J. Basic. Appl. Sci. Res., 3(1s)388-392, 2013
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ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Calculation of Produced Radioisotopes and Burn up in the Miniature Neutron Source Reactor Using Radioactive Decay Equations

Seyed Alireza Mousavi Shirazi[1*], Mohammad Sadegh Shafeie Lilehkouhi[2]

[1] Full Time Faculty Member, Department of Physics, Islamic Azad University, South Tehran Branch, Tehran, Iran
[2] M.Sc Student in Nuclear Reactor Engineering, Department of Nuclear Engineering, Islamic Azad University, Science and Research Branch, Tehran, Iran

Received: June 10 2013
Accepted: July 5 2013

ABSTRACT

In this paper, the amounts of some produced radioisotopes such as radiomedicines and actinides, which are either fissile or fertile, in the Miniature Neutron Source Reactor (MNSR) are calculated using radioactive decay equations within one year continues operation of reactor with the neutron flux: 10^9n/cm^2.sec.
In order to calculate the values of produced radioisotopes, the variations of nucleuses densities of radionuclides have been written through all the differential equations of atom densities variations, then the amounts of the produced radioisotpes at the core of this reactor have been computed by solving the mentioned equations through numerical method and also using the MATLAB software, according to the type of applied fuel and its enrichment percentage (UAL_4 with 90.2 %) within one year. In addition, the burnup of reactor's fuel has been calculated based on the obtained results.
KEYWORDS: Burnup; Decay equations; Mass; Radioisotope; Reactor.

Nomenclature
r: enrichment of fuel
m_f: mass of fuel
m_{ff}: mass of fissile materials
λ: decay constant
φ: neutron flux ($n/cm^2.s$)
σ_a: absorption cross section
σ_c: capture cross section
Y: fission yield
P: power of reactor (*Watt*)
N: atomic density (*atom*)
M: atomic mass (*gr*)
A: Avogadro's number

1. INTRODUCTION

The prototype of Miniature Neutron Source Reactor (MNSR) is based on SLOWPOKE reactor (Safe Low Power Critical Experiment). The SLOWPOKE reactor is as a research reactor, which has been designed and made in 1960 by AECL (Atomic Energy of Canada Limited). The MNSR is a thermal pool tank reactor with enriched fuel and light water moderator, so that the most important applications of it include: neutron activation analysis (N.A.A), research works, training, production of radioisotopes and material testing. The critical mass of this reactor is very low, in order to hold the reactor in the steady state, the beryllium plates are loaded upon the reactor core. The core of this reactor is submerged at a water pool having dimensions: (2.5m×6m) [1].

After fueling at reactor core and operation of it, the radioisotopes and various actinides are produced, therefore calculation of them amongst radiomedicines is very significant within a long interval.
Some of radioisotopes with short half life may be utilized in medical radiation course for the sake of imaging or as radiomedicine generators. Some of applications of radiomedicines are: radiotherapy and brachytherapy in treatment of cancerous tumors and also industrial utilization such as thickness determination and assessment of material quality [2].
There are two important radioisotopes including: Mo^{99} and Tc^{99m}, so that these are applied to tag the radiomedicines in nuclear medicine [3].
The main technical characteristics of MNSR reactor are shown in Table 1:

Corresponding author: Seyed Alireza Mousavi Shirazi, Department of Physics, Islamic Azad University, South Tehran Branch, Tehran, Iran. Emails: a_moosavi@azad.ac.ir, alireza_moosavi@yahoo.com (Tel: +98-9122027059)

Table 1. The main characteristics of MNSR reactor [1]

Characteristic	Description
Type of reactor	Pool-tank
Maximum thermal power	30KW
Fuel	UAl_4
Enrichment	90.2%
Core shape	Cylindrical with circle shape of fuel
Mass of U^{235} in fuel	1000gr
Diameter of core	23cm
Height of core and fuel rods	23cm
Diameter of fuel pins	4.3mm
Fuel shape	Pencil shape fuel pins
Number of fuel rods	243
Operation time of reactor on maximum power	2.5 hours
Life time of fuel rod	10 years
flux of thermal neutron on maximum power inside of fission chamber	10^{12} *n/cm².sec*
Flux of thermal neutron on maximum power outside of fission chamber	5×10^{11} *n/cm².sec*

2. METHODOLOGY

The MNSR reactor is never continuously able to operate more than 2.5 hours in the maximum nominal power that is: 30KW, but it might be unceasing operated on neutron flux: 10^9 *n/cm².sec* within one year. The diagrams of both production and decay of radioisotopes produced in the MNSR core are illustrated in Figures 1 and 2. [4, 5]:

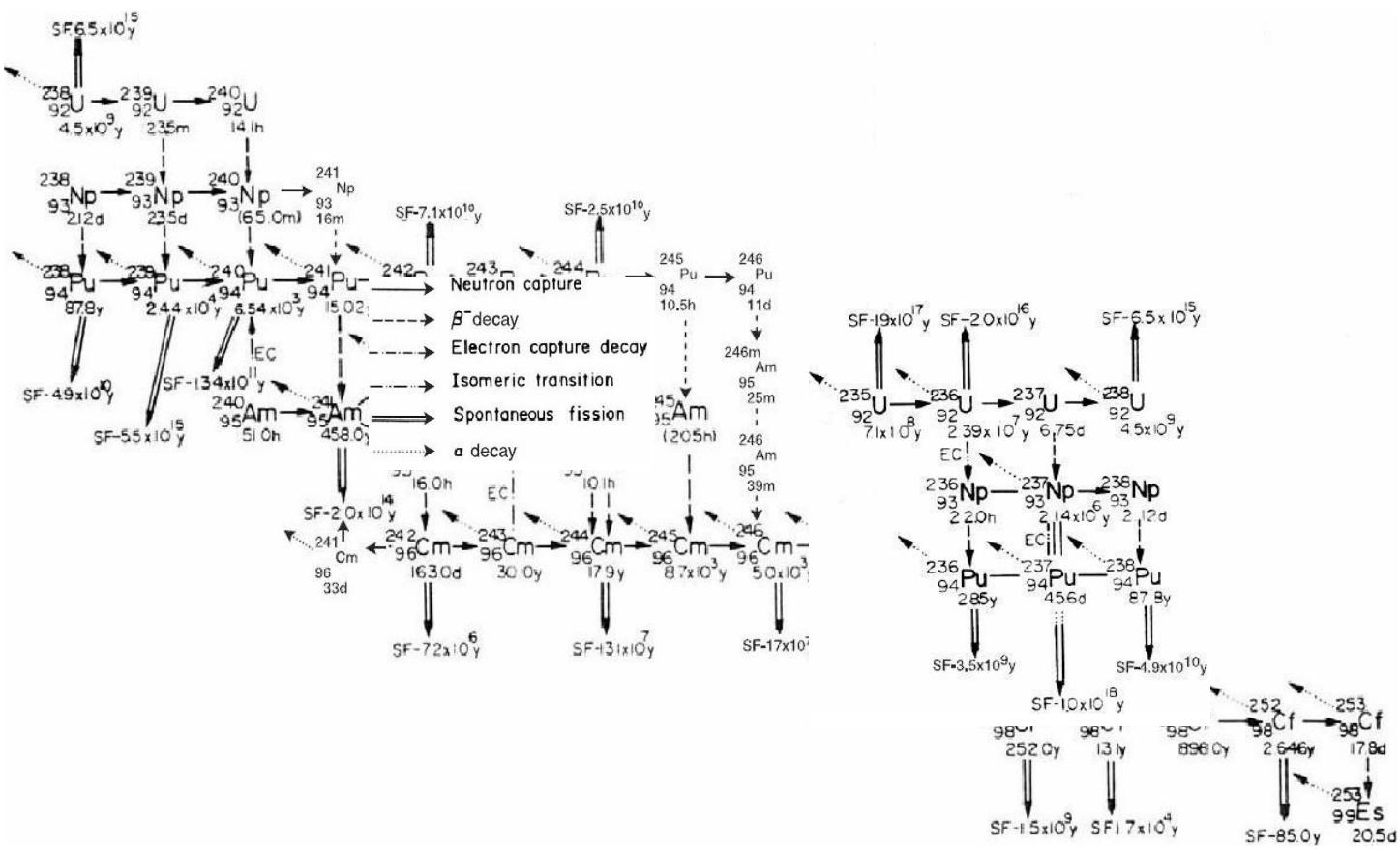

Figure 1. The diagram of production and decay of radioisotopes

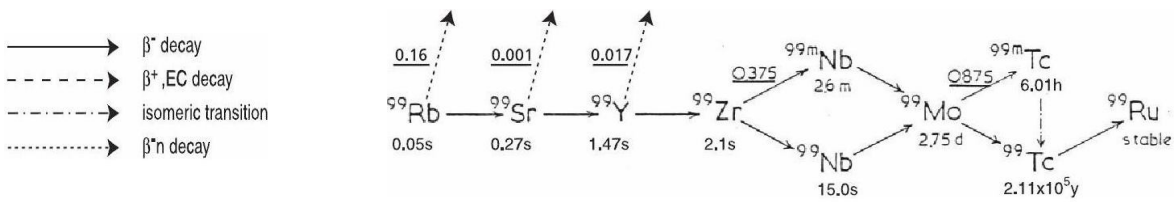

Figure 2. The diagram of decay and production of fission products

In the general state, it can be written:

$$\frac{dX_i}{dt} = \sum_1^n l_{ij}\lambda_j X_j(t) + \varphi \sum_{k=1}^n f_{ik}\sigma_k X_k(t) - (\lambda_i + \sigma_i \varphi) X_i(t) \qquad i,\ j = 1,2,...,\ n \ [5] \qquad (1)$$

Therefore the chain radioactive decay differential equations of atomic density variations are written for each of radioisotopes among radiomedicines, based on the diagram of production and decay of radioisotopes. These equations are shown as follows:

$$\frac{\partial U^{235}}{\partial t} = -\sigma_a^{U^{235}} \cdot \varphi \cdot U^{235} \qquad (2)$$

$$\frac{\partial U^{236}}{\partial t} = \sigma_\gamma^{U^{235}} \cdot \varphi \cdot U^{235} - \sigma_a^{U^{236}} . \varphi . U^{236} \qquad (3)$$

$$\frac{\partial U^{237}}{\partial t} = \sigma_\gamma^{U^{236}} \cdot \varphi \cdot U^{236} + \sigma_a^{U^{238}} . \varphi . U^{238} - \lambda_{U^{237}} . U^{237} \qquad (4)$$

$$\frac{\partial U^{238}}{\partial t} = -\sigma_a^{U^{238}} \cdot \varphi \cdot U^{238} \qquad (5)$$

$$\frac{\partial U^{239}}{\partial t} = \sigma_\gamma^{U^{238}} \cdot \varphi \cdot U^{238} - \sigma_a^{U^{239}} . \varphi . U^{239} - \lambda_{U^{239}} . U^{239} \qquad (6)$$

$$\frac{\partial Np^{236}}{\partial t} = \sigma_a^{Np^{237}} \cdot \varphi \cdot Np^{237} - \sigma_a^{Np^{236}} . \varphi . Np^{236} - \lambda_{Np^{236}} . Np^{236} \qquad (7)$$

$$\frac{\partial Np^{237}}{\partial t} = \lambda_{U^{237}} . U^{237} - \sigma_a^{Np^{237}} . \varphi . Np^{237} \qquad (8)$$

$$\frac{\partial Np^{238}}{\partial t} = \sigma_\gamma^{Np^{237}} \cdot \varphi \cdot Np^{237} - \sigma_a^{Np^{238}} . \varphi . Np^{238} - \lambda_{Np^{238}} . Np^{238} \qquad (9)$$

$$\frac{\partial Np^{239}}{\partial t} = \lambda_{U^{239}} . U^{239} - \sigma_a^{Np^{239}} . \varphi . Np^{239} - \lambda_{Np^{239}} . Np^{239} \qquad (10)$$

$$\frac{\partial Pu^{238}}{\partial t} = \lambda_{Np^{238}} . Np^{238} - \sigma_a^{Pu^{238}} . \varphi . Pu^{238} \qquad (11)$$

$$\frac{\partial Pu^{239}}{\partial t} = \sigma_\gamma^{Pu^{238}} \cdot \varphi \cdot Pu^{238} - \sigma_a^{Pu^{239}} . \varphi . Pu^{239} + \lambda_{Np^{239}} . Np^{239} \qquad (12)$$

$$\frac{\partial Pu^{240}}{\partial t} = \sigma_\gamma^{Pu^{239}} \cdot \varphi \cdot Pu^{239} - \sigma_a^{Pu^{240}} . \varphi . Pu^{240} + \sigma_\gamma^{U^{239}} . \varphi . U^{239} + \sigma_\gamma^{Np^{239}} . \varphi . Np^{239} \qquad (13)$$

$$\frac{\partial Pu^{241}}{\partial t} = \sigma_\gamma^{Pu^{240}} \cdot \varphi \cdot Pu^{240} - \sigma_a^{Pu^{241}} . \varphi . Pu^{241} - \lambda_{Np^{241}} . Np^{241} \qquad (14)$$

$$\frac{\partial Pu^{241}}{\partial t} = \sigma_\gamma^{Pu^{241}} \cdot \varphi \cdot Pu^{241} - \sigma_a^{Pu^{242}} . \varphi . Pu^{242} \qquad (15)$$

$$\frac{\partial Am^{241}}{\partial t} = \lambda_{Pu^{241}} . Pu^{241} - \lambda_{Am^{241}} . Am^{241} - \sigma_a^{Am^{241}} . \varphi . Am^{241} \qquad (16)$$

$$\frac{\partial Rb^{98}}{\partial t} = \frac{Y_{Rb^{98}}.P}{\gamma} - \lambda_{^{98}Rb}.Rb^{98} \qquad (17)$$

$$\frac{\partial Rb^{99}}{\partial t} = \sigma_{\gamma}^{Rb^{98}}.\varphi.Rb^{98} - \lambda_{Rb^{99}}.Rb^{99} \qquad (18)$$

$$\frac{\partial Sr^{99}}{\partial t} = \frac{Y_{Sr^{99}}.P}{\gamma} + \lambda_{Rb^{99}}.Rb^{99} - \lambda_{Sr^{99}}.Sr^{99} \qquad (19)$$

$$\frac{\partial Y^{99}}{\partial t} = \frac{Y_{Y^{99}}.P}{\gamma} + \lambda_{Sr^{99}}.Sr^{99} - \lambda_{Y^{99}}.Y^{99} \qquad (20)$$

$$\frac{\partial Zr^{99}}{\partial t} = \frac{Y_{Zr^{99}}.P}{\gamma} + \lambda_{Y^{99}}.Y^{99} - \lambda_{Zr^{99}}.Zr^{99} \qquad (21)$$

$$\frac{\partial Nb^{99}}{\partial t} = \frac{Y_{Nb^{99}}.P}{\gamma} + 0.625\,\lambda_{Zr^{99}}.Zr^{99} - \lambda_{Nb^{99}}.Nb^{99} \qquad (22)$$

$$\frac{\partial Nb^{99m}}{\partial t} = \frac{Y_{Nb^{99m}}.P}{\gamma} + 0.375\,\lambda_{Zr^{99}}.Zr^{99} - \lambda_{Nb^{99m}}.Nb^{99m} \qquad (23)$$

$$\frac{\partial Mo^{99}}{\partial t} = \frac{Y_{Mo^{99}}.P}{\gamma} + \lambda_{Nb^{99m}}.Nb^{99m} + \lambda_{Nb^{99}}.Nb^{99} - \lambda_{Mo^{99}}.Mo^{99} \qquad (24)$$

$$\frac{\partial Tc^{99m}}{\partial t} = \frac{Y_{Tc^{99m}}.P}{\gamma} + 0.875\,\lambda_{Mo^{99}}.Mo^{99} - \lambda_{Tc^{99m}}.Tc^{99m} \qquad (25)$$

3. RESULTS

After writing all the chain radioactive decay differential equations, these equations are numerically solved using the MATLAB software and through the initial information of primary fuel mass, thereby obtaining the atomic density of each of considered radioisotopes sequentially and consequently obtaining the fuel burnup [6]. Finally, the mass of each produced radioisotope (*m*) is obtained, according to obtained atomic density of all the mentioned radioisotopes and also based on this formula: $m = \frac{N.M}{A}$ [7].

The masses of radioisotopes and radiomedicines produced in the MNSR core after one year operation are shown in Table 2:

Table 2. The obtained masses of radioisotopes produced in the MNSR core

$m_{Rb^{98}} = 6.51\times10^{-11}\,gr$	$m_{U^{238}} = 108.92\,gr$
$m_{Rb^{99}} \approx 0\,gr$	$m_{U^{239}} = 4.002\times10^{-10}\,gr$
$m_{Sr^{99}} = 5.54\times10^{-9}\,gr$	$m_{Np^{236}} = 1.59\times10^{-10}\,gr$
$m_{Y^{99}} = 5.01\times10^{-7}\,gr$	$m_{Np^{237}} = 3.08\times10^{-5}\,gr$
$m_{Zr^{99}} = 3.7\times10^{-6}\,gr$	$m_{Np^{238}} = 7.8\times10^{-19}\,gr$
$m_{Nb^{99}} = 3.72\times10^{-6}\,gr$	$m_{Np^{239}} = 3.8\times10^{-6}\,gr$
$m_{Nb^{99m}} = 1.21\times10^{-4}\,gr$	$m_{Pu^{238}} = 9.3\times10^{-17}\,gr$
$m_{Mo^{99}} = 1.064\,gr$	$m_{Pu^{239}} = 4.08\times10^{-6}\,gr$
$m_{Tc^{99m}} = 0.114\,gr$	$m_{Pu^{240}} = 3.78\times10^{-11}\,gr$
$m_{U^{235}} = 999.335\,gr$	$m_{Pu^{241}} = 3.21\times10^{-16}\,gr$
$m_{U^{236}} = 2.75\times10^{-3}\,gr$	$m_{Pu^{242}} \approx 0\,gr$
$m_{U^{237}} = 8.2\times10^{-7}\,gr$	

4. CONCLUSION

The obtained results from calculation indicate that the MNSR reactor on continuous neutron flux: $10^9 n/cm^2.sec$, which is a low neutron flux, spends only 0.0665% of primary fresh fuel after one year operation. Therefore, more than 99% of existent fuel remains entirely at reactor core. According to Table 2, as it is observed, due to being low neutron flux, the amounts of some produced radioisotopes are very slight.

The fuel burnup after one year operation of reactor can also be obtained through the results.

Since the produced energy per fission of all the nucleuses of 1gr U^{235} is $8.195\times10^{10} J$, the produced power during 24 hours will be: 9.5×10^5 *Watt*. Therefore:

$$\text{Burnup} = 9.5\times10^5 m_{ff} = 9.5\times10^5 r.m_f \qquad (26)$$

According to above equation, the fuel burnup of MNSR reactor will be: 1731*Watt.d/gr*, whereas the initial applied mass of U^{235} in MNSR, which is as the main fissile, is 1kg and its spent amount is: 0.665*gr*.

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