The Prediction of Motion of an Artificial Satellite

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ABSTRACT

In this paper the solution of an artificial satellite motion under the influence of Earth’s gravitational field in terms of Euler parameters that was solved using recurrent power series solution which is one of the semi-analytical solutions. Applications of the method enable anyone to predict the motion of the artificial satellite in any conic section. This expected because the only force affecting on the motion of artificial satellite is Earth’s gravitational field.

KEYWORDS: satellite- semi analytical solution- Euler parameters.

INTRODUCTION

Orbit prediction of artificial satellites become one of the most important problems at present, due to their wide applications in scientific researches, mission planning and military purposes, … etc.

There are many transformations have emerged in the literature in the recent past, one of the famous of them is the set of Eulerian redundant parameters [1], [2],[3] [4],[5]and [6]. These parameters combine between orbit dynamics and rigid body dynamics, and they are valid for any type of orbital motion. In this paper, we use a recurrent power series solution of the Eulerian perturbed differential equations of motion depending on Taylor series expansion to get the classical elements of the satellite at any time.

FORMULATION OF THE PROBLEM

Generally, the equations of motion of an artificial satellite are given generally as

\[ \ddot{x} + \frac{\mu}{r^3} x = \vec{P} \]  

(2.1)

where \( x \) is the relative position vector, \( r = \left| \dot{x} \right| \), \( \mu \) is the Earth's gravitational constant, \( \vec{P} \) is the all perturbing forces, which equals to \(- \frac{\partial V}{\partial \dot{x}} + \vec{P}^* \), hence \( \vec{P}^* \) is the resultant of all non-conservative perturbing forces, and \( V \) is the perturbed time-independent potential, which can be expressed as

\[ V = \mu \sum_{i=2}^{\infty} R^i J_i \left( \frac{1}{r} \right)^{i+1} L_i \left( \frac{x_3}{r} \right) \]

where \( R \) is the Earth's equatorial radius, \( J_i \) is the non-dimensional coefficient of the Earth's potential and \( L_i \) is the Legendre polynomial of order \( i \).

In our case the only force acting on an artificial satellite is that due to the Earth’s oblateness, so we have \( \vec{P}^* = 0 \),

\[ V = \frac{3}{2} q_2 x_3^2 r^{-5} - \frac{1}{2} q_2 r^{-5} \]  

(2.2.1)

where \( q_2 = \mu R^2 J_2 \), \( r = \sqrt{x_1^2 + x_2^2 + x_3^2} \).

Eq.(2.1) are the basic classical equations of motion of artificial satellites and the corresponding one in terms of the Euler parameters with \( \tilde{\phi} \) (perturbed true anomaly) as independent variable [3],

\[ y'_1 = 0.5 \left( w_1 y_4 + w_3 y_2 \right), \]  

(2.3.1)

\[ y'_2 = 0.5 \left( w_1 y_3 - w_3 y_1 \right), \]  

(2.3.2)

\[ y'_3 = 0.5 \left( -w_1 y_2 + w_3 y_4 \right), \]  

(2.3.3)

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\[ y'_4 = 0.5 \left( -y_1 y_3 - w_3 y_3 \right), \quad (2.3.4) \]
\[ y'_5 = y_6, \quad (2.3.5) \]
\[ y'_6 = -y_5 + \frac{1}{y_7} \left( \frac{P_\xi}{\mu y_5^2 y_7} + \frac{2V}{\mu y_5 y_7} - \frac{g_2 y_6}{2 y_7} \right), \quad (2.3.6) \]
\[ y'_7 = g_2 \left( \frac{2w_1 P_\eta^*}{\mu y_5^3} + \frac{2}{y_5^4 \sqrt{\mu} y_7} \frac{\partial V}{\partial t} - \frac{2 y_6}{\mu y_5^3} \left( \frac{\partial V}{\partial x} \cdot \vec{x} \right) + 2V \right), \quad (2.3.7) \]
\[ y'_8 = \frac{1}{y_5^2 \sqrt{\mu y_7}}, \quad (2.3.8) \]

where
\[
\begin{align*}
w_1 &= \frac{P_\xi}{\mu w_3 y_5 y_7}, \\
w_3 &= \sqrt{1 - \frac{2V}{\mu y_5^2 y_7}}, \\
P_\xi &= C_{11} P_1 + C_{12} P_2 + C_{13} P_3, \\
P_\eta^* &= C_{21} P_1^* + C_{22} P_2^* + C_{23} P_3^*, \\
P_{\xi}^* &= C_{31} P_1 + C_{32} P_2 + C_{33} P_3, \\
C_{11} &= y_1^2 - y_2^2 - y_3^2 + y_4^2, \\
C_{12} &= 2(y_1 y_2 + y_3 y_4), \\
C_{13} &= 2(y_1 y_3 - y_2 y_4), \\
C_{21} &= 2(y_1 y_2 - y_3 y_4), \\
C_{22} &= y_1^2 + y_2^2 - y_3^2 + y_4^2, \\
C_{23} &= 2(y_2 y_3 + y_1 y_4), \\
C_{31} &= 2(y_1 y_3 + y_1 y_4), \\
C_{32} &= 2(y_2 y_3 - y_1 y_4), \\
C_{33} &= y_1^2 - y_2^2 + y_3^2 + y_4^2, \\
x_i &= r C_{ii} = C_{ii} / y_3, \quad (2.9)\end{align*}
\]

hence we denote the differentiation with respect to the independent variable \( \tilde{\phi} \) by a prime (\( ' \)), since
\[
(\prime) = \frac{d}{d\tilde{\phi}} (\_). \]

**RECURRENT POWER SERIES SOLUTION:**

In this section, recurrent power series solution of Eqs.(2.3) will be gone in the three following steps.

Rewriting the equations of motion in terms of \( y \)'s only as
\[y'_1 = 3Q_2y^5_3y_4y_5y_9 - 3Q_2y^4_1y_2y^2_4y_5y_9 + \\
6Q_2y^3_1y^2_3y_4y_5y_9 - 6Q_2y^2_1y^3_2y^2_4y_5y_9 + \\
3Q_2y_1y^2_2y_3y_4y_5y_9 - 3Q_2y^5_2y^2_4y_5y_9 - \\
3Q_2y^5_1y^2_3y_4y_5y_9 + 3Q_2y_2y^3_4y_5y_9 - \\
6Q_2y_1y^3_3y^2_4y_5y_9 + 6Q_2y_2y^2_3y^4_4y_5y_9 - \\
3Q_2y_1y^3_3y^5_4y_5y_9 + 3Q_2y_2y^6_4y_5y_9 - \\
3Q_2y^2_1y_2y^3_3y^2_5y_5y_9 + 6Q_2y_1y^2_2y^3_3y_4y_5y_9 - \\
3Q_2y^2_2y^3_4y^2_5y_5y_9 + 0.25Q_2y_2y_5y_9 + 0.5y_2,\]

\[(3.1.1)\]

\[y'_2 = 3Q_2y^5_3y^2_5y_5y_9 - 3Q_2y^4_1y_2y_3y_4y_5y_9 + \\
6Q_2y^3_1y_2y^2_3y^2_5y_5y_9 - 6Q_2y^1_2y^3_3y^2_4y_5y_9 + \\
3Q_2y^2_1y^2_3y^2_5y_5y_9 - 3Q_2y^5_2y^2_3y_4y_5y_9 - \\
3Q_2y^1_1y^6_3y^5_4y_5y_9 + 3Q_2y_2y^5_3y_4y_5y_9 - \\
6Q_2y_1y^3_3y^2_4y^2_5y_5y_9 + 6Q_2y_2y^3_3y^4_4y_5y_9 - \\
3Q_2y_1y^3_3y^4_3y^2_5y_5y_9 + 3Q_2y_2y^3_3y^5_4y_5y_9 + \\
3Q_2y^2_1y^3_3y^2_5y_5y_9 - 6Q_2y^1_2y_2y^3_3y_4y_5y_9 + \\
3Q_2y^1_1y^2_2y^2_3y^3_5y_9 - 0.25Q_2y_1y^5_5y_9 - 0.5y_1,\]

\[(3.1.2)\]

\[y'_3 = -3Q_2y^5_1y^2_3y_3y_5y_9 + 3Q_2y^4_1y_2^2y_4y_5y_9 - \\
6Q_2y^3_1y^2_3y^3_5y_5y_9 + 6Q_2y^1_2^2y^4_3y_4y_5y_9 - \\
3Q_2y_1y^5_2y_3y_5y_9 + 3Q_2y^6_2y_4y_5y_9 + \\
3Q_2y^1_1y_2y^5_3y^5_4y_5y_9 - 3Q_2y^2_2y^2_3y_4y_5y_9 + \\
6Q_2y_1y_2y^3_3y^2_4y_5y_9 - 6Q_2y^2_2y^2_3y^2_4y_5y_9 + \\
3Q_2y_1y_2y^5_3y^4_4y_5y_9 - 3Q_2y^2_2y^5_3y_4y_5y_9 - \\
3Q_2y^2_1y^2_3y^4_5y_5y_9 + 6Q_2y_1y^2_2y^2_3y^4_5y_5y_9 - \\
3Q_2y^2_2y^2_3y^5_5y_5y_9 + 0.25Q_2y_4y_5y_9 + 0.5y_4,\]

\[(3.1.3)\]
\[ y'_4 = -3Q_2 y_1^6 y_3 y_5 y_9 + 3Q_2 y_1^5 y_2 y_4 y_5 y_9 - 6Q_2 y_1^4 y_2^2 y_3 y_5 y_9 + 6Q_2 y_1^3 y_2^3 y_4 y_5 y_9 - 3Q_2 y_1^2 y_2^4 y_3 y_5 y_9 + 3Q_2 y_1 y_2^5 y_4 y_5 y_9 + 3Q_2 y_1^2 y_3 y_5 y_9 - 3Q_2 y_1 y_2 y_4 y_5 y_9 + 6Q_2 y_1 y_2 y_3 y_4^2 y_5 y_9 - 6Q_2 y_1 y_2 y_3^2 y_4 y_5 y_9 + 3Q_2 y_1 y_2 y_3 y_4 y_5 y_9 - 3Q_2 y_1 y_2 y_3 y_4 y_5 y_9 + 3Q_2 y_2 y_3 y_4^2 y_5 y_9 - 0.25Q_2 y_3 y_5 y_9 - 0.5 y_3, \] 

\[ y'_{5} = y_{6}, \] 

\[ y'_{6} = -y_5 + y_9 - 6Q_2 y_1^2 y_3^2 y_5^2 y_9 + 12Q_2 y_1 y_2 y_3 y_4 y_5^2 y_9 - 6Q_2 y_1 y_2 y_3 y_4^2 y_5 y_9 + 0.5Q_2 y_1^2 y_3^2 y_6 y_9 + 12Q_2 y_1 y_2 y_3 y_4^2 y_6 y_9 - 6Q_2 y_1 y_2 y_3 y_4 y_6 y_9 + 0.5Q_2 y_6 y_9, \] 

\[ y'_{7} = 12Q_2 y_1 y_3 y_6 - 24Q_2 y_1 y_2 y_3 y_4 y_6 + 12Q_2 y_2 y_4 y_5 y_6 - Q_2 y_6, \] 

\[ y'_{8} = \mu^{-1/2} y_{10} r_2, \] 

where \( Q_2 = q_2 / \mu, \) \( y_9 = 1 / y_7, \) \( y_{10} = 1 / y_{7}^{1/2}, \) \( r_2 = r. \)

Substituting with new variables to reduce the order of Eqs.(3.1). So, we let

\[ y_4 = y_1 y_1, i = 1(1)4; \quad y_{9} = y_1 y_9, i = 5, 6; \]
\[ e_i = y_{10} r_5; \quad e_6 = y_6 y_6; \quad u = y_{13} y_{69}; \]
\[ w = y_{34} y_{86}; \quad z_4 = y_{14} y_{14}, i = 1(1)4; \quad z_6 = y_{16} y_{16}, i = 1(1)4; \]
\[ x_{i1} = y_{14} y_{14}, i = 1(1)4; \quad x_{11} = y_{14} y_{14}, i = 1(1)4; \]
\[ b_{i2} = y_{24} y_{24}, i = 1(1)4; \quad b_{12} = y_{24} y_{14}, i = 1(1)4; \]
\[ c_{i2} = y_{24} y_{24}, i = 1(1)4; \quad c_{12} = y_{24} y_{14}, i = 1(1)4; \]
\[ d_i = d_{i1} y_6, i = 1(1)3; \quad d_{11} = d_{11} y_6, i = 1(1)3; \]
\[ d_{4} = d_{4} y_{69}, d_{6} = d_{6} y_{69}. \]
Using the previous substitutions into Eqs.(3.12), we get the following first order differential set in eight unknowns
\[ y'_1 = Q_2 c_{14} - Q_2 c_{24} - 3Q_2 x_{12} + 6Q_2 x_{32} - 3Q_2 x_{22} + 0.25Q_2 e_2 + 0.5y_2, \] 
\[ y'_2 = Q_2 c_{13} - Q_2 c_{23} + 3Q_2 x_{11} - 6Q_2 x_{31} + 3Q_2 x_{21} - 0.25Q_2 e_1 - 0.5y_1, \] 
\[ y'_3 = -Q_2 c_{12} + Q_2 c_{22} - 3Q_2 x_{14} + 6Q_2 x_{34} - 3Q_2 x_{24} + 0.25Q_2 e_4 + 0.5y_4, \] 
\[ y'_4 = -Q_2 c_{11} + Q_2 c_{21} + 3Q_2 x_{13} - 6Q_2 x_{33} + 3Q_2 x_{23} - 0.25Q_2 e_3 - 0.5y_3, \] 
\[ y'_5 = y_6, \] 
\[ y'_6 = -y_3 + y_9 - 6Q_2 x_{15} + 12Q_2 x_{35} - 6Q_2 d_4 + 0.5Q_2 e_3 + Q_2 d_6 + 0.5e_6, \] 
\[ y'_7 = -2Q_2 d_5 - Q_2 y_6, \] 
\[ y'_8 = \mu^{-1/2} r_3. \] 

Let us define the eight Taylor expansions as follows
\[
h_i = \sum_{n=1}^{\infty} H_i^{(n)} \tilde{\phi}^{-n} ; \quad i = 1, 2, ..., 8 \tag{3.4}
\]
and
\[
h'_i = \sum_{n=1}^{\infty} n H_i^{(n+1)} \tilde{\phi}^{-n} ; \tag{3.5}
\]
where we have used small letters (h) for the unknown variables and capital letters (H) for the coefficients in their Taylor series expansion.

Now, let us define the product of two infinite power series. If a and b are two infinite power series such that
\[
a = \sum_{i=1}^{\infty} A^{(i)} S^{-i},
\]
\[
b = \sum_{i=1}^{\infty} B^{(i)} S^{-i}.
\]

Then, it is easy to show that
\[
c = a b = \sum_{n=1}^{\infty} C^{(n)} S^{-n},
\]
where
\[
C^{(n)} = \sum_{i=1}^{\infty} A^{(i)} B^{(n-i+1)}.
\]

Substituting Eqs.(3.4 and 3.5) into Eqs.(3.3), and using the rule of the product of two power series, and equating coefficients of equal powers of $\tilde{\phi}$ in both sides of each of the resulting equations, then we get the coefficients of the following recurrence formulae
\[ nY_{1}^{n+1} = Q_2 c_{14} - Q_2 c_{24} - 3Q_2 x_{12} + 6Q_2 x_{32} - 3Q_2 x_{22} + 0.25Q_2 e_2 + 0.5Y_2, \] 
\[ nY_{2}^{n+1} = Q_2 c_{13} - Q_2 c_{23} + 3Q_2 x_{11} - 6Q_2 x_{31} + 3Q_2 x_{21} - 0.25Q_2 e_1 - 0.5Y_1, \] 
\[ nY_{3}^{n+1} = -Q_2 c_{12} + Q_2 c_{22} - 3Q_2 x_{14} + 6Q_2 x_{34} - 3Q_2 x_{24} + 0.25Q_2 e_4 + 0.5Y_4, \] 
\[ nY_{4}^{n+1} = -Q_2 c_{11} + Q_2 c_{21} + 3Q_2 x_{13} - 6Q_2 x_{33} + 3Q_2 x_{23} - 0.25Q_2 e_3 - 0.5Y_3, \] 
\[ nY_{5}^{n+1} = Y_6, \] 
\[ nY_{6}^{n+1} = -Y_3 + Y_9 - 6Q_2 x_{15} + 12Q_2 x_{35} - 6Q_2 d_4 + 0.5Q_2 e_3 + Q_2 d_6 + 0.5E_6, \] 
\[ nY_{7}^{n+1} = -2Q_2 d_5 - Q_2 Y_6, \] 
\[ nY_{8}^{n+1} = \mu^{-1/2} r_3. \]
then we can compute the coefficients (capital letters) to get small letters which are the unknown variables (y’s).

4. COMPUTATIONAL DEVELOPMENTS:
In this section, the computational developments of the formulations of section 3 will be considered.

4.1 Computation Of The Initial Values:
Knowing the position and velocity vectors $\mathbf{x}_0$ and $\mathbf{x}_0'$ at the instant time ($t = 0$), we have the initial values of the y’s as follows

1) $r_0 = \sqrt{x_{01}^2 + x_{02}^2 + x_{03}^2}$,
2) $y_5 = 1/r_0$,
3) $\dot{r}_0 = y_5 (x_{01}\dot{x}_{01} + x_{02}\dot{x}_{02} + x_{03}\dot{x}_{03})$,
4) $V_0 = 1.5\mu R^2 J_2 y_5^2 x_{03} - 0.5\mu R^2 J_2 y_5^2$,
5) $y_7 = (x_{01} + \dot{x}_{02} + \dot{x}_{03} - \ddot{r}_0^2 + 2V_0)/(\mu y_5^2)$,
6) $y_6 = -\dot{r}_0 / \sqrt{\mu y_7}$,
7) $C_{i\ell} = -x_{\ell0}y_5$, $i = 1(1)3$,
8) $p = (x_{01} + \dot{x}_{02} + \dot{x}_{03} - \ddot{r}_0^2)/(\mu y_5^2)$,
9) $C_{2i} = (\dot{x}_{0i}/y_5 - \dot{r}_0 x_{0i})/\sqrt{\mu p}$,
10) $C_{31} = C_{12} - C_{23}$,
11) $C_{32} = C_{13} - C_{23}$,
12) $C_{33} = C_{11} - C_{22}$,
13) $y_4 = 0.5/\sqrt{1 + C_{11} + C_{22} + C_{33}}$,
14) $y_1 = (C_{23} - C_{32})/(4y_4)$,
15) $y_2 = (C_{31} - C_{13})/(4y_4)$,
16) $y_3 = (C_{12} - C_{21})/(4y_4)$,
17) $y_8 = t_0$.

Computation Of The Step Size
The related equation between the step size $\Delta t$ of the time $t$ and the step size $\Delta\tilde{\phi}$ of perturbed true anomaly is

$$\Delta\tilde{\phi} = \Delta t \cdot y_5^2 \sqrt{\mu y_7}.$$

Computation Of Accuracy Check
The accuracy of the computed values of the y’s at any time could be checked using the relation

$$y_1^2 + y_2^2 + y_3^2 + y_4^2 = 1.$$

RESULTS AND DISCUSSION
As we mentioned in the abstract that we’ll take the numerical example of the Indian satellite RS-1 at about 300 Km height (Sharma and Mani, 1985) which remained in its orbit for 371 days, its initial position and velocity components are

$$\mathbf{X} = (1626.742, 6268.094, -1776.018) \text{ Km},$$
\[ \vec{\dot{X}}_0 = (-5.920522, 0.239214, -5.15883) \text{ Km/sec} \]

at epoch 20 July 1980, where its one orbital revolution is elapsed 1.588352085 hrs. Since the adopted physical constant are

\[ R = 6378.135 \text{ Km}, \quad \mu = 398600.8 \text{ Km/sec}^2, \]

and the Earth’s zonal harmonic coefficient \( J_2 \) equal 1.0826157 \times 10^{-3}.

Using the above values to compute the position and velocity components, i.e., the six elements, and the accuracy check at any time. The results tabulated in the following table which shows the variation in the elements (a, e, i, \( \Omega \), \( \omega \)) and the check relation even 10 days for 250 day. Because of [7] have a perfect prediction up to 250 day and after that day the prediction is not good. So, we did our work for the same interval of days.

Computed osculating orbital elements.

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<th>Mean Solar Day</th>
<th>a (km)</th>
<th>e</th>
<th>i</th>
<th>( \Omega )</th>
<th>( \omega )</th>
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CONCLUSION

From the table we can conclude that, from 0 up to 210 mean solar days, there are obviously decay in the two elements (a, e), but the other elements (i, \( \Omega \), \( \omega \)) show lightly change. This expected because the only force affecting on the motion of artificial satellite is Earth’s gravitational field. This force slightly affect on the elements (i, \( \Omega \), \( \omega \)) where these elements are strongly affected by the other forces like drag, solar radiation pressure, etc.

Also, the table shows that the accuracy check is always nearly to one, i.e., the predictions of the components of position and velocity (the elements) of the artificial satellite is very good.

After 210 up to 250 mean solar days the accuracy check is not good. Depending on this result the satellite will go-down after 210 day. This result is nearly to that coming from [7] and [6]

To get more accurate prediction of the motion of the artificial satellite we will be taken into account the whole other forces affecting on the motion.

REFERENCES


