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Acceptable Solutions of Relativistic Solitons of a Gaussian Laser Pulse in a Preformed Plasma Channel

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ABSTRACT

The propagation of an intense laser beam in a preformed plasma channel is studied. Considering a propagating Gaussian laser pulse in a relativistic plasma channel which has a parabolic density profile, the evolution equation of the laser spot size is derived including the effects of ponderomotive self-channeling, preformed channel focusing and relativistic self focusing. Also, in order to investigate the conditions for the existence of electromagnetic solitary waves, solutions of the envelope equation are discussed in terms of a relativistic effective potential for the laser spot size. Furthermore, some solitary wave solutions are illustrated numerically.

KEYWORD: Plasma Channel, Relativistic Solitons, Electromagnetic Solitary Waves, Preformed Channel Focusing.

1- INTRODUCTION

The nonlinear interaction of plasmas with high intensity lasers is of great current interest [1-5]. The possibility of reaching extreme power levels with such setups is one of the promising aspects of laser-plasma systems [6], and also holds the potential of overcoming the laser intensity limit $\approx 10^{25}\ W/cm^2$ [7]. As the field strengths approaches the critical Schwinger field $E_{\text{crit}} \approx 10^{16}\ V/cm$ [8], there is possibility of photon-photon scattering, even within a plasma [9], as the ponderomotive force due to the intense laser pulse gives rise to plasma channels [10].

In fact, the main nonlinear effects in the propagation of intense electromagnetic pulses through a plasma arise from the relativistic variation of electron mass (relativistic nonlinearity) and from the perturbation in the electron density which takes place because of the ponderomotive forces due to the radiation fields (strict nonlinearity). Both these effects change the effective dielectric constant of the plasma medium for the propagation of the electromagnetic wave and lead to a coupling between the transverse electromagnetic wave and the longitudinal waves of the plasma medium.

The study of formation and propagation of relativistic electromagnetic solitary waves and their effects on the plasma due to highly nonlinear processes of strong electromagnetic wave coupling with the plasma wave is important to understand many aspects of laser-plasma interaction such as fast ignition scheme, laser wake field acceleration and laser overdense penetration [11-14].

It is well known that the characteristic distance for propagation of a directed radiation beam in vacuum is the Rayleigh range, Z_R. On the other hand, although a laser pulse in a uniform plasma can guide itself by the effect of relativistic self-focusing and ponderomotive self-channeling, the diffraction would dominate over these effects when the laser power is smaller than the critical power $P_{\text{c}} = 17(\omega_0/\omega_{\text{p}})^2 (GW)$ where ω_{p} is the plasma frequency and ω_0 is the laser frequency. It has been shown that a preformed plasma channel can prevent diffraction and allow the propagation of an intense laser pulse through many Rayleigh lengths without disruption.

The aim of this paper is to investigate the existence of the relativistic solitary waves of a Gaussian laser pulse in a preformed plasma channel with a parabolic density profile. The organization of this paper is as follows: In Sec. 2, considering the appropriate equations, we obtain the differential equation describing the evolution of the laser spot size. The governing equation presents the effects of ponderomotive self-channeling, preformed channel focusing and self-focusing with relativistic corrections. In Sec. 3, we use this equation to discuss the solutions and some solitary wave solutions are illustrated numerically. Sec. 4 summarizes the finding of this study.

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2- EVOLUTION EQUATIONS

The normalized vector potential with a slowly varying complex envelope for a circularly polarized laser pulse propagating in a plasma channel with a parabolic density profile of the form $n(r)=n_0\left(1+r^2/r_{\text{ch}}^2\right)$ can be written as [15]

$$\mathbf{a}(r,z,t)=\frac{1}{2}a(r,z,t)\left(\hat{\mathbf{e}}_x+i\hat{\mathbf{e}}_y\right)\exp\left[\mathrm{i}\left(k_0 z-\omega_0 t\right)\right]+\text{c.c} \tag{1}$$

Where n_0 is the initial axial electron density, r_{ch} the effective channel radius and $a(r,z,t)$ is the complex amplitude. Also, k_0 and ω_0 are the laser centre wave number and the frequency, respectively. In relativistic regime, using Coulomb gauge $\nabla\cdot\mathbf{a}=0$, the wave equation for the laser field can be written as

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{a}=k_p^2\left(1+\gamma_V\,\frac{\left(1-\beta_0 v\right)}{\gamma_0\left(\beta_0-v\right)^2}\frac{r^2}{r_{ch}^2}-\frac{|\mathbf{a}|^2}{2}+\gamma_V^2\,\frac{\left(1-\beta_0 v\right)^2}{\gamma_0^2\left(\beta_0-v\right)^4}\nabla_\perp^2\frac{|\mathbf{a}|^2}{2}\right)\mathbf{a} \tag{2}$$

where $k_p=\omega_p/c$ is the plasma wave number and $\gamma_V=\left(1-V^2/c^2\right)^{-1/2}$ is the 'effective relativistic factor' associated with the velocity of the wave and should not be confused with the relativistic factor, $\gamma=\left(1-u^2/c^2\right)^{-1/2}$ related to the fluid velocity of the plasma. Also, $\beta=u/c$, $v=V/c$ and the subscript 0 represent quantities at infinity. It is necessary to mention that, in deriving equation (2), the long pulse limit, i.e., $\omega_p\tau_L>>1$ is used [$\omega_p=\left(4\pi\, n_0 e^2/m_0\right)^{1/2}$ and τ_L are the plasma frequency and the laser pulse duration, respectively]. It is worth to note that in the weakly relativistic limit, equation (2) reduces to its counterpart in [15]. Substituting Eq. (1) into Eq. (2) and assuming that the complex amplitude of the vector potential has a solution with the Gaussian transverse profile as

$$a(r,z)=a_r(z)\exp\left[-r^2/r_s^2(z)\right]\exp\left[i\left(b(z)r^2+\phi(z)\right)\right] \tag{3}$$

the relativistic equation describing the evolution of the laser spot size is given as

$$\frac{\partial^2 r_s}{\partial z^2}=\frac{\gamma_V\left(1-\beta_0 v\right)}{\gamma_0\left(\beta_0-v\right)^2}\frac{\left(1-p\right)}{r_s^3}-N_c r_s-\frac{\gamma_V^2\left(1-\beta_0 v\right)^2}{\gamma_0^2\left(\beta_0-v\right)^4}\frac{a_0^2}{2r_s^5} \tag{4}$$

where $a_r(z), r_s(z), b(z)$ and $\phi(z)$ are the real amplitude, spot size, spatial chirp parameter and phase shift of the laser pulse respectively. Also, $p=k_p^2 a_0^2 r_0^2/16$ is the normalized laser power, $N_c=\gamma_0^2 k_p^2 r_0^4/4r_{ch}^2$ is a parameter related to the effect of reformed channel focusing and the dimensionless variables $z/Z_R\to z$ and $r_s/r_0\to r_s$ where $Z_R=k_0^2 r_0^2/2$ is the Rayleigh length, are used. Now, considering a collimated incident laser pulse, i.e., $b_0=\left(\partial r_s/\partial z\right)_{z=0}=0$ and the initial condition $r_s=1$ at $z=0$ and integrating Eq. (4) once gives

$$\frac{1}{2}\left(\frac{\partial r_s}{\partial z}\right)^2+V\left(r_s\right)=0 \tag{5}$$

where

$$V\left(r_s\right)=\frac{\gamma_V\left(1-\beta_0 v\right)}{\gamma_0\left(\beta_0-v\right)^2}\frac{\left(1-p\right)}{2r_s^2}+\frac{1}{2}N_c r_s^2-\frac{\gamma_V^2\left(1-\beta_0 v\right)^2}{\gamma_0^2\left(\beta_0-v\right)^4}\frac{a_0^2}{8r_s^4}-V_0 \tag{6}$$

in which

$$V_0=\frac{\gamma_V\left(1-\beta_0 v\right)\left(1-p\right)}{2\gamma_0\left(\beta_0-v\right)^2}+\frac{N_c}{2}-\frac{\gamma_V^2\left(1-\beta_0 v\right)^2 a_0^2}{8\gamma_0^2\left(\beta_0-v\right)^4} \tag{7}$$

We note that for $\beta_0 = 0$, equations (4)-(7) obviously reduce to the expressions for the non relativistic limits.

3- SOLUTION AND RESULTS

The evolution equation, Eq. (5), can be used to study the variation of the spot size of the laser beam. The roots of the relativistic effective potential function can be easily found using the solutions of a cubic equation. Solving $V(r_s) = 0$ gives three solutions as

$$r_{s1} = 1 \tag{8}$$

$$r_{s2} = \left[\left(N_p + \sqrt{N_p^2 - 16 N_c a_0^2}\right)\Big/ 8N_c\right]^{1/2} \tag{9}$$

$$r_{s3} = \left[\left(N_p - \sqrt{N_p^2 - 16 N_c a_0^2}\right)\Big/ 8N_c\right]^{1/2} \tag{10}$$

where

$$N_p = \frac{4\gamma_V (1-\beta_0 v)(1-p)}{2\gamma_0 (\beta_0 - v)^2} - a_0^2 \tag{11}$$

three cases can be considered:

(a) if $p > 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, where $\eta = \dfrac{\gamma_V (1-\beta_0 v)}{\gamma_0 (\beta_0 - v)^2}$, the equation has three real roots: $r_{s1} = 1$ and $r_{s2} = r_{s3} = \sqrt{\eta a_0 / 2\sqrt{N_c}} > 1$.

(b) if $p < 1 - N_c - \eta^2 a_0^2/2$, $V(r_s) = 0$ has three distinct real roots $r_{s3} < r_{s2} < r_{s1} = 1$;

(c) if $1 - N_c - \eta^2 a_0^2/2 \le p \le 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, three kinds of cases can be considered:

(c1) if $N_c = N_c^*$, where the critical channel parameter $N_c^* = \eta^2 a_0^2/4$, $V(r_s) = 0$ has three real roots: twofold root $r_{s1} = r_{s2} = 1$ and $r_{s3} < 1$.

(c2) if $N_c > N_c^*$, three types can be discussed as follows:

(c2.1) if $p > 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, $V(r_s) = 0$ has only one real root, i.e., $r_{s1} = 1$;

(c2.2) if $1 - N_c - \eta^2 a_0^2/2 < p < 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, $V(r_s) = 0$ has three real roots: twofold root $r_{s2} < r_{s1} < r_{s3}$;

(c2.3) if $p = 1 - N_c - \eta^2 a_0^2/2$, then $V(r_s) = 0$ has three unequal rael roots: $r_{s1} = 1 < r_{s3} < r_{s2}$;

(c3) if $N_c < N_c^*$, the following results are given:

(c3.1) if $p = 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, $V(r_s) = 0$ has triple root, i.e., $r_{s1} = r_{s2} = r_{s3} = 1$

(c3.2) if $1 - N_c - \eta^2 a_0^2/2 < p < 1 - \eta a_0 \sqrt{N_c} - \eta^2 a_0^2/4$, $V(r_s) = 0$ has only one real root, i.e., $r_{s1} = 1$;

(c3.3) if $p = 1 - N_c - \eta^2 a_0^2/2$, $V(r_s) = 0$ has three real roots: twofold roots: $r_{s1} = 1$ and $r_{s2} = r_{s3} = \sqrt{\eta a_0 / 2\sqrt{N_c}} < 1$;

Figures (1-9) show the variations of the potential, V for various values of p and N_c corresponding to the cases (a)-(c3.3) respectively. In all cases, fix parameters, $v = 0.4, \beta = 0.8$ and $a_0 = 0.3$ are considered.

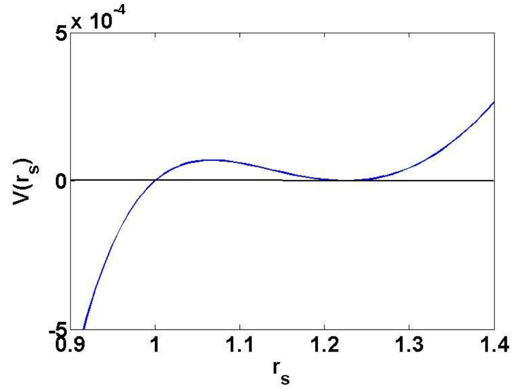

Figure 1. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.3, p = 0.85$.

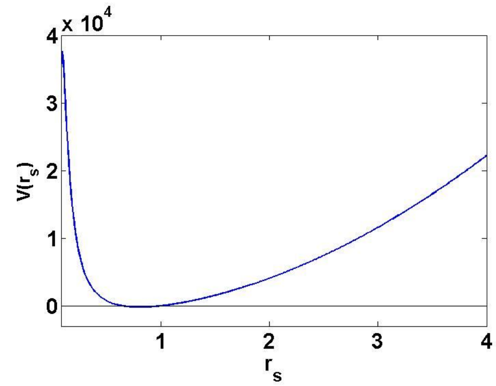

Figure 2. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.3, p = 0.15$.

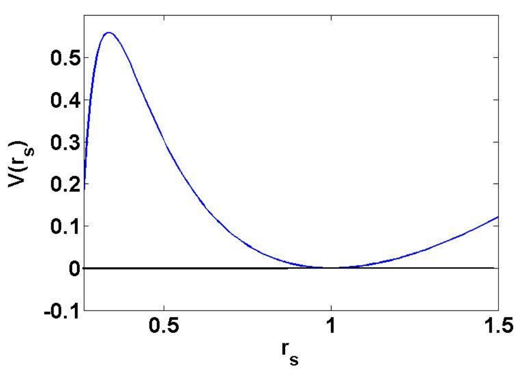

Figure 3. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.04, p = 0.95$.

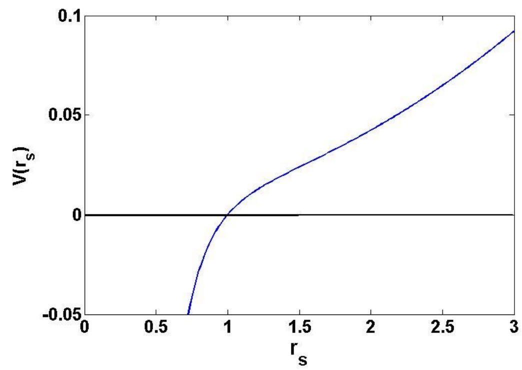

Figure 4. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.4, p = 0.75$.

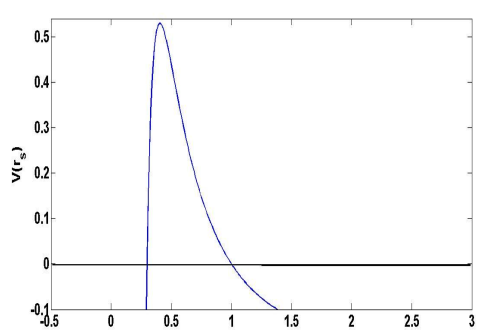

Figure 5. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.45, p = 0.9$.

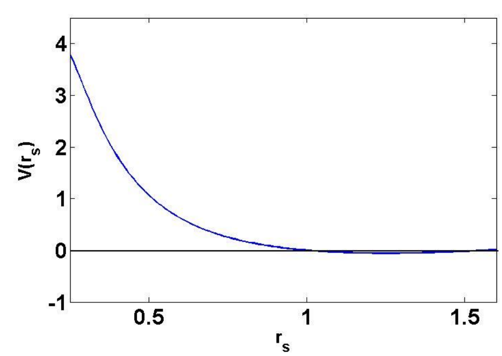

Figure 6. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.45, p = 0.65$.

In figure (3), the position $r_s = 1$ is stable. In this case, (c1), the particle will be at rest. This case could be related to a constant spot size.

In figures (1, 4, 5, 8, 9), the position $r_s = 1$ is unstable. These cases correspond to the catastrophic focusing. In fact, in this position, the particle will move to the position $r_s \to 1$ for the certain parameters introduce in these cases.

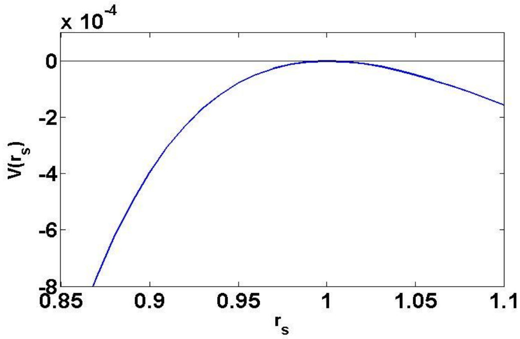

Figure 7. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.02, p = 1.01$.

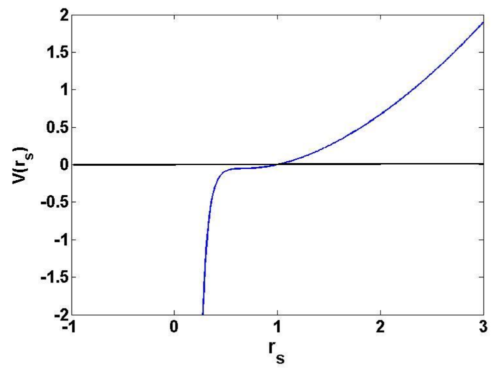

Figure 8. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.02, p = 1.02$.

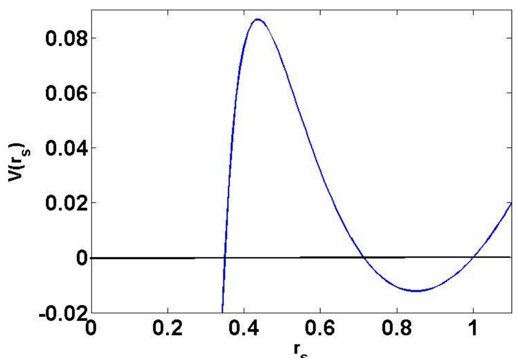

Figure 9. Potential $V(r_s)$ as a function of spot size r_s for $N_c = 0.02, p = 1$.

Also, as is clear from figures (2, 6), the particle will move periodically between r_{s1} and r_{s2} in cases (b) and (c2.3) which shows the characteristic feature of periodic solutions. Finally, in the case of (c3.1), figure (7), the particle is in critical state.

4- SUMMARIES

In this paper, assuming a circularly polarized Gaussian laser pulse propagating in a plasma channel with a parabolic density profile, we obtained a relativistic effective potential and its governing equation. Then, by analyzing the differential equation of the pulse spot size, we investigated the conditions for the existence of electromagnetic solitary waves. Finally, we illustrated some solitary wave solutions numerically.

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