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# Ranking in DEA Using Different Efficiency Levels

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**ABSTRACT**

This study proposes a DEA (data envelopment analysis) method based upon using different efficiency levels. Firstly, all efficient DMUs (decision making units) should be deleted from the original PPS (production possibility set). Then, a new PPS is constructed based on remaining DMUs which were inefficient already. After that each efficient DMU under assessment, will be added to that new PPS solely (it means that this addition must be in the absence of the other efficient DMUs). The alteration caused to the new PPS and its DMUs would be as a criterion to assess and rank that added efficient one. Two case studies are presented to demonstrate the practically of our proposal.
**KEYWORDS*:*** DEA; Ranking; Efficiency level

## 1 INTRODUCTION

The applicability of DEA has been quickened greatly by appearance of Charnes, Cooper and Rhodes's work as the CCR model [4], that is applied to measure the efficiency score of DMUs. However, the applicability DEA in different contexts, but it cannot provide itself complete information to do discriminate between efficient DMUs. So, a huge number of papers have been developed in DEA to make a discrimination among efficient DMUs. Here, we briefly mention some of them. As one of them we can refer to the method named as cross-efficiency matrix initiated by Sexton et al. [7]. The super-efficiency method that is referred to as an extended DEA measure (EDM) was originated by Andersen and Petersen [2]. As another ranking method is one in which a common set of weights is developed for the performance indices of DEA efficient DMUs [6]. Another method referred to as benchmarking approach was developed by Torgersen et al. [8]. A method initiated by Bardhan et al. [3] exists by which the ranking of inefficient units are carried out. As seen in DEA literature, some methods are suitable just to rank extreme efficient DMUs [1,5]. This follows from the fact that the existence of extreme efficient DMUs weakens or even eliminates the effect of non-extreme efficient DMUs in their PPS. But in our proposal, we delete all efficient DMUs firstly. After that, we define a new PPS ($PPS_{new}$) that is constructed by just inefficient DMUs. Then an efficient DMU noticed to be under assessment would be added to the $PPS_{new}$ lonely, i.e. in the absence of the other efficient DMUs. Then all alterations that would be occurred by adding that efficient DMUs to the DMUs into the $PPS_{new}$ would be as a criterion to rank that efficient DMU. In fact by deleting all efficient DMUs at fifst, we give this chance to each non-extreme efficient DMU to show its effect on the PPSnew in order to rank that DMU. Then by this viewpoint, our method would be practicable to rank all types of efficient

DMUs. The remainders is organized as follows. Section 2 shows a preliminary of DEA. Section 3 exhibits our proposal. Section 4 shows some explanatory graphical examples with two case studies. Section 5 involves conclusions.

### 2 Preliminaries of DEA

Suppose there are n DMUs in a set called $M_0 := \{DMU_j \,/\, j \in J_0\}$ where $J_0 := \{1, 2, ...., n\}$. Now based on the members of the $M_0$, a set called production possibility set (PPS) is defined as follows:

$$PPS = \{(x, y) / x \geq 0 \text{ can produce } y \geq 0\}. \qquad (1)$$

Then if the variable returns to scale technology (VRS, hereafter) is assumed, the production possibility set based on the members of $M_0$ in (1) can be defined as follows:

$$T_0^{VRS} = \left\{ (x, y) / x \geq \sum_{j \in J_0} \lambda_j x_j , y \leq \sum_{j \in J_0} \lambda_j y_j , \sum_{j \in J_0} \lambda_j = 1 , \lambda_j \geq 0, \ j \in J_0 \right\}. \qquad (2)$$

Therefore, the dual form of the BCC model to assess the $DMU_p$ can be written as Eg. (3):

$$\theta_p^* = \min \quad \theta_p \qquad (3)$$

$$s.t. \quad (\theta_p x_p, y_p) \in T_0^{VRS},$$

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Suppose $\varepsilon$ is a small non-Archimedean number, in this case the expanded form (3) will be rewritten as follows:

$$\begin{aligned}
\theta_p^* = \min \quad & \theta_p - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\
s.t. \quad & -\sum_{j\in J_0} \lambda_j x_{ij} + \theta_p x_{ip} - s_i^- = 0, && i = 1, ...., m, \\
& \sum_{j\in J_0} \lambda_j y_{rj} - s_r^+ = y_{rp}, && r = 1, ...., s, \\
& \sum_{j\in J_0} \lambda_j = 1, \\
& \lambda_j \geq 0, && j \in J_0 \\
& s_i^- \geq 0, && i = 1,..., m, \\
& s_r^+ \geq 0, && r = 1,...., s, \\
& \theta_p && is \ free
\end{aligned}$$

If $M_k := \{DMU_j / j \in J_k\}$ in which $J_k \subseteq J_0$ then $T_k^{VRS}$ as a PPS based on members of $M_k$ with VRS technology will be as follow:

$$T_k^{VRS} = \left\{ (x, y) / x \geq \sum_{j\in J_k} \lambda_j x_j , y \leq \sum_{j\in J_k} \lambda_j y_j , \sum_{j\in J_k} \lambda_j = 1 , \lambda_j \geq 0, \ j \in J_k \right\}. \quad (5)$$

**Definition 2.1.** Suppose that $E_0 := \{DMU_j / \theta_j^* = 1, \ s^{-*} = 0, \ s^{+*} = 0, \ j \in J_0\}$ where $(\theta_j^*, s^{-*}, s^{+*}, \lambda^*)$ is an optimal solution of model (4) to evaluate $DMU_j$ , then we call $E_0$ as the 0th-level of efficiency.

**Definition 2.2.** Suppose that $E_k := \{DMU_j / \theta_j^* = 1, \ s^{-*} = 0, \ s^{+*} = 0, \ j \in J_k\}$ where $(\theta_j^*, s^{-*}, s^{+*}, \lambda^*)$ is an optimal solution of model (6) to evaluate $DMU_j$ as follows .

$$\begin{aligned}
\theta_p^* = \min \quad & \theta_p - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\
s.t. \quad & -\sum_{j\in J_k} \lambda_j x_{ij} + \theta_p x_{ip} - s_i^- = 0, && i = 1, ...., m, \\
& \sum_{j\in J_k} \lambda_j y_{rj} - s_r^+ = y_{rp}, && r = 1, ...., s, \\
& \sum_{j\in J_k} \lambda_j = 1, \\
& \lambda_j \geq 0, && j \in J_k \\
& s_i^- \geq 0, && i = 1,..., m, \\
& s_r^+ \geq 0, && r = 1,...., s, \\
& \theta_p && is \ free
\end{aligned}$$

Then we define $E_k$ as the kth-level of efficiency.

**3 Our proposal**

As mentioned earlier, some methods are suitable just to rank extreme efficient DMUs. This follows from the fact that the appearance of extreme efficient DMUs weakens or even eliminates the effect of non-extreme efficient DMUs. But in our proposal by removing all types of efficient DMUs first as, and then by adding lonely (in the absence of the other efficient

DMUs) an efficient DMU under assessment, we give this chance to each non-extreme efficient DMU to show its effect on the $PPS_{new}$. In the following, we express the proposed method. To develop our proposal we make some ordered steps as follows:

**1.** $M_1 := M_0 \setminus E_0$. *If* $M_1 = \phi$, *then go to step* (5).

**2.** To reach the 1th-level of efficiency, i.e. $E_1 := \{DMU_j \in M_1 / \theta_j^* = 1.0\ s^{-*} = 1.0,\ s^{+*} = 0\}$,
use the model (5) to evaluate the members of $M_1$. For this aim, set $k = 1$ to have $J_1 = \{ j / DMU_j \in M_1\}$ instead of the $J_k$ in that model.

**3.** $M_2 := M_1 \setminus E_1$. *If* $M_2 = \phi$, *then go to step* (5).

**4.** To reach the 2th-level of efficiency, i.e. $E_2 := \{DMU_j \in M_2 / \theta_j^* = 1.0, s^{-*} = 0,\ s^{+*} = 0\}$
use the model (5) to evaluate the members of $M_2$. To this, set $k = 2$ in order to have $J_2 = \{ j / DMU_j \in M_2\}$ instead of the $J_k$ in that model and then go to step (7).

**5.** Use $DMU_{NIP}$ as a hypothetical DMU, NIP means negative ideal point, whose input-output vectors as ($X_{NIP}$ , $Y_{NIP}$ ) are defined as follows:

$$x_{iNIP} = \max_{j \in J_0} x_{ij}, \quad i = 1,...m, \qquad (6)$$
$$y_{rNIP} = \min_{j \in J_0} y_{rj}, \quad r = 1,...s,$$

**6.** If $M_1 = \phi, then\, \overline{E} := \{DMU_{NIP}\}$ and go to step (8), otherwise if $M_1 \neq \phi\, but\, M_2 = \phi$, then $\overline{E} := \{DMU_{NIP}\} \cup E_1$, then go to step (8).

**7.** $\overline{E} := U_{i=1,2} E_i$ and $\overline{R} := \{ j / DMU_j \in \overline{E}\}$.

**8.** $\overline{M}_1 = \overline{E} \cup M_1$ and $\overline{R}_1 := \{ j / DMU_j \in \overline{M}_j\}$ and $\tilde{n} = card\ (\overline{M}_1)$ (card mcans cardinal).

**9.** $\widehat{M}_q := \overline{M}_1 \cup \{DMU_q\}$ and $\widehat{R}_q := \{ j / DMU_j \in \widehat{M}_q\}$ that $q \in R_0$.

**10.** Set $i = 0$ and re-evaluate $DMU_l$ for $l \in \overline{R}_1$ via the following:

$$\begin{aligned}
\gamma_{l,q}^* = \max \quad & \sum_{i=1}^{m} s_{il}^- + \sum_{r=1}^{s} s_{rl}^+ \\
s.t. \quad & -\sum_{j \in \widehat{R}_q} \lambda_j x_{ij} + x_{il} - s_{il}^- = 0, && i = 1, ....,m, \\
& \sum_{j \in \widehat{R}_q} \lambda_j y_{rj} - s_{rl}^+ = y_{rl}, && r = 1, ...., s, \\
& \lambda_j \geq 0, && j \in \widehat{R}_q, \\
& s_{il}^- \geq 0, && i = 1,...,m, \\
& s_{rl}^+ \geq 0, && r = 1,...., s,
\end{aligned}$$

and $i = i + I$ . If $i \neq \tilde{n}$ , then return to step 10.

**11**. Then after calculating the $\gamma_{l,q}^*$ for all $l \in \overline{R}_1$ , the ranking index for an efficient $DMU_q$ $(q \in R_0)$ is defined as $\gamma_q^* = \sum_{l \in \overline{R}_1} \gamma_{l,q}^*$ .

**Hint 1.** Note that two cases probably will happen: 1) If $M_2 \neq \phi$ , then $\overline{E} \subseteq M_1$ and therefore, $\overline{M}_1 = M_1$. 2) If $M_1 = \phi\ or\ M_2 = \phi$ , then a hypothetical DMU, $DMU_{NIP}$ , should be defined. Therefore, the card $(\overline{M}_1) = card\,(M_1) + 1$.

**Definition 3.1.** $\gamma_q^* > \gamma_p^*$ if and only if the performance of $DMU_q$ is better than $DMU_p$.

**Theorem 3.2**. $\gamma_q^* > 0$ , $\forall q \in R_0$.

Proof. Obviously $\gamma_q^* \geq 0, \forall q \in R_0$.. Assume that $\exists q_0 \in R_0$ such that $\gamma_{q0}^* = 0$ , then $\sum_{l \in \bar{R}_1} (\sum_{i=1}^{m} s_{il}^{*-} + \sum_{r=1}^{s} s_{rl}^{*+}) = 0$ . it implies that $\nexists$ $DMU_l \notin \bar{E}_1$ such that $(-x_{q0}, y_{q0}) \geq\neq (-x_l, y_l)$ Considering the definition of $E_0$ and $\bar{E}_1$ results in $DMU_{q0} \notin E_0$ This contradiction completes the proof.

**Theorem 3.3.** if $M_1 = \phi$, *then* $\gamma_q^* = \sum_{i=1}^{m} s_i^{\tilde{-}} + \sum_{r=1}^{s} s_r^{\tilde{+}}$ where $s_i^{\tilde{-}} = x_{i,NIP} - x_{i,q}$ and $s_r^{\tilde{+}} = y_{r,q} - y_{r,NIP}, \forall i, r.$

Proof . If $M_1 = \phi$, *then* $\bar{E} := \{DMU_{NIP}\}$. Nothing the $\bar{E}$ and also the first and the second constraints of Eq.(5) results in $\gamma_q^* = \sum_{l \in \bar{R}_1} (\sum_{i=1}^{m} s_{il}^{-} + \sum_{r=1}^{s} s_{rl}^{+}) = \sum_{i=1i}^{m} s_i^{\tilde{-}} + \sum_{r=1}^{s} s_r^{\tilde{+}}$ and completes the proof.

## 4 Examples

### 4.1 Explanatory graphical examples

In this section we describe the above-mentioned steps graphically.

Note that there may be three cases: case 1. $M_1 = \phi$; case 2. $M_1 \neq \phi$ but $M_2 = \phi$ case 3. $M_2 \neq \phi$. For clarifying these three cases, we use three distinct figures as Fig 1., Fig 2. and Fig 3. as follows.

Firstly, note the Fig 1. where there exist 5 DMUs making up the $M_0 = \{DMU_1, DMU_2, ..., DMU_5\}$. In this case, $E_0 = \{DMU_1, DMU_2, ..., DMU_5\}$, then $M_1 = M_0 \setminus E_0 = \phi$ .Therefore, we have to define a $DMU_{NIP}$ as an artificial DMU to assess the efficient DMUs belonging to the $E_0$ to rank them.

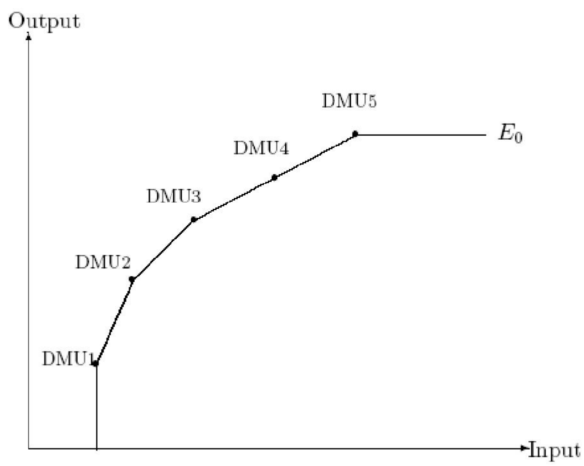


Fig. 1. A figure to illustrate the case 1.

Note the Fig. 2 where there exist 6 $DMU_s$ constructing up the $M_0 = \{DMU_1, DMU_2, ..., DMU_6\}$. We obtain $E_0 = \{DMU_1, DMU_2, ..., DMU_5\}$ as same as the Fig. 1, and $M_1 = M_0 \setminus E_0 = \{DMU_6\}$ . Since $M_2 = M_1 \setminus E_1$ and here $E_1 = M_1 = \{DMU_6\}$ , then $M_2 = \phi$ . In addition, the restoration of each member of the $E_0$ to the $PPS_{new}$ cannot change the position of the unique member of $E_1$ (it is provable by showing the $\gamma_q^* = 0$ for $q \in R_0 = \{1,2,......5\})$ . Therefore, the definition of the $DMU_{NIP}$ would be necessary to assess the members of $E_0$ to rank them.

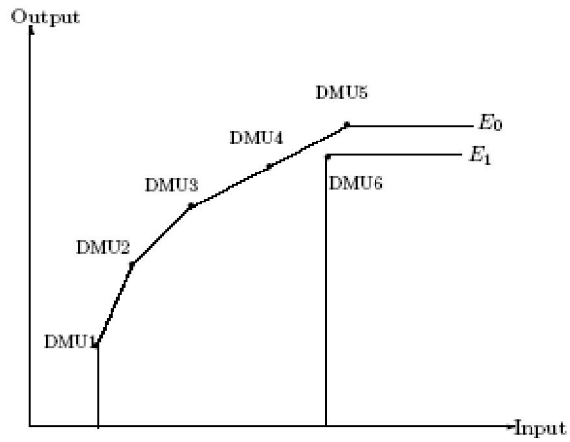


Fig. 2. A figure to illustrate the case 2.

Note the Fig. 3. where there exist 10 DMUs making up the $M_0 = \{DMU_1, DMU_2, ..., DMU_{10}\}$ . It is obtained that $E_0 = \{DMU_1, DMU_2, ..., DMU_5\}$ like those two figures 1 and 2. Therefore, $M_1 = M_0 \setminus E_0 = \{DMU_6, DMU_7, ..., DMU_{10}\}$. Also we obtain $E_1 = \{DMU_6\}$, then $M_2 = M_1 \setminus E_1 = \{DMU_7, DMU_8, ..., DMU_{10}\}$. Since $M_2 \neq \phi$ the definition of the $DMU_{NIP}$ would not be necessary to assess the members of $E_0$ to rank them.

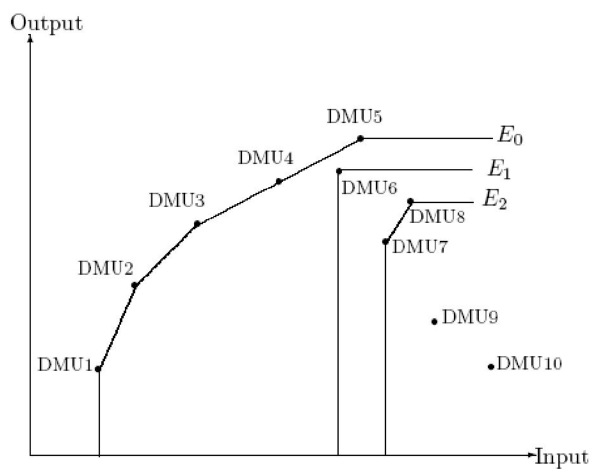


Fig. 3. A figure to illustrate the case 3.

**4.2 Examples with real data**

Case with $M_2 = \phi$ .

Using the data gathered in Table (1) shows the power of the proposed algorithm to rank all efficient DMUs. Moreover, Table (2) compares the results of the algorithm with some previous ranking models.

Table 1. DMU's data (extracted from [1])

| DMUs | Inputs | | Outputs | |
|---|---|---|---|---|
| | $I_1$ | $I_2$ | $O_1$ | $O_2$ |
| A | 150 | 0.2 | 14000 | 3500 |
| B | 400 | 0.7 | 14000 | 21000 |
| C | 320 | 1.2 | 42000 | 10500 |
| D | 520 | 2.0 | 28000 | 42000 |
| E | 350 | 1.2 | 19000 | 25000 |
| F | 320 | 0.7 | 14000 | 15000 |

Table 2. Results of the algorithm and a few ranking models

| $\gamma^*$ | | CCR | | BCC | | CEA | | EDM | |
|---|---|---|---|---|---|---|---|---|---|
| C | 6.8E+4 | A | 1.000 | A | 1.000 | A | 1.000 | A | 200.000 |
| D | 5.2E+4 | B | 1.000 | B | 1.000 | B | 0.916 | B | 140.000 |
| A | 4.8E+4 | C | 1.000 | C | 1.000 | D | 0.916 | C | 140.000 |
| B | 4.8E+4 | D | 1.000 | D | 1.000 | C | 0.842 | D | 113.000 |

It must be noted that DEA models usually have no more than $\frac{n}{2}$ while n DMUs have been selected to assess. Otherwise, the number of efficient DMUs will be unreasonably large. A similar case happens in the above example where 6 DMUs exist such that four DMUs become efficient, so belong to $E_0$ and the rest of them belong to $E_1$. According to the algorithm, the definition of the $DMU_{NIP}$ would be necessary. The detailed information relating to the algorithm's steps are explained as follows.

Here, $M_0 = \{DMU_A, DMU_B.DMU_c, DMU_D, DMU_E, DMU_F\}$ then solving the model (4) relating to $M_0$ provides the 0th-level of efficiency, i.e. $E_0 = \{DMU_A, DMU_B, DMU_C, DMU_D\}$ As step 1 $, M_1 = M_0 \setminus E_0 = \{DMU_E, DMU_F\}$ . As step 2, $E_1 = \{DMU_E, DMU_F\}$, then $M_2 = M_1 \setminus E_1 = \phi$ In this case $DMU_{NIP}$ is needed to define whose input and output indices are $X_{NIP}$ = (520, 2) and $Y_{NIP}$ = (14000, 3500) and $\overline{E} = E\{DMU_{NIP}\}$ . Finally, continuing the other steps yields the results listed in Table (2).

**Case with $M_2 \neq \phi$**

Here the data of 20 bank branches are listed in Table (3) and applied by the algorithm. Following the steps of the algorithm yields: $M_1 = \{DMU_j / j \in J_1\}$ whose $J_1 = \{2,3,5,6,8,9,10,11,13,14,16,18,19\}$. Also $E_1 = \{DMU_2, DMU_3, DMU_5, DMU_6, DMU_8, DMU_9, DMU_{11}, DMU_{16}\}$ and $E_2 = \{DMU_{13}\}$. Since $M_2 = M_1 \setminus E_1$, obviously $M_2 \neq \phi$ and then $DMU_{NIP}$ is not needed to define. Moreover, it holds $\overline{E} = E_1 \cup E_2$. Since $\overline{M_1} = M_1$, we have $\hat{M}_q = M_1 U \{DMU_q\}$ whose $q \in R_0 = \{1,4,7,12,15,17,20\}$ .

Following the steps completely provides the results listed in Table (4). It can be seen that the forth branch is the best one and the first branch is the worst one among the branches.

Table 3. DMU's data (extracted from [6])

| DMU | input 1 | input 2 | input 3 | output 1 | output 2 | output 3 |
|---|---|---|---|---|---|---|
| 1 | 0.950 | 0.700 | 0.155 | 0.190 | 0.521 | 0.293 |
| 2 | 0.796 | 0.600 | 1.000 | 0.227 | 0.627 | 0.462 |
| 3 | 0.798 | 0.750 | 0.513 | 0.228 | 0.970 | 0.261 |
| 4 | 0.865 | 0.550 | 0.210 | 0.193 | 0.632 | 1.000 |
| 5 | 0.815 | 0.850 | 0.268 | 0.233 | 0.722 | 0.246 |
| 6 | 0.842 | 0.650 | 0.500 | 0.207 | 0.603 | 0.569 |
| 7 | 0.719 | 0.600 | 0.350 | 0.182 | 0.900 | 0.716 |
| 8 | 0.785 | 0.750 | 0.120 | 0.125 | 0.234 | 0.298 |
| 9 | 0.476 | 0.600 | 0.135 | 0.080 | 0.364 | 0.244 |
| 10 | 0.678 | 0.550 | 0.510 | 0.082 | 0.184 | 0.049 |
| 11 | 0.711 | 1.000 | 0.305 | 0.212 | 0.318 | 0.403 |
| 12 | 0.811 | 0.650 | 0.255 | 0.123 | 0.923 | 0.628 |
| 13 | 0.659 | 0.850 | 0.340 | 0.176 | 0.645 | 0.261 |
| 14 | 0.976 | 0.800 | 0.540 | 0.144 | 0.514 | 0.243 |
| 15 | 0.685 | 0.950 | 0.450 | 1.000 | 0.262 | 0.098 |
| 16 | 0.613 | 0.900 | 0.525 | 0.115 | 0.402 | 0.464 |
| 17 | 1.000 | 0.600 | 0.205 | 0.090 | 1.000 | 0.161 |
| 18 | 0.634 | 0.650 | 0.235 | 0.059 | 0.349 | 0.068 |
| 19 | 0.372 | 0.700 | 0.238 | 0.039 | 0.190 | 0.111 |
| 20 | 0.583 | 0.550 | 0.500 | 0.110 | 0.615 | 0.764 |

Table 4. Results of the algorithm

| DMU | $\gamma^*$ | BCC |
|---|---|---|
| 4 | 8.2323 | 1.000 |
| 7 | 7.0726 | 1.000 |
| 20 | 5.4844 | 1.000 |
| 12 | 5.4441 | 1.000 |
| 15 | 4.2020 | 1.000 |
| 17 | 4.1333 | 1.000 |
| 1 | 4.1112 | 1.000 |

## 5 Conclusions

This study proposed an algorithm which had several important features. First and the foremost, it was the ability of the algorithm in ranking all efficient DMUs, extreme and non-extreme efficient ones. In addition, the algorithm was feasible and stable in all cases. The algorithm was illustrated via 11 steps. The algorithm was based on the removal of all efficient DMUs simultaneously, from the original PPS and also the introduction of an $DMU_{NIP}$ , in two cases mentioned before, $DMU_{NIP}$ was a hypothetical unit which had been defined to keep the practicality of the algorithm. After the removal of all efficient DMUs , the PPSnew was introduced based on the other remaining units including inefficient DMUs or perhaps, $DMU_{NIP}$ . For the evaluation of an intended-to-rank efficient DMU, this unit was added to the PPSnew. Then, amount of changes occurred to the members of the $\overline{E}$ from their efficiency viewpoint after the addition of an efficient DMU concerned was as the criterion of ranking that efficient DMU. In order to explain these steps, 3 different figures were depicted that each figure was relevant to one probable case. Finally, 2 numerical examples with real data were mentioned.

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