned an inventory routing problem with backlog orders and stochastic demands such that A single vehicle transportation service to retailers. They proposed an analytical model and solve it using simulation and the heuristic method. Other topology researchers studied inventory routing problem with a limited time horizon

(Savelsbergh and Song, 2008 [16-19]). (Abdelmaguid et al, 2006 [20]) have introduced a single depot inventory routing problem with backlog orders and absolute demand storage in which several vehicles traveling by the possibility of homogeneous. They are useful heuristic based genetic algorithm for solving the problem. Another useful heuristic for the problem has been proposed recently, and they have been improving their resolution (Abdelmaguid et al, 2009 [21]). Several articles studied on inventory routing problem in the supply network. (Liu and Chen, 2011 [27]) considered the problem of network and offered for providers and suppliers an affordable pricing. Furthermore, (Liu and Chen, 2012 [26]) considered the issue in the supply network and proposed variable neighborhood search for solving it.

As noted, multi depot inventory routing problem have been studied in several cases. In all these cases it is assumed that no order has delayed and retailers usually use single depot (Andersson et al, 2009 [9]). Since the two main hypothesis investigated in several papers are violated in the real world, in this paper we consider the problem of new hypotheses to come closer to the real conditions. To solve the problem, we developed a mixed integer mathematical model that allows backlog orders from multi depot. Mathematical model is to find the best combination of transportation costs, deferred maintenance, and small examples for orders. To solve the problem with many clients in real terms, the problem is broken into two main sub-problems and then we solved each with a separate level. In the proposed method, the problem is searched at the macro level by a parallel genetic algorithm. To determine the optimality of decisions for macro-level problem, the problem is solved by a combinatorial algorithm. In the following, we describe the proposed solution. Other parts of this paper are as follows: In Section 2, a mathematical model is developed for the problem. Algorithm is described in section 3 and the computational results are given in section 4. The conclusions are contained in section 5.

2. Problem defining and the mathematical model

In our model we consider two-level supply chain in which the retailer is set by the flow of services from different transport capacities that are homogeneous. In the first case, a central depot serves to a set of retailers that are geographically dispersed and have a specific request. Retailers have limited storage capacity and cannot store any of the products that they want. The problem is a multi period problem with finite time horizon. Retailer's demand in each period is assumed to be known and must be met by the end of the specified period. If the delivery is delayed by the depot, they will face with a fine called the cost of backlog orders. Retailer that has been delayed will serve the next periods. The main goal of the inventory routing problem is finding the appropriate balance between inventory costs (holding and shortage) and shipping costs. In multi depot problem, central depot, is replaced with multi depot. Other assumptions remain unchanged. Assumptions of routing equipment transportation problem in both cases are the same as the previous models.

In this episode we have a mixed integer mathematical model for multi depot inventory routing problem with backlog orders. First, the sets, constants, parameters and decision variables used in the model are introduced:

Sets:

R: retailer set

D: depot set

T: time period set

K: vehicle set

Constants and Parameters:

c_{ij} : Travel cost from retailer/depot i to retailer/depot j for $\forall i, (RUD)$ satisfying the triangular inequality:

$$c_{ik} + c_{kj} \geq c_{ij} \quad (1)$$

π_i : Backlog cost per period per unit for $\forall i \in R$

h_i : holding cost per period per unit for

$\forall i \in R$

d_{it} : Demand of retailer i in period t for $\forall i \in (RUD), \forall t \in T$

v_i : Storage capacity of retailer i for

$\forall i \in R$

s_{ik} : Time to serve retailer i by vehicle k for $\forall i \in R, \forall k \in K$ t_{ij} : Travel time from the retailer / supplier to retailer i / j Supplier

t_{ij} : travel time from retailer/ supplier i to retailer/supplier j for $\forall i, j \in (RUD)$

b_i : Initial amount of inventory of retailer i for $\forall i \in R$

L: Length of each time period

Q: Vehicle capacity

M: A big positive constant

N: Maximum number of the retailers

Decision variables

For $\forall i, \{RUD\}, \forall t \in T, \forall k \in K, X_{ijkt}$ is equal to one if vehicle k travels from node i to node j in period t , otherwise it is zero.

$I_{i,t}^-$: Backlog amount incurred by retailer i in period t , for $\forall i \in R, \forall t \in T$

$I_{i,t}^+$: Inventory amount in retailer i store in period t , for $\forall i \in R, \forall t \in T$

W_{ikt} : The amount delivered to retailer i by vehicle k in period t , for $\forall i \in R, \forall k \in K, \forall t \in T$

$U_{i,t}$: If a vehicle is serving retailer i , the number of the other retailers that should be visited by that vehicle after customer i in each period of time. It should be noted that $U_{i,t}$ is unique for $\forall i \in R, \forall t \in T$.

The mathematical model has been developed based on a mathematical model of (Abdelmaguid et al 2009) is given by:

$$\text{Min } z = \left(\sum_{i \in RUD} \sum_{j \in RUD} \sum_{k \in K} \sum_{t \in T} c_{ij} X_{ijkt} \right) + \left(\sum_{i \in R} \sum_{t \in T} \pi_i I_{i,t}^- \right) + \left(\sum_{i \in R} \sum_{t \in T} h_i I_{i,t}^+ \right) \quad (2)$$

Subject to:

$$\sum_{k \in K} \sum_{j \in RUD} X_{ijkt} \leq 1 \quad \forall i \in R, \forall t \in T \quad (3)$$

$$X_{ijkt} \leq \sum_{p \in D} X_{pikt} + M(1 - U_{i,t}) \quad \forall i, j \in R, \quad i \neq j, \forall t \in T, \forall k \in K \quad (4)$$

$$\sum_{\substack{i \in RUD \\ n \neq i}} X_{inkt} - \sum_{\substack{j \in RUD \\ n \neq j}} X_{njkt} = 0 \quad \forall k \in K, \forall n \in (R \cup D), \forall t \in T \quad (5)$$

$$\sum_{i \in RUD} \sum_{j \in RUD, i \neq j} S_{i,k} X_{ijkt} + \sum_{i \in RUD} \sum_{j \in RUD, i \neq j} t_{ij} X_{ijkt} \leq L \quad \forall k \in K, \forall t \in T \quad (6)$$

$$I_{i,t-1}^+ + I_{i,t}^- = I_{i,t}^+ + d_{it} + I_{i,t-1}^- \quad \forall i \in R, \forall t \in T, \forall k \in K \quad (7)$$

$$I_{i,t}^+ \leq V_i \quad \forall i \in R, \forall t \in T, \forall k \in K \quad (8)$$

$$W_{jkt} \leq Q \sum_{i \in RUD} X_{ijkt} \quad \forall j \in R, \forall k \in K, \forall t \in T \quad (9)$$

$$\sum_{j \in R} W_{jkt} \leq Q \quad \forall k \in K, \forall t \in T \quad (10)$$

$$U_{i,t} - U_{j,t} + \sum_{k \in K} (N-1) X_{ijkt} \leq (N-1) - 1 \quad \forall i, j \in R, i \neq j, \forall t \in T \quad (11)$$

$$U_{i,t} \leq N - 1 \quad \forall i \in R, \forall t \in T \quad (12)$$

$$U_{i,t} \geq 1 \text{ and integer} \quad \forall i \in R, \forall t \in T \quad (13)$$

$$X_{ijkt} \in \{0,1\} \quad \forall i, j \in (R \cup D), \quad i \neq j, \forall k \in K, \forall t \in T \quad (14)$$

$$W_{ikt} \geq 0 \quad \forall i \in R, \forall k \in K, \forall t \in T \quad (15)$$

$$I_{i,t}^+ \geq 0 \quad \forall i \in R, \forall t \in T \quad (16)$$

$$I_{i,0}^+ = b \quad \forall i \in R \quad (17)$$

$$I_{i,t}^- \geq 0 \quad \forall i \in R, \forall t \in T \quad (18)$$

The objective function Eq. (2) minimizes the total cost of transportation, backlog and holding inventories. The shortage and inventory carrying costs are calculated by the net backlog and inventory amount in the retailer's store at the end of each period. Equation (3) reassures that each retailer is visited only once by only one vehicle in each time period. Equation (4) indicates that each vehicle starts its trip from a depot and returns to the same depot during each time period. Equation (5) explains the ordinary flow conservation relationship. (according to both Eq. (4) and (5)). It is implied by Eq. (6) that the transfer time between retailers and suppliers does not exceed the length of a time period which is considered to be an eight hour time period. All the retailers are supposed to be served by the end of that period. Equation (7) is the inventory balance equation. Equation (8) indicates that the inventory of each retailer cannot exceed its storage capacity. Equation (9) and (10) define upper bounds on the delivery amounts by the vehicle capacity. Equations (11)-(13) are the MTZ sub-tour elimination relationships (Tucker et al, 1960).

Equations (14)-(16) and equation (18) are the domain constraints. Equation (17) indicates the initial amount of inventory at the retailer’s store.

The resulting model, called MDIRPB(Multi Depot Inventory Routing Problem Backlogging) , is mixed integer linear program and can be thought of as a combination of MDVRP (Multi Depot Vehicle Routing Problem) and an inventory control model. Because of the underlying VRP component, the problem is NP-hard and its complexity grows exponentially by increasing the number of retailers, depots, time periods. The tighter the due dates and capacity limitations, the more difficult the problem becomes to solve.

3.The solution method

One of the key decision in inventory routing problem is to determine how much to deliver to each retailer in each period and by which vehicle (w_{ikt}). Once this decision has been made, the inventory and backlog amounts can be determined using Equations (7) and (8). Then, a VRP can be solved for each period to find the best possible routes. Many researchers decompose an IRP into two sub problems: an inventory control sub-problem (SUB1) and a vehicle routing sub-problem (SUB2) (see for instance, [22, 23]). We will separate the problem into two main sub-the amount of w_{ikt} is computed problems. In the first sub-problem amount of w_{ikt} is computed in the same genetic structure, and then the second sub-problem uses these amounts to find the most appropriate routes in each period. Distribution is determined by the balance between the costs of transportation , inventory and backlog order. The heuristic combined with the parallel genetic algorithm is presented to solve MDIRPB.

3.1. MDVRP solution procedure

Vehicle routing problem is one of the key components of inventory routing problem. Therefore, the algorithm used for solving the underlying VRP should be an efficient one. In this paper, we employ an efficient algorithm for solving multi-depot vehicle routing problem. The method decomposes the multi-depot problem to several single depot ones following the ideas in (Gillet and Miller, 1971) . Then, each VRP is solved using an efficient saving based algorithm of (Clarke and Wright 1964). Afterwards, the solution is improved using a nearest neighborhood heuristic (NNH). The combination of these three algorithms helps us find a good solution for MDVRP part of MDIRPB.

3.2.Parallel genetic algorithm for solving MDIRPB

In this paper, genetic algorithm and a specific coding structure is presented for solving MDIRPB. The structure of the problem inventory (SUB1) is a model of ($R*T$) gene is a gene that if (r, t) of period t and r is the distributor. If the gene is a demand distribution for the period t to t+a-1 in period t is sent. Fig.1 provides the answer for SUB1.

	T1	T2	T3	T4	T5	T6
R1	0	0	2	1	0	0
R2	3	0	1	0	3	0
R3	0	2	2	1	0	2
R4	3	2	1	2	1	1
R5	1	2	3	0	0	2

Figure 1. Structure of the gene SUB1

The initial solution is randomly generated. A single point mutation is a random change in the number of genes and integration with genetic operators has been used to cut a few corners. The roulette wheel is used for operator selection. Order value for each chromosome in the t period marked by issues related MDVRP hybrid algorithm presented in Section MDVRP solution procedure is solved. The fitness function is the sum of inventory and transportation costs will be calculated and included in the process of parallel genetic algorithm is used.

Many ways to implement parallel genetic algorithm are presented. The most widely used method in multi-deme model by (Gross, 1985) proposed that it is used for problem solving MDIRP. Repeat the process of algorithm is described below.

Step 1. P of the population mutation rate, population size and different integration occurs.

Step 2. Chromosomes are initialized by different communities.

Step 3. i = 1

Step 4. For p = 1 .. P repeat

Step 4.1. Fit for Chromosomes are calculated function values of p. (To calculate the fitness function must MDVRP combined for all periods presented to be solved by the hybrid algorithm)

Step 4.2. Operators of selection, mutation, and integrate apply over the p.

Step 4.3. The next generation is determined.

Step 5. If t% Epoch =0

Step 5.1. Migration on topology

Step 5.2. Replacement strategy based on immigration

Step 6. $i = i + 1$

Step 7., If $i < I$ go to step 4.

Step 8. End

The communities formed in parallel computing and parallel operators, it also occurs.

4. Computational results

To test the efficiency of the proposed heuristic method, IRPB problems have been solved in two cases: single depot IRPB and multi depot IRPB. The first case of samples [21] is used for comparison and for the second case of random sampling is used. Since there is no way to solve MDIRPB, in this article is used from the provided lower bound of CPLEX 12.2 software for MIP model. The heuristic method has been coded and compiled in MATLAB (version 2012) running under a core i7 processor with 4GB RAM. For single depot there are three scenarios proposed in [21]. The first and second scenarios contain 12 problem types. For each type, they generate and solve five samples. Each sample is shown by six digits. The first one represents the scenario number. The second and third ones are for customer count, and the fourth one is for time period count. The fifth one stands for the vehicle count, and the last one is the sample number in that problem type. Table 1, shows some samples of results is compared to the (Abdelmaguid et al, 2009) results .The overall results show that the proposed algorithm had a better balance between inventory costs and provide transportation. Table 2 shows the results multi-depot IRPB. As can be seen in this table, the results obtained by parallel genetic algorithm are very close to those of Cplex.

Fig.2,3 the average cost for solved examples show the algorithm under study. Graphs clearly show that the proposed algorithm than gives Abdelmaguid et al algorithm contributed more to the cost of inventory. This algorithm with a more appropriate balance between inventory costs and transportation, in addition to reducing costs and generating solutions are more appropriate.

Figure 2. Comparison of the average cost of the parallel genetic algorithm and the ETCH-H algorithm

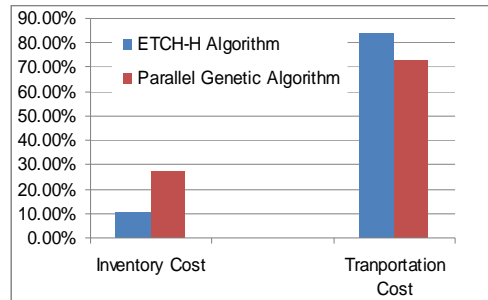
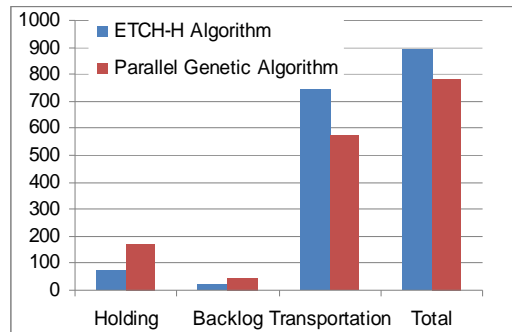


Figure 3. Comparison of various algorithms in terms of cost



Figures 5 and 6 show parallel genetic algorithms for the problem solving of process 8-30272-3. As Figure 4 shows, the convergence of the algorithm is good. Figures 5 and 6, the results show that the algorithm for the period of 1 to 7.

5. Conclusion

This paper investigates multi-depot inventory routing problems allowing order backlogs. Due to the inevitability of backlogging in real world problems, and the existence of more than one supplier depot in most cases, the combination of these two features in an integrated model is of practical importance. The method developed herein helps to decrease the cost in comparison with the case where single depot IRP with no. To solve the problem, a mixed integer problem model proposed and parallel genetic algorithm is used to solve it. Algorithm computational results with the results in the single depot (Abdelmaguid et al 2009) compared the multiple depot at the lower bound of the obtained results are compared with CPLEX 12.2. The results show the efficiency of parallel genetic algorithm is suitable. As future research, we can assume MDIRPB synchronization problems and issues to be considered in the scheduling chain. Also MDIRPB developed exact solution algorithms or heuristics to solve by other methods or meta-heuristics.

Considering the problem of network providing IRP can also be used to model the real issues. IRP problem is to manage the distribution of food, particularly in major cities of capillary distribution is used.

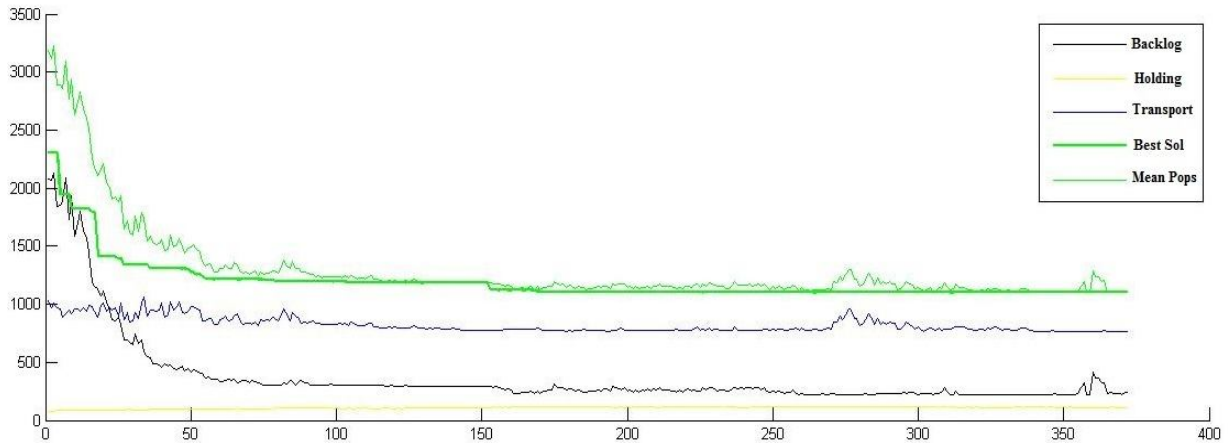


Figure 4. The process of convergence of the parallel genetic algorithm

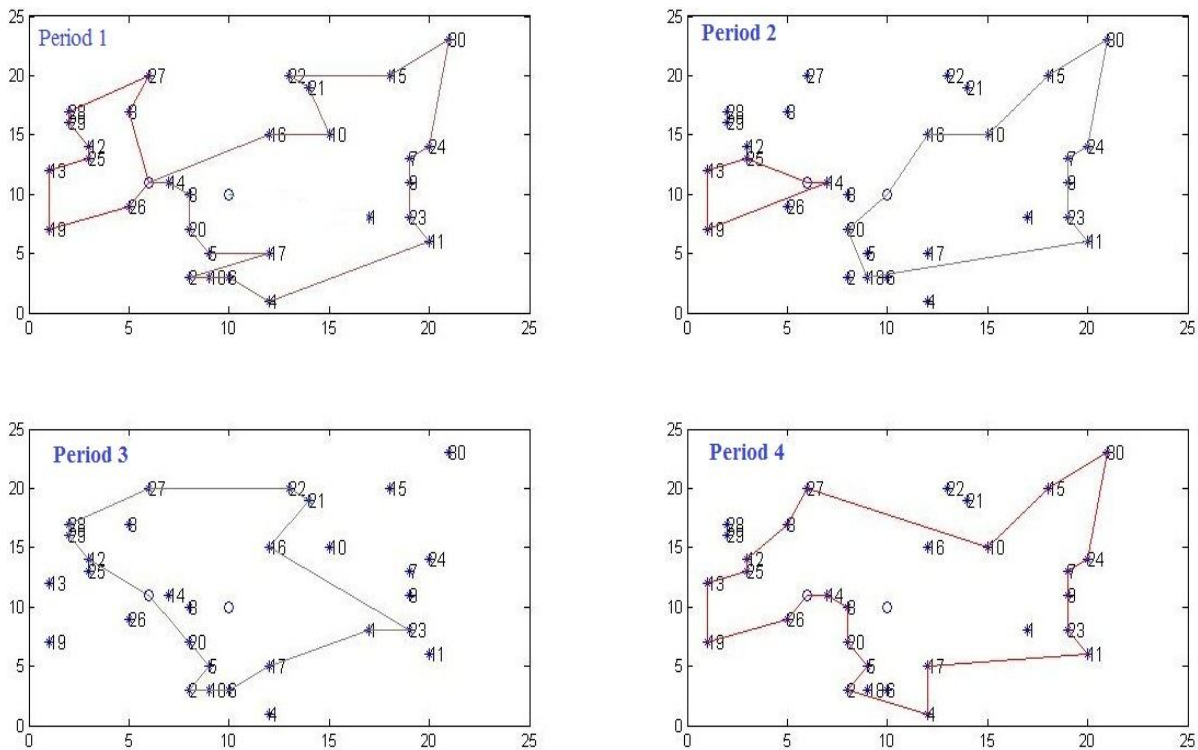


Figure 5. The proposed distribution for the period of 1 to 4

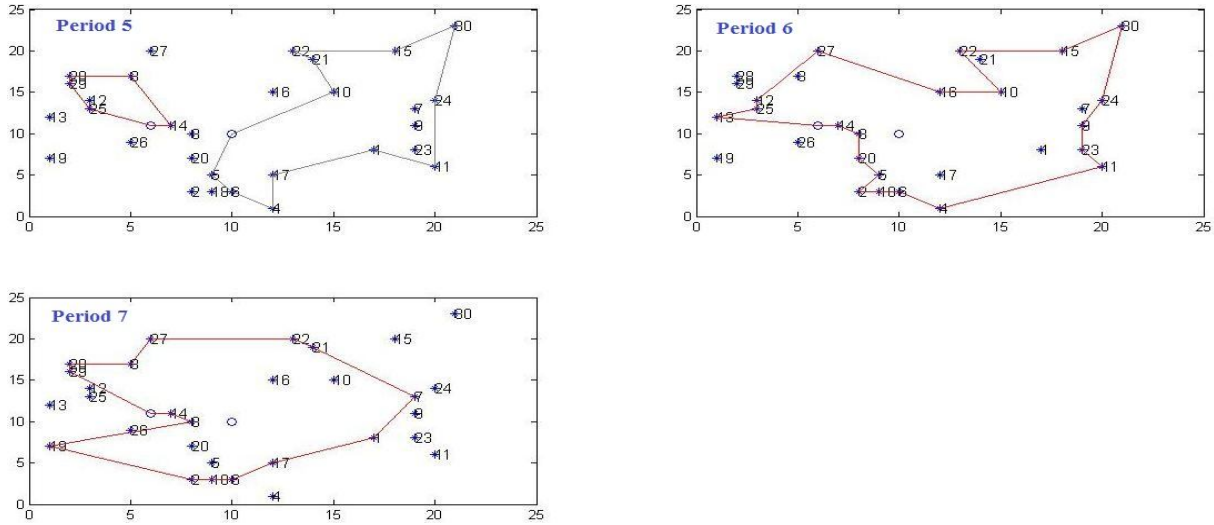


Figure 6. The proposed distribution for the period of 5 to 7

Table 1. Comparison of parallel genetic algorithm with ETCH-H algorithm in IRPB

Problem	Parallel Genetic Algorithm				ETCH-H Algorithm			
	Transp.	Backlog	Holding	Total	Transp.	Backlog	Holding	Total
1-1072-1	297.708	0	124.9	422.6	351	0	149.97	500.97
1-1072-2	291.41	0	148.09	439.5	436	0	126.52	562.52
1-1072-3	283.14	0	137.47	420.61	342	0	114.39	456.39
1-1072-4	284.08	0	108.49	392.57	453	0	74.34	527.34
1-1072-5	294.53	0	122.43	416.98	441	0	103.68	544.68
2-1572-1	854.13	93.21	233.71	1181.1	1194	0	51.92	1245.92
2-1572-2	718.18	18.76	239.09	976.03	1137	72.41	25.56	1234.97
2-1572-3	899.02	130.27	198.01	1227.3	1244	40.5	65.88	1350.38
2-1572-4	869.1	22.8	207.19	1099.1	1080	15.65	99.43	1195.08
2-1572-5	782.36	59.32	229.83	1071.5	1180	188.07	20.89	1388.96
3-3027-1	565.86	75.65	171.16	812.67	781	16.25	51.05	848.3
3-3027-2	579.83	26.78	181.33	787.94	766	6.18	55.4	827.58
3-3027-3	641.13	45.52	137.87	824.52	83	0	51.16	934.16
3-3027-4	562.7	81.77	164.98	809.45	827	10.47	50.61	888.08
3-3027-5	643.16	94.65	149.54	887.35	870	0	31.59	901.59
Average	571.089	43.2486667	170.273	784.615	745.66667	23.302	71.4927	893.795
Share Of The Total Cost	72.79 %	27.21 %			83.43 %	10.61 %		

Table 2. Comparison of parallel genetic algorithm with lower bound obtained by CPLEX 12.2 in MDIRPB

Problem	Cplex				Parallel Genetic Algorithm					
	Time	Best Bound	Objective Function	Optimality Gap	Holding	Backlog	Transp.	Time	Total	Optimality Gap
05-2-7-1	00:31:47	227.18	227.19	0.00%	50.85	0	184.11	00:00:02	230.268059	1.36%
05-2-7-2	00:03:45	148.64	148.66	0.01%	31.23	0	120.8	00:00:01	149.41426	0.52%
05-2-7-3	02:23:29	213.378	213.4091	0.01%	72.32	0	143.34	00:00:02	216.90148	1.65%
10-2-5-1	02:34:05	213.76	227.23	6.30%	65.76	0	164.48	00:00:03	214.781989	0.48%
10-2-5-2	02:44:03	230.61	253.53	9.94%	76.27	0	161.08	00:00:07	231.448954	0.36%
10-2-5-3	02:35:05	229.89	246.91	7.40%	63.95	0	176.9	00:00:05	235.368635	2.38%
Average	01:48:42	210.576	219.48818	3.95%	60.0633	0	158.452	00:00:03	2313.030563	1.13%

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