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Comparing 10 Different Interpolation Methods Used to Determine Geoid-quasigeoid Separation (Case Study in Iran)

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ABSTRACT

Interpolation method is one of the common methods to determine different geodetic quantities. In this study, geoid-quasigeoid separation in Alborz mountainous region, Kavir plain region, and Khuzestan coastal region in Iran is calculated by using 10 interpolation methods, then the obtained results in these regions are compared. Boundary values in three studied regions are the geoid-quasigeoid separation values obtained from the differences between normal and orthometric heights. Then, the geoid-quasigeoid separation values in internal points of each region are calculated by using the interpolation methods. After comparing the separation values obtained through the interpolation methods in the internal points of each studied region with the separation values obtained from the difference between normal and orthometric heights in the same points, it is observed that the best interpolation method is "triangulation with linear interpolation method" in Alborz mountainous region, and "radial basis function" in kavir and Khuzestan regions. Also the "nearest neighbor method" has the least potential for this purpose in these regions.

KEY WORDS: interpolation, geoid-quasigeoid separation, Root Mean Square(RMS), geoidal undulation, height anomaly

1- INTRODUCTION

In physical geodesy, accurate calculation for the separation between the geoid and the quasigeoid is of special importance, for transforming the geoidal undulation (N) to height anomaly (ζ) and/or the orthometric height (H^O) to normal height (H^N) and vise versa (see fig .1). Accordingly we can write [6]:

$$H^O + N = H^N + \zeta \tag{1}$$

or

$$N - \zeta = H^N - H^O \tag{2}$$

The orthometric height (H^O) and the normal height (H^N) are defined as follows [5]:

$$H^O = \frac{C}{\bar{g}} \tag{3}$$

$$H^N = \frac{C}{\bar{\gamma}} \tag{4}$$

Where $C = W_0 - W$ is the geopotential number, i.e., the geoid's potential value (W_0) minus the potential value (W) at a computational point P. $\bar{g}$ is the mean actual gravity along the plumb line from the surface of the earth down to the geoid. $\bar{\gamma}$ is the mean normal gravity along the normal from the telluroid to the surface of the reference ellipsoid, or equivalently from the surface of the earth down to the quasi-geoid.

In recent years, several works in this subject were published worldwide. Featherstone and Kirby [3] calculated geoid-quasigeoid separation in Australia using the approximate relation represented by Heiskanen and Moritz [5]. In this calculation, they used Bouguer anomaly which was determined in the respective points of network. Sadiq et.al. [8] calculated the mentioned quantity in Pakistan using the calculation of terms involving first and second order orthometric heights. They showed that the first term in the relation for calculation the geoid-quasigeoid separation, which includes the Bouguer anomaly contains the largest amount, while the second term of this relation, which

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includes the vertical gravity anomaly gradient may be considered only in mountainous regions with ragged topography and can be reached the values in order to centimeters. Flury and Rummel [4] used the effect of topographic masses in an accurate compact formula for calculation of the geoid-quasigeoid separation in two mountainous regions on Alps. Sjöberg [9] presented a strict formula for calculation of the geoid–quasigeoid separation. He demonstrated that his strict formula though differs formally from the formula used by Flury and Rummel [4], the obtained results are matched with their results to a satisfactory extent. Mehramuz et.al. [7] calculated the separation for two mountainous and two flat regions in Iran, using the leveling data and adjusted gravity values. Also they determined the respective quasigeoid using the geoidal undulation obtained by applying EGM 2008 Global Geopotential Model in these regions.

Interpolating geoid-quasigeoid separation values in different regions, particularly, in regions where there is little terrestrial data due to lack of facilities and inaccessibility is of great importance. Different studies have been performed in this subject. For example Erol and Celik [2] determined local geoid using GPS/leveling data with "Kriging" and "inverse distance" interpolation methods in Turkey (Izmir region). Chi-Shung et.al. [1] compared different interpolation models, in the case of topographic height. Zhang and Wei [10] compared four interpolation methods including "inverse distance", "linear interpolation", "Shepard interpolation", and "Chebyshev interpolation" . They showed that "Chebyshev interpolation method" has more stability and accuracy for quasigeoid determination.

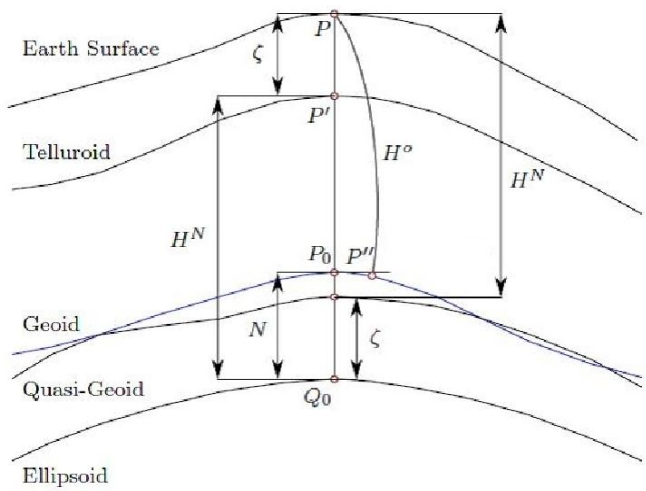

Figure 1 : Geoidal undulation(N) , height anomaly(ζ) , normal height(H^N) , orthometric height(H^O) and the surface of telluroid , quasi-geoid ,geoid and reference ellipsoid

In this research, geoid – quasigeoid separation values are calculated in the internal points of the studied regions using 10 interpolation methods. For this purpose, boundary separation values in these regions are considered as the separation values obtained from the differences between normal and orthometric heights. Finally, interpolated separation values in these internal points are compared with the previously determined separation values in the same points.

2- Determination of geoid-quasigeoid separation using 10 interpolation methods

In this research, in the first step, the orthometric height (H^o) and normal height (H^N) are calculated by relations (3) and (4) in the points of Iranian National Cartographic Center(NCC) (see figure.2), then, in these points the geoid-quasigeoid separation values are obtained from the differences between the calculated normal and orthometric heights ($H^N - H^O$) (see figure.3). then, three regions are selected in size of ($1^\circ \times 1^\circ$) (see figure.4) and considering the resolution of the defined regular network in these regions ($5' \times 5'$), the network in all three under-study regions includes 121 internal points and 48 boundary points (see figure.5). Statistical data related to these regions are presented in table 1.

Table 1.Statistical data related to the three under-study regions

Region	Longitude(λ°)	Latitude(φ°)	Mean elevation of the region ($\bar{H}$(m))
Alborz	$51 \leq \lambda \leq 52$	$35 \leq \varphi \leq 36$	1443
Kavir plain	$54 \leq \lambda \leq 55$	$34 \leq \varphi \leq 35$	769
Khuzestan	$48 \leq \lambda \leq 49$	$30 \leq \varphi \leq 31$	3

By calculating orthometric and normal heights in these points, it is possible to determine geoid-quasigeoid separation ($H^N - H^o$) in 169 points in each of these regions. Now, the respective quantity can be calculated in all internal points (121 points) in these regions by using the interpolation of the boundary geoid-quasigeoid separation values (48 values). The applied interpolation methods are: Inverse distance to a power , Kriging , Minimum curvature , Natural neighbor , Nearest neighbor , Polynomial regression , Radial basis function , Triangulation with linear interpolation , Moving average , Local polynomial .
The results are shown in figures 6, 7 and 8.

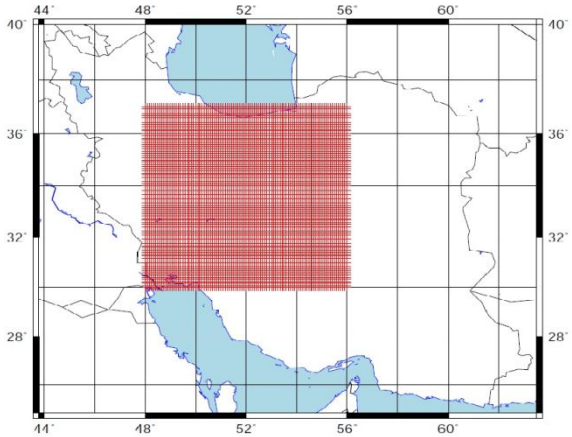

Figure 2: Distribution of the points of NCC network in Iran

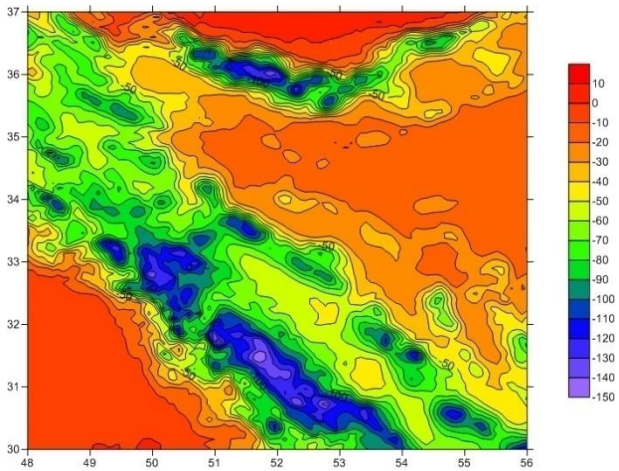

Figure 3: Map of the separation between geoid and quasi-geoid in centimeter obtained from the differences between normal and orthometric heights in the points of NCC network in Iran (Contour Interval is 10 cm.)

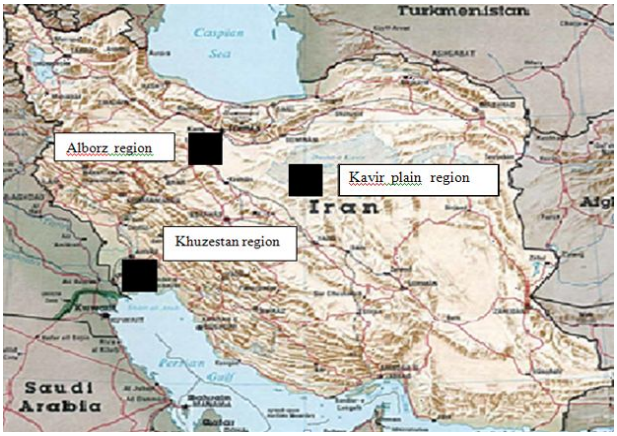

Figure 4: Position of Alborz, Kavir plain and Khuzestan regions in Iran

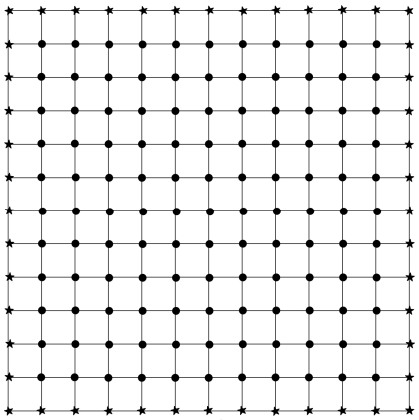

Figure 5: Location of boundary and internal points of the used network in the three under-study regions (boundary points: *, internal points: •)

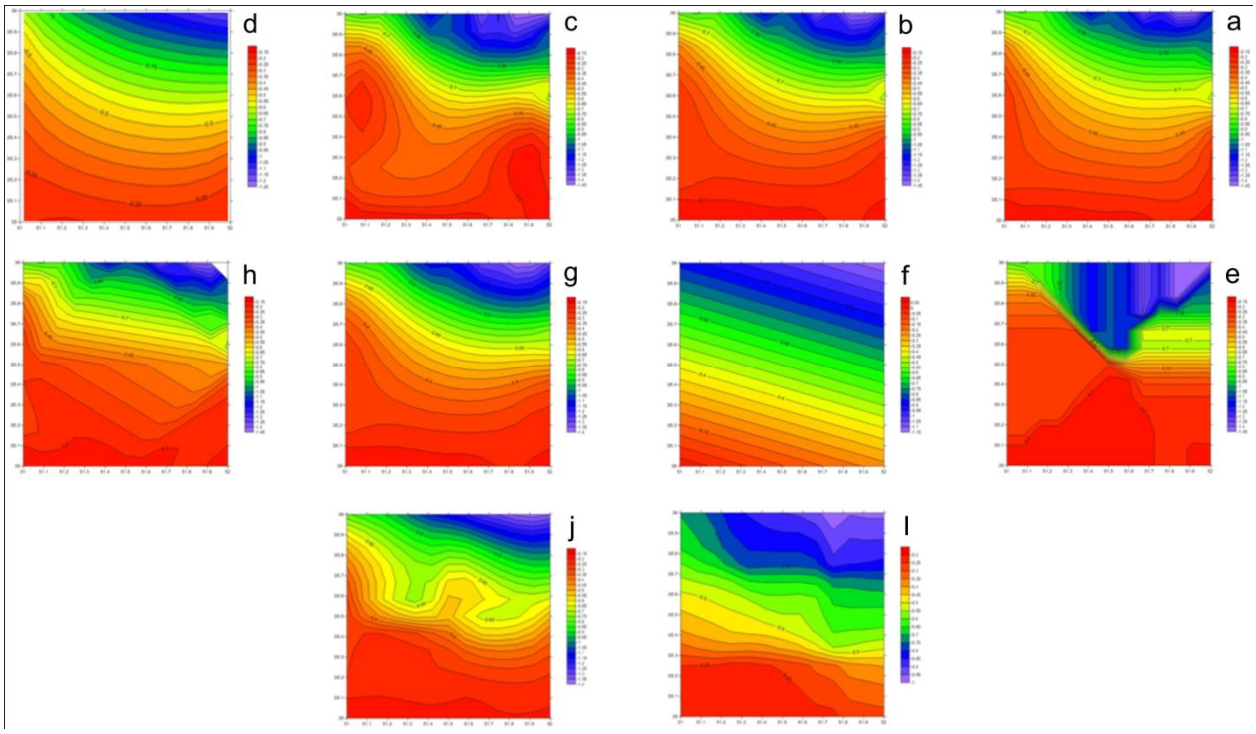

Figure 6: Map of geoid-quasigeoid separation in centimeter in Alborz region obtained by:
a)Inverse distance to a power , b) kriging , c) Minimum curvature , d) Natural neighbor , e) Nearest neighbor , f) Polynomial regression , g) Radial basis function, h)Triangulation with linear interpolation , i) Moving average , j) Local polynomial interpolation method

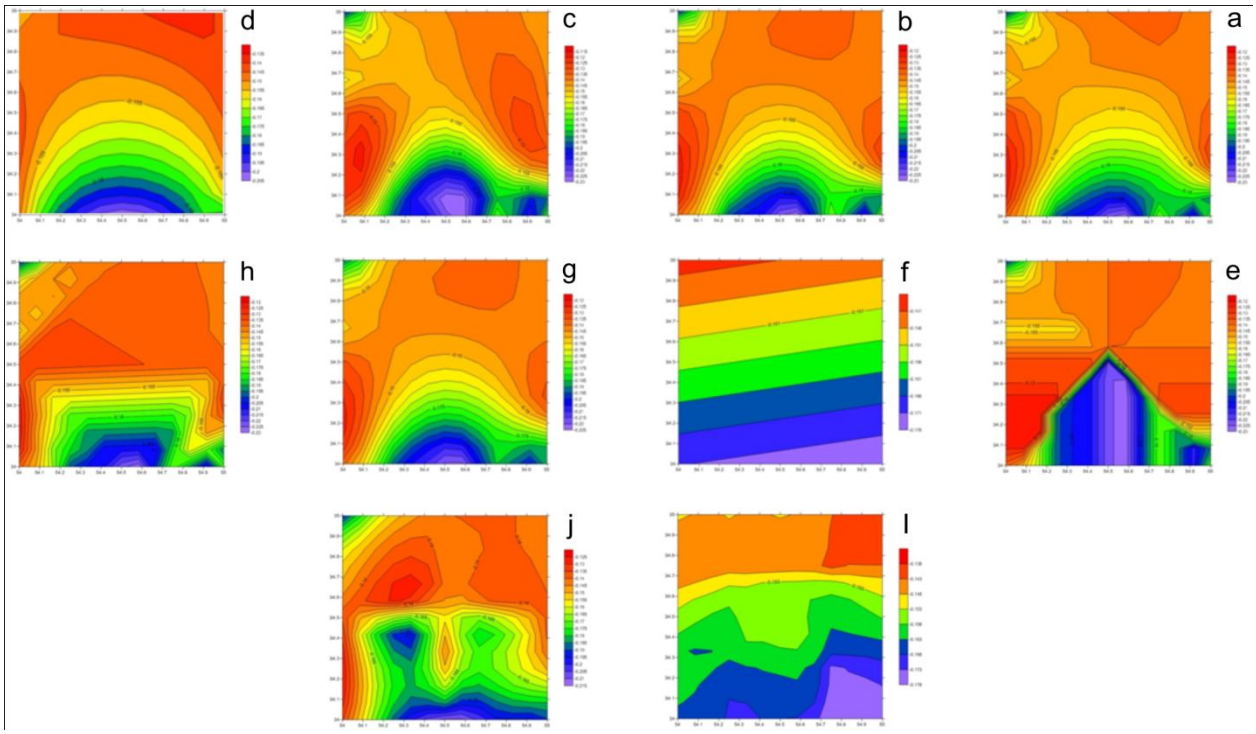

Figure 7: Map of geoid-quasigeoid separation in centimeter in Kavir plain region obtained by:
a)Inverse distance to a power , b) kriging , c) Minimum curvature , d) Natural neighbor , e) Nearest neighbor , f) Polynomial regression , g) Radial basis function, h)Triangulation with linear interpolation , i) Moving average , j) Local polynomial interpolation method

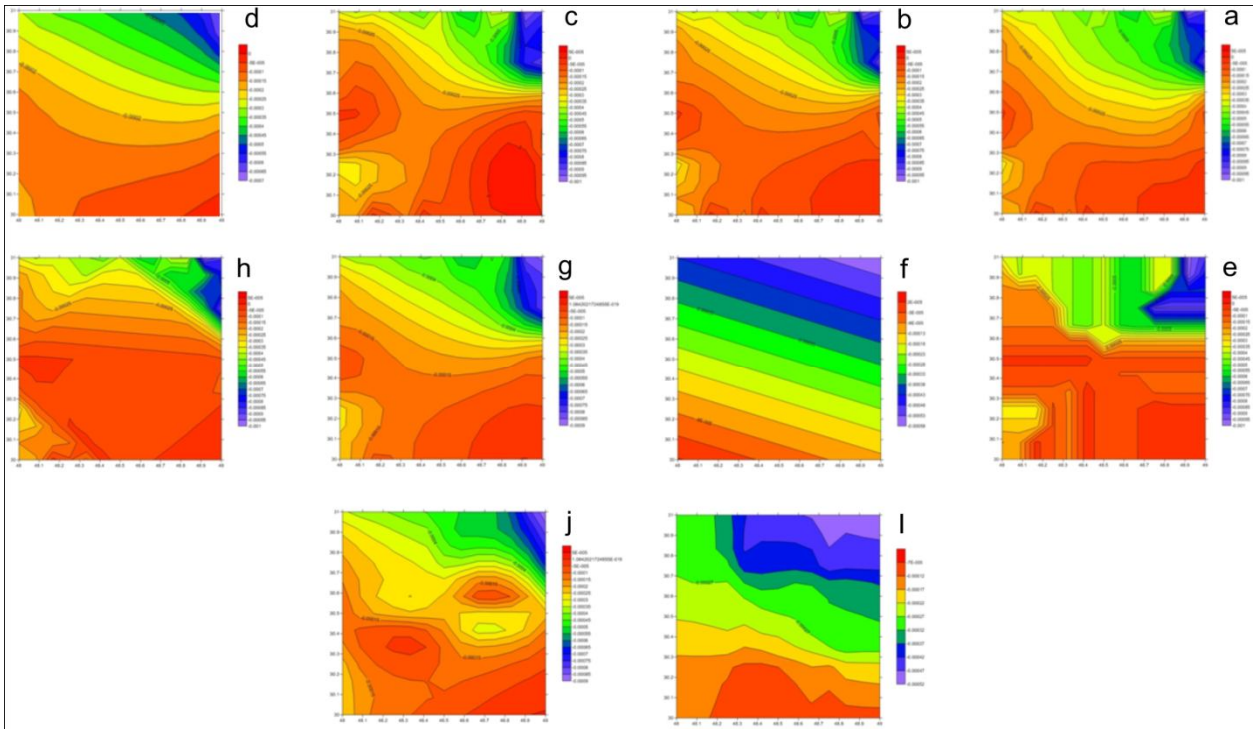

Figure 8: Map of geoid-quasigeoid separation in centimeter in Khuzestan region obtained by:
a)Inverse distance to a power , b) kriging , c) Minimum curvature , d) Natural neighbor , e) Nearest neighbor , f) Polynomial regression , g) Radial basis function, h)Triangulation with linear interpolation , i) Moving average , j) Local polynomial interpolation method

3- Comparing the interpolation methods in three studied regions

In this step, in order to compare the interpolation methods which are used in the mentioned regions**,** minimum, maximum, mean, and Root Mean Square (RMS) of the difference between geoid-quasigeoid separation values obtained by using the interpolation methods and those obtained from the differences between normal and orthometric heights in the internal points of three studied regions are calculated (Tables 2, 3, 4)

Table 2: Minimum, maximum, mean, and Root Mean Square (RMS) of the difference between the geoid-quasigeoid separation values calculated by using the interpolation methods and those obtained from the differences between normal and orthometric heights in the internal points of Alborz region

INTERPOLATION METHOD	Min(cm)	Max(cm)	Mean(cm)	RMS(cm)
Inverse Distance to a Power	-15.46	41.35	15.07	18.75
Kriging	-15.75	40.63	13.28	17.47
Minimum Curvature	-18.49	44.72	12.99	18.25
Natural Neighbor	-17.58	39.68	13.58	17.70
Nearest Neighbor	-28.11	89.82	12.99	26.31
Polynomial Regression	-12.58	40.40	16.01	21.72
Radial Basis Function	-16.07	40.59	13.26	17.44
Triangulation with Linear Interpolation	-10.44	31.48	9.86	13.67
Moving Average	-27.29	37.36	15.07	21.33
Local Polynomial	-19.71	46.91	13.49	19.14

Table 3: Minimum, maximum, mean, and Root Mean Square (RMS) of the difference between the geoid-quasigeoid separation values calculated by using the interpolation methods and those obtained from the differences between normal and orthometric heights in the internal points of Kavir plain region

INTERPOLATION METHOD	Min(cm)	Max(cm)	Mean(cm)	RMS(cm)
Inverse Distance to a Power	-0.11	3.29	1.03	1.33
Kriging	-0.17	3.23	0.75	1.18
Minimum Curvature	-1.39	4.87	0.67	1.65
Natural Neighbor	-0.34	3.51	1.06	1.39
Nearest Neighbor	-1.95	8.04	1.22	2.87
Polynomial Regression	-2.16	4.17	1.01	1.54
Radial Basis Function	-0.19	3.21	0.74	1.17
Triangulation with Linear Interpolation	-1.67	4.50	0.63	1.59
Moving Average	-2.68	3.86	1.06	1.58
Local Polynomial	-1.69	5.08	0.98	1.79

Table 4: Minimum, maximum, mean, and Root Mean Square (RMS) of the difference between the geoid-quasigeoid separation values calculated by using the interpolation methods and those obtained from the differences between normal and orthometric heights in the internal points of Khuzestan region

INTERPOLATION METHOD	Min(cm)	Max(cm)	Mean(cm)	RMS(cm)
Inverse Distance to a Power	-0.017	0.032	0.007	0.012
Kriging	-0.019	0.025	0.003	0.00894
Minimum Curvature	-0.028	0.035	0.003	0.0104
Natural Neighbor	-0.018	0.023	0.003	0.0090
Nearest Neighbor	-0.023	0.046	0.005	0.015
Polynomial Regression	-0.025	0.029	-0.008	0.0139
Radial Basis Function	-0.019	0.025	0.003	0.00893
Triangulation with Linear Interpolation	-0.029	0.019	-0.002	0.0101
Moving Average	-0.026	0.026	0.008	0.0129
Local Polynomial	-0.030	0.031	0.004	0.0125

The results obtained from figures 6, 7, 8 and tables 2, 3, 4 show that the interpolation methods in Kavir plain and Khuzestan regions have more accurate results. In addition to, based on the calculated Root Mean Square (RMS) values in these regions, it is found that, in Alborz mountainous region “Triangulation with linear interpolation” (with RMS=13.67 cm) and in Kavir plain and Khuzestan regions, “radial basis function” (with RMS=1.17 and 0.00893 cm respectively) are the best interpolation methods for calculation of the geoid-quasigeoid separation. Moreover “nearest neighbor” (with RMS = 26.31, 2.87, and 0.015 cm in Alborz, Kavir plain and Khuzestan regions respectively) leads to the weakest results in the above-mentioned regions.

4-Conclusion

In this article, geoid-quasigeoid separation values are calculated in the internal points of three regions including Alborz mountainous region, Kavir plain region and Khuzestan region by interpolating the boundary geoid-quasigeoid separation values obtained from the differences between normal and orthometric heights. Then, the separation values obtained from 10 interpolation methods, in the internal points of these regions are compared with the separation values directly calculated from the differences between normal and orthometric heights in the same points.

The results of this study are summarized as follows:

- The results illustrated in table 2 show that it is not suitable to calculate geoid-quasigeoid separation by using the interpolation methods in mountainous regions, instead, such relations as the relation represented by Flury and Rummel [4] or the strict formula represented by Sjöberg [9] should be used for this purpose in the regions with ragged topography.
- Results from tables 2, 3 and 4 illustrate that, since using “radial basis function” and “Kriging” methods leads to an insignificant value for Root Mean Square (RMS), both methods are worth to be used in three

studied regions while "nearest neighbor" method has the least efficiency for this purpose. The reason is that Root Mean Square (RMS) for this method is considerable in the studied regions.

- Results from figures 6, 7 and 8 and statistical analysis represented in tables 2, 3 and 4 show that the results obtained by applying interpolation methods depend on different parameters. So, generally, no interpolation method can be considered as the best method. In fact, in order to select the best interpolation method, conditions of each region should be analyzed in detail. For example in this research , the results obtained by applying interpolation methods depend on several parameters such as the dimensions of the respective region, the number of boundary and internal points of the region, the distance between network points, and the topography of the region. It should be noted that any change in the mentioned parameters causes the change in the results obtained by applying interpolation methods.

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