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Stability and Tracking in the New Chaotic System Using Backstepping Method

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ABSTRACT

Chaos is one of the most important phenomenons based on complex nonlinear dynamics. The control problem of a new chaotic system is investigated. This system is a novel three dimension autonomous (3D) chaotic system with a quadratic exponential nonlinear term. Backstepping method is used to control new chaotic system in two participate sections, stabilization and tracking reference input. The numerical simulations are presented to demonstrate the effectiveness of the proposed controllers.
KEYWORDS: New Chaotic System, Quadratic Exponential, Backstepping Method, Stabilization, Tarcking Reference Input.

1. INTRODUCTION

Chaos, as a very interesting nonlinear phenomenon has been intensively studied in science and engineering. Chaotic system is a very complex nonlinear dynamical system. controlling these complex chaotic dynamics for engineering applications has emerged as a new and attractive field and has developed many profound theories and methodologies. Nowadays, many different techniques and methods have been proposed to achieve chaos control. The linear feedback control, speed feedback control and nonlinear doubly-periodic function feedback control were used to stable a new hyperchaos [1]. In [2], a hyperchaotic Lorenz system was constructed via state feedback control. In [3] based on the sliding mode concept, a nonlinear delayed feedback control was proposed for stabilizing the unstable periodic orbits (UPOs) of chaotic systems. Adaptive feedback linearization control technique for chaos suppression in a chaotic system was proposed in [4]. In [5], recursive backstepping technique has been used to control chaos in two different 4-D chaotic systems, namely: Lorenz–Stenflo system and a new 4-D system with three cross product, which was recently introduced by Qi et al. [6] proposed an identification-based adaptive backstepping control (IABC) for the chaotic systems. In [7], a robust intelligent backstepping tracking control (RIBTC) system using an adaptive cerebellar model articulation controller (CMAC) was proposed for a class of uncertain non-linear chaotic systems. In [8], an adaptive feedback control method was presented to stabilize a class of chaotic systems. Generalized Backstepping Method (GBM) have been introduced in [9,10] to stabilize a class of MIMO chaotic systems. In [11], the problem of controlling chaos in Arneodo chaotic system was considered. In order to suppress chaos and regulate the system around one of its unstable equilibrium points, three different methods (feedback linearization, sliding mode control and backstepping design) were used. [12] considered the nonsingular terminal sliding mode control for chaotic systems with uncertain parameters or disturbances. The problem of chaos control in the nonlinear Bloch equations was considered based on a modifed active control technique [13]. The stability conditions in fractional order hyperchaotic systems have been obtained using linear feedback control technique [14].The paper is organized as follows: Section 2 describes new chaotic system. Section 3. Presents backstepping method. Section 4. Backstepping controller is designed to stability new chaotic system. Section 5. Backstepping controller is designed to tracking any reference input. Section 6. Simulated result of work. Section 7. Provides the conclusion.

New 3D Autonomous Chaotic System

A new 3D autonomous chaotic system [15] is expressed as follows.

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= bx - cxz \\ \dot{z} &= e^{xy} - dz \end{aligned} \tag{1}$$

where a,b,c,d are all constants coefficients assuming that $a,b,c,d > 0$ and x, y, z are the state variables. System (1) can generate a new chaotic attractor for the parameters $a = 10, b = 40, c = 2, d = 2.5$ with the initial conditions $[2.2,2.4,28]^T$. Dynamical behavior including is displayed in Figure 1 and 2.

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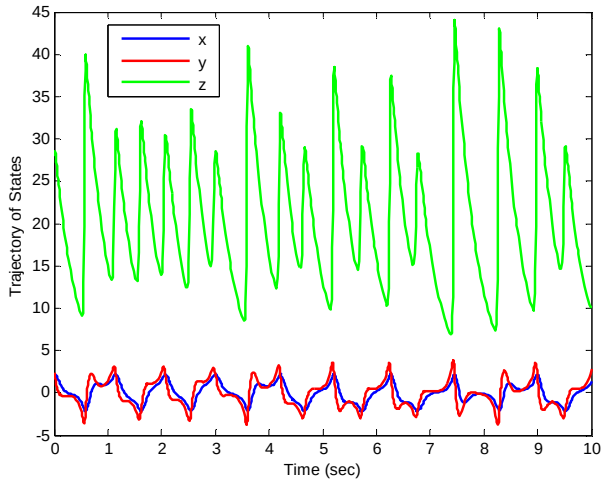

Fig.1. Phase portrait of system (1).

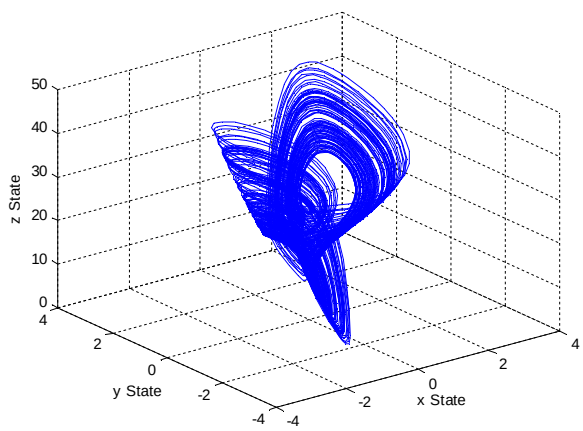

Fig.2. States trajectory variation of system (1).

Backstepping Design for Strict Feedback System

Consider the strict-feedback nonlinear system as follow.

$$\dot{x}_i = f_i(x_1, \cdots, x_i) + g_i(x_1, \cdots, x_i)x_{i+1}\,; 1 \le i \le n-1$$
$$\dot{x}_n = f_n(x) + g_n(x)u \qquad (2)$$

Where $x = [x_1, \cdots, x_n]^T$, $f_i(0)$ and $g_i(0)$ are smooth functions with $f_i(0) = 0$ and $g_i(0) \neq 0$.

Step 1.

Considering the first subsystem of equation (2), we take x_2 as a virtual control input and choose.

$$x_2 = \frac{1}{g_1(x_1)}[u_1 - f_1(x_1)] \qquad (3)$$

The first subsystem is changed to be $\dot{x}_1 = u_1$. Choosing $u_1 = -k_1x_1$ with $k_1 > 0$, the origin of the first subsystem $x_1 = 0$ is asymptotically stable and the corresponding Lyapunov function is

$V_1(x_1) = {x_1^2}/{2}$, equation (3) is changed to equation (4).

$$x_2 = \varphi_1(x_1) = \frac{1}{g_1(x_1)}[-k_1x_1 - f_1(x_1)] \qquad (4)$$

Step 2.

Considering (x_1, x_2), take x_3 as a virtual control input and choose.

$$x_3 = \frac{1}{g_2(x_1,x_2)}[u_2 - f_2(x_1, x_2)] \qquad (5)$$

The (x_1, x_2) subsystem is changed to equation (6).

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2$$
$$\dot{x}_2 = u_2 \quad (6)$$

Which is in the form of integrator backstepping, so the control law u_2 is as follow.

$$u_2 = -\frac{\partial V_1}{\partial x_1} g_1(x_1) - k_2[x_2 - \varphi_1(x_1)] + \frac{\partial \varphi_1}{\partial x_1}[f_1(x_1) + g_1(x_1)x_2] \quad (7)$$

Where $k_2 > 0$. This control law asymptotically stabilizes $(x_1, x_2) = (0,0)$ and Lyapunov function is as equation (8).

$$V_2(x_1, x_2) = V_1(x_1) + \frac{1}{2}[x_2 - \varphi_1(x_1)]^2 \quad (8)$$

Substituting equation (7) into equation (5) gives.

$$x_3 = \varphi_2(x_1, x_2) = \frac{1}{g_2}\left[-\frac{\partial V_1}{\partial x_1} g_1 - k_2[x_2 - \varphi_1] + \frac{\partial \varphi_1}{\partial x_1}(f_1 + g_1 x_2) - f_2\right] \quad (9)$$

The remaining step can be deduced by analogy. Until step n, we shall determine the actual control law $u = \varphi_n(x)$, which can asymptotically stabilize equation (1) [16].

Stabilization of The New Chaotic System

Backstepping method is used to bring the states x, y, z to the origin point $(0,0,0)$. In order to control system (1), we add a control input u to the 3rd equation of it. Then the controlled system is.

$$\dot{x} = a(y - x)$$
$$\dot{y} = bx - cxz \quad (10)$$
$$\dot{z} = e^{xy} - dz + u$$

Step 1.

Consider the first subsystem of equation (10).

$$\dot{x} = a(y - x) \quad (11)$$

Construct the joint Lyapunov function.

$$V_0(x) = \frac{1}{2}x^2 \quad (12)$$

Take y as a virtual control input and choose.

$$y = \varphi_0(x) = -k_1 x \quad (13)$$

Step 2.

Consider (x, y) of equation (10).

$$\dot{x} = a(y - x)$$
$$\dot{y} = bx - cxz \quad (14)$$

Take z as a virtual control input and choose.

$$z = \varphi_1(x, y) = \frac{\frac{\partial \varphi_0}{\partial x}\dot{x} - \frac{\partial V_0}{\partial x} - k_2(y - \varphi_0) - bx}{-cx} \quad (15)$$

And take the Lyapunov function as.

$$V_1(x, y) = V_0 + \frac{1}{2}(y - \varphi_0)^2 \quad (16)$$

Step 3.

Consider all system.

$$\dot{x} = a(y - x)$$
$$\dot{y} = bx - cxz \quad (17)$$
$$\dot{z} = e^{xy} - dz + u$$

Take u as an actual control input and choose.

$$u = \varphi_2(x, y, z) = \frac{\partial \varphi_1}{\partial x}\dot{x} + \frac{\partial \varphi_1}{\partial y}\dot{y} - \frac{\partial V_1}{\partial y} - k_3(z - \varphi_1) - e^{xy} + dz \quad (18)$$

And Lyapunov function as.

$$V_2(x, y, z) = V_1 + \frac{1}{2}(z - \varphi_1)^2 \quad (19)$$

Tracking Any Desired Trajectory

In this section, we will find a control law u so that a scaler output $x(t)$ of the new system (1) can track any desired trajectory $r(t)$. let $\bar{x}$ be the deviation between the output x and the desired trajectory $r(t)$. i.e. $\bar{x} = x - r(t)$. therefore; the equation (1) would be converted to equation (20), as follows.

$$\dot{\bar{x}} = \dot{r} - a(y - r + \bar{x})$$
$$\dot{y} = b(r - \bar{x}) - c(r - \bar{x})z \quad (20)$$

$\dot{z} = e^{(r-\bar{x})y} - dz + u$

Step 1.

Consider the first subsystem of equation (20).

$$\dot{\bar{x}} = \dot{r} - a(y - r + \bar{x}) \tag{21}$$

Construct the joint Lyapunov function.

$$V_0(\bar{x}) = \frac{1}{2}\bar{x}^2 \tag{22}$$

Take y as a virtual control input and choose.

$$y = \varphi_0(\bar{x}) = r - \bar{x} + \frac{\dot{r}}{a} + \frac{k_1\bar{x}}{a} \tag{23}$$

Step 2.

Consider $(\bar{x}, y)$ of equation (10).

$$\begin{aligned} \dot{\bar{x}} &= \dot{r} - a(y - r + \bar{x}) \\ \dot{y} &= b(r - \bar{x}) - c(r - \bar{x})z \end{aligned} \tag{24}$$

Take z as a virtual control input and choose.

$$z = \varphi_1(\bar{x}, y) = \frac{\frac{\partial\varphi_0}{\partial\bar{x}}\dot{\bar{x}} - \frac{\partial V_0}{\partial\bar{x}} - k_2(y - \varphi_0) - b(r - \bar{x})}{-c(r - \bar{x})} \tag{25}$$

And take the Lyapunov function as.

$$V_1(\bar{x}, y) = V_0 + \frac{1}{2}(y - \varphi_0)^2 \tag{26}$$

Step 3.

Consider all system.

$$\begin{aligned} \dot{\bar{x}} &= \dot{r} - a(y - r + \bar{x}) \\ \dot{y} &= b(r - \bar{x}) - c(r - \bar{x})z \\ \dot{z} &= e^{(r-\bar{x})y} - dz + u \end{aligned} \tag{27}$$

Take u as an actual control input and choose.

$$u = \varphi_2(\bar{x}, y, z) = \frac{\partial\varphi_1}{\partial\bar{x}}\dot{\bar{x}} + \frac{\partial\varphi_1}{\partial y}\dot{y} - \frac{\partial V_1}{\partial y} - k_3(z - \varphi_1) - e^{(r-\bar{x})y} + dz \tag{28}$$

And Lyapunov function as.

$$V_2(\bar{x}, y, z) = V_1 + \frac{1}{2}(z - \varphi_1)^2 \tag{29}$$

Numerical Simulation

This section presents numerical simulations the new chaotic system. The backstepping method is used as an approach to control new chaotic system. The simulations are given in the following three cases:

- Case 1 : Stabilization to the Origin Point$(0,0,0)$,
- Case 2 : Tracking Step Input $r(t) = 1$,
- Case 3 : Tracking Refrence Input $r(t) = 2 - 2e^{-t}$.

The parameters of backstepping controller are listed in table 1.

Table 1. Parameters of Backstepping Controller

	k_1	k_2	k_3
Case 1	30	25.5	30
Case 2	27	20	30
Case 3	10	30	23

Figure 3 shows that x state of system (10) can be stabilized with the control law u (18) to the origin point$(0,0,0)$. Figure 4 shows that y state of system (10) can be stabilized with the control law u (18) to the origin point$(0,0,0)$. Figure 5 shows that z state of system (10) can be stabilized with the control law u (18) to the origin point$(0,0,0)$. Figure 6 shows the control law u (18) to the origin point$(0,0,0)$. Figure 7 shows that the scalar output $x(t)$ can track the desired trajectory $r(t) = 1$ with the control input u (28). Figure 8 shows that the scalar output $x(t)$ can track the desired trajectory $r(t) = 2 - 2e^{-t}$ with the control input u (28).

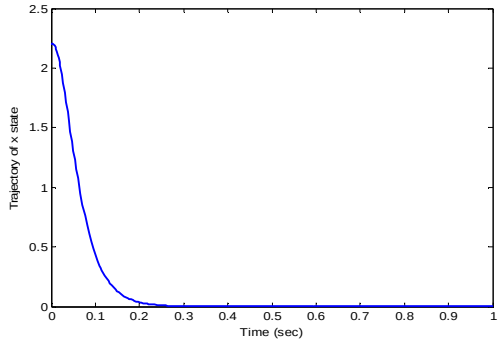

Fig.3. Time response of the state x.

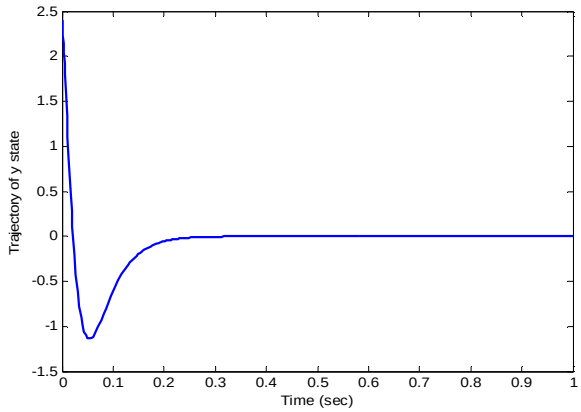

Fig.4. Time response of the state y.

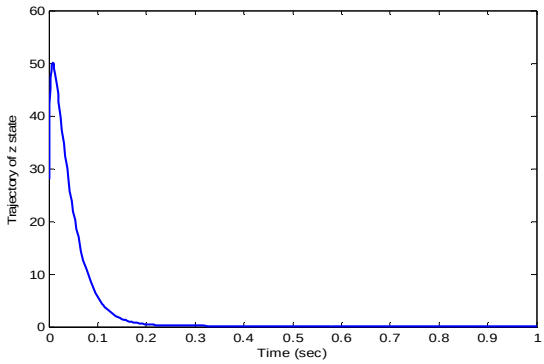

Fig.5. Time response of the state z.

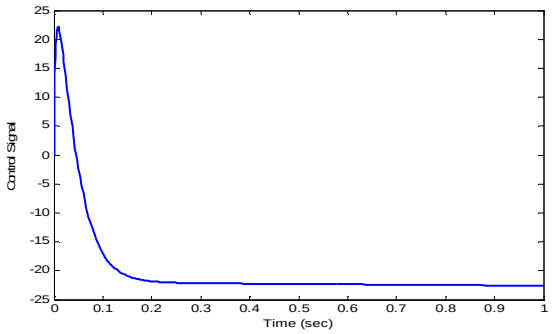

Fig.6. Time response of the control input (18).

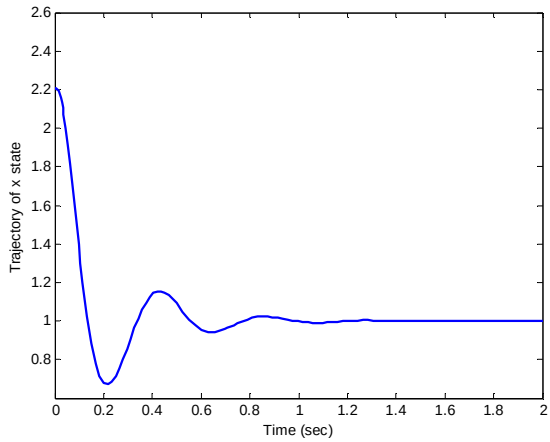

Fig.7. Time response of the state x to track $r(t) = 1$.

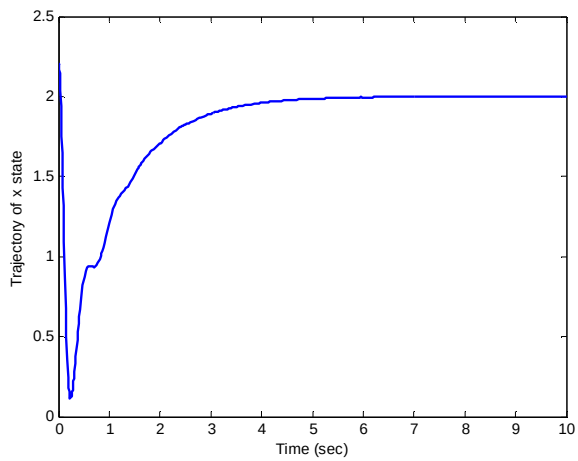

Fig.8. Time response of the state x to track $r(t) = 2 - 2e^{-t}$.

Conclusion

The control problem of a new chaotic system is investigated. This system is a novel three dimension autonomous (3D) chaotic system with a quadratic exponential nonlinear term. In this letter by using the backstepping mode concept, two controller are proposed for stabilizing and tracking any reference input in the new chaotic system. Furthermore, numerical simulations are presented to verify the effectiveness of the proposed controllers.

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