J. Basic. Appl. Sci. Res., 2(8)7454-7458, 2012
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ISSN 2090-4304
Journal of Basic and Applied Scientific Research
www.textroad.com

Value-at-Risk and Extreme Value Distribution for Financial Returns of Pakistani Firms

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ABSTRACT

This paper is first effort to measure the stock market risk of Pakistani financial data by modern risk management tools like extreme value theory. We compare the results from this modern tool with Basel accord proposed risk measure and found former method better one. We believe that using better risk measuring tools helps to manage the stock risk in a better way. This boosts the investor's confidence and economic growth as well.
KEYWORDS: Financial Econometrics, Financial risk management, Financial Time Series.

1. INTRODUCTION

This study is the direct consequence of the evidences of the instability of financial system like the late global crises of October 2008, which renders the existing financial models absurd(Nawaz & Afzal, 2011). Allan Green Span explained this crisis as "Tsunami of the Financial Markets that occurs once in a Century".

Stock market is the backbone of an economy(Aghabaki, Molaei, & Maleki, 2012).In Pakistani stock exchanges normally three techniques are used to measure stock risk. These are variance-covariance, Risk Metrics and historical simulations. The issue is that these techniques have one assumption in common that they assume that stock price follows Gaussian distribution. We need such a risk management system that can cope with turmoil events that normally have serious consequences. These techniques failed in the financial crises. This motivates us to use the extreme value theory which has strong foundation in mathematical modeling.

This study observes the tails of Pakistani stock returns through extreme value theory. This theory is a modern risk management tool.

2. Risk Measures

We will discuss two risk measures Value-at-Risk (VaR) and the return level.

VaR is a method of assessing risk that uses standard statistical techniques used routinely in other technical fields. In other words VaR summarizes the worst losses over a target horizon that will not be exceeded with a given level of confidence (Jorion, 2007).Mathematically VaR is defined as $VaR_p = F^{-1}(1-p)$. In this equation F^{-1}can be given as the inverse of underlying distribution.

Return level can be defined as $R_n^k = H^{-1}(1-1/k)$. Return level is the expected level to be exceeded in one out of *k* period. Each period will be of length *n*.

3. Extreme Value Theory

Extreme Value Theory (EVT) has two important results. One is that extrema of a given series under certain conditions converges to the Gumbel, Frechet or Weibull distribution. These distributions have a standard form called generalized extreme value (GEV) distribution.

Second result relates to the modeling of excess return over a given threshold. It has been observed that the limiting distribution of theses excess returns will follow a generalized Pareto distribution (GPD) for a high threshold. EVT is used to estimate at a very high quantile say for 99.9% and above.

3.1 The GEV distribution(Fisher & Tippett, 1928), (Gnedenko, 1943) result

Let X_i denotes a sequence of random variables and $\{X_1, \ldots, X_n\} \sim iid$ (*iid* stands for independent and identically

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distributed) such that the Maxima $X_n = Max\left(X_1, \ldots, X_n\right)$ converges to

$$\mathbf{H}_{\xi,\mu,\sigma}(x) = \begin{cases} e^{-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{\frac{-1}{\xi}}} & \text{if } \xi \neq 0 \\ e^{-e^{\frac{-(x-\mu)}{\sigma}}} & \text{if } \xi = 0 \end{cases}$$

$$\text{While} \quad 1 + \xi\frac{(x-\mu)}{\sigma} > 0.$$

This implies that the asymptotic distribution of the maxima always belongs to the GEV distribution whatever the original distribution would be. The parameter ξ ,is called the tail index, indicates the tail thickness of the given distribution. The parameters μ and σ corresponds to the scalar and tendency respectively.

When the $\xi = 0$, then the distribution H will be a Gumbel type. When the $\xi < 0$ then the H will be Weibull distribution and for $\xi > 0$, the H will be Frechet distribution. Since financial data is a fat tail data, the Frechet distribution has been found to be the most appropriate to the fat-tailed financial data (Embrechts, Klüppelberg, & Mikosch, 1999).

3.2 The excess beyond a threshold (Pickands, 1975), (Balkema & De Haan, 1974) result

The second result of EVT involves the estimation of the conditional distribution of the excess beyond a high threshold. Let X be a random variable with a distribution F and a threshold u such that $u > x_F$. then F_u will be the distribution of excesses of Xover u. This can be show in the figure 1.

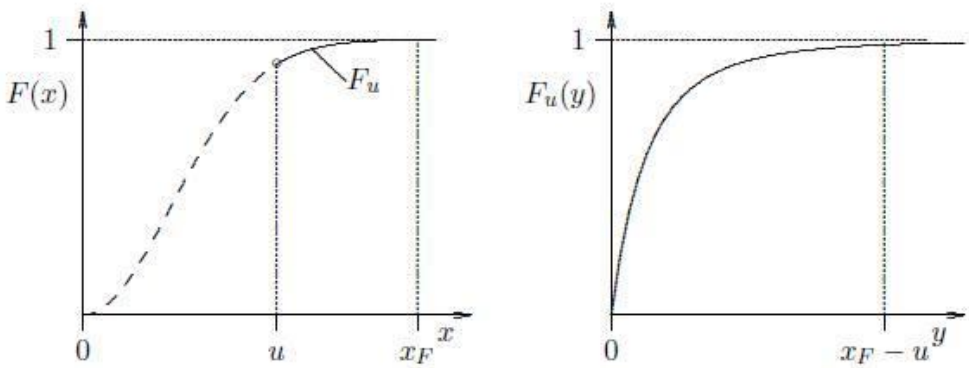


Fig 1. Distributional function *F* and conditional distribution function F_u.

The F_u is said to be the conditional excess distribution function

$$F_u(y) = P\left(X - u \leq y \mid X > u\right), \quad 0 \leq y \leq x_F - u$$

Where X is a random variable, u is the threshold, $y = x - u$ are the excesses. The right end point of the given distribution $F$is $x_F \leq \infty$.

Once the threshold is estimated as a result of VaR calculation, the conditional distribution F_u will be approximated as a GPD. Then we can write:

$$F_u(y) \approx G_{\xi,\sigma}(y), \; u \to \infty,$$

Where:

$$\mathbf{G}_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma}y\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

This distribution *G*will be model excess beyond a given threshold u, where u is supposed to be sufficiently large to satisfy the condition $u \to \infty$(Castillo & Hadi, 1997).

4. Modeling the Fat Tails of Stock Return

We are going to model the tail of the daily KSE-100 (Karachi Stock Exchange) index returns for the period from 29–06–1993 to 27–05–2009.Figure 2 shows the plot of the *n* = 3,762 observed daily returns.

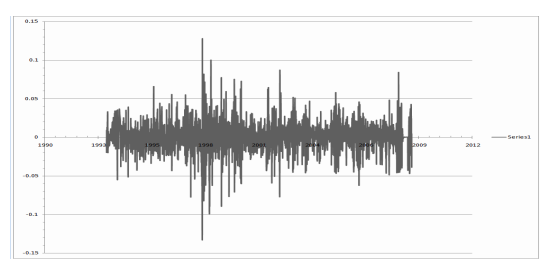


Fig 2: Daily returns of the KSE 100 index.

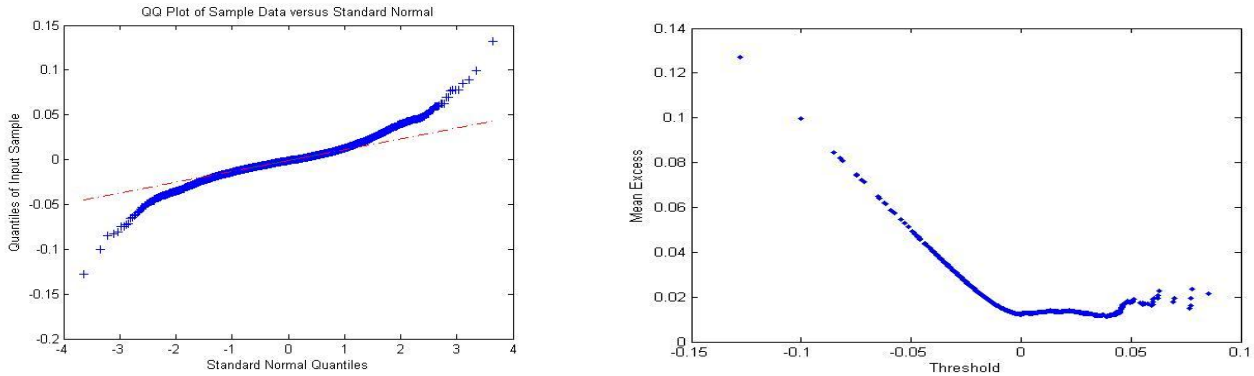
4.1 EXPLANATORY DATA ANALYSIS

The main explanatory tools used in EVT (Extreme Value Theory) are the quantile –quantile plot (QQ plot) and the sample excess plot (ME plot).

As QQ plot is not straight that means that the underlying distribution is a fat-tailed distribution. This is the case of the QQ plot of all the KSE returns against the normal (Figure 3a).

Sample mean excess plot is another graphical tool for selection of the threshold. It will be linear in the case of GPD. Fig 3b shows this plot corresponding to our data.

Now first we consider Peaks over Threshold (POT) method and then method of block maxima.



(a) (b)

Fig 3: (a) QQ plot against normal, (b) Sample mean excess plot.

4.2 The POT Method

The distribution beyond the threshold is show in the Figure (6a), which is a generalized Pareto distribution.The maximum likelihood estimates for GPD are $\hat{\xi} = 0.0502$and $\hat{\sigma} = 0.0120$.Putting p=0.01, in the following equation,

$$\mathsf{VaR_p} = \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left\{\left(\frac{n}{N_u}p\right)^{-\hat{\xi}} - 1\right\}$$

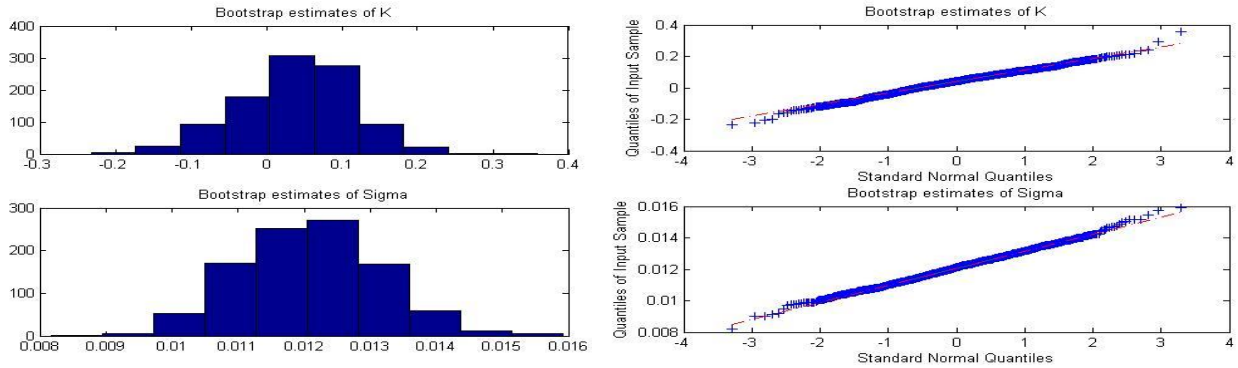
gives the resultant VaR=0.0493. Where n is the total number of observations, N_u is the number of observations beyond the threshold u.McNeil and Saladin (1997) method discussed this method in detail.

The estimates from the point, the maximum likelihood (ML) andthe bootstrap (BS) confidence intervals are presented in Table 1.

Table 1: For the POT method point estimates and 95% ML and BS confidence intervals.

	Lower Bound		Point Estimate	Upper Bound	
	BS	**ML**	**ML**	**ML**	**BS**
xi	-0.0797	-0.0803	0.0502	0.1808	0.2167
sigma	0.0101	0.0098	0.0120	0.0145	0.0142
VaR	0.0445	0.0440	0.0493	0.0563	0.0566

Also the parameter standard errors are obtained by asymptotic covariance matrix alternatively.Which is 0.666 and 0.0012 for ξ, σrespectively.



(a) (b)

Fig 4: (a) QQ plot for estimates of tail index (b) and $\log(\sigma)$.

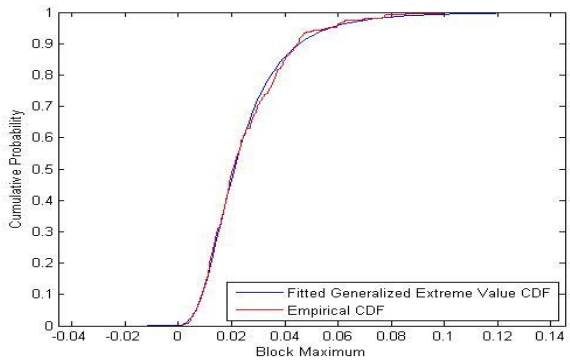
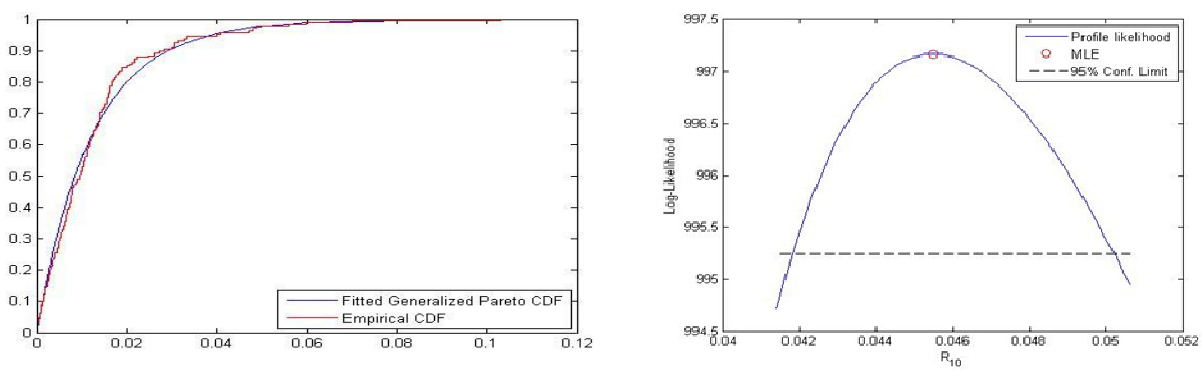


Fig 5: Sample distribution of maxima corresponding to fitted GEV.

Also we use bootstraps simulation method to generate 1,000 replicates of datasets through resampling from data. GP distribution is to be fitted on each one. The estimates then were saved. The resultant histogram of these bootstraps replicates are shown in figure (4a). It shows that estimates of *k* are a bit asymmetric while estimates of sigma are skewed to right. For further probing into sigma estimates, we estimate the parameter and standard error on log scale. The resultant bootstrap estimates for ξ, $\log(\sigma)$ are approximately normal as shown in figure (4b).



(a) (b)

Fig 6: (a) GPD fitted to the exceedances above the threshold 0.0292, (b) Relative profile log likelihood and 95% confidence interval for estimated R^{10} estimated with the method of block maxima.

4.3 Method of Block Maxima

The procedure for EVT is that first of all, the given sample is divided into 345 overlapping subsamples, say a period of one week as in our case. The absolute value of minima out of each subsample is collected. The limiting distribution out of theses normalized extrema will be the standard GEV distribution.

The log-likelihood estimates we obtain are $\hat{\xi} = 0.1672$, $\hat{\sigma} = 0.0104$, and $\hat{\mu} = 0.0170$. . The resultant GEV distribution of our sample data is shown in Fig (5).

As the Return Level is a measure as;

$$\mathbf{R}^k = H^{-1}_{\xi,\sigma,\mu}\left(1 - \frac{1}{k}\right)$$

Putting the value of estimates of ξ and $\sigma$by log-likelihood method, we have

$$\widehat{\mathbf{R}}^k = \begin{cases} \hat{u} - \frac{\hat{\sigma}}{\hat{\xi}}\left(1 - \left(-\log\left(1 - \frac{1}{k}\right)\right)^{-\xi}\right) & \text{if } \xi \neq 0 \\ \hat{u} - \hat{\sigma} log\left(-\log\left(1 - \frac{1}{k}\right)\right) & \text{if } \xi = 0 \end{cases}$$

Here if we assume *k* is10, then R^{10} will be 0.0455. It means the maximum potential loss for a week will exceed 4.55% in one out of ten weeks on average.

Here we use a likelihood based method instead of asymptotic approximation to find the confidence limit for more accurate results. Figure (6b) gives the plot of the relative profile log-likelihood. In the case, $\alpha = 0.05$, the estimate for R^{10} will be [0.0407*;* 0.0502]. The confidence interval for GEV parameters using 1,000 bootstraps samples are shown in the Table (1).

Table 2: Point estimates and 95% maximum ML and BS confidence intervals.

	Lower Bound		Point Estimate	Upper Bound	
	BS	ML	ML	ML	BS
xi	0.0889	0.0754	0.1672	0.2591	0.2489
sigma	0.0095	0.0095	0.0104	0.0114	0.0115
R10	0.0407	0.0418	0.0455	0.0502	0.0517

5. Conclusion

We presented some methods of EVT to analyze financial data also with the scope of illustrating how powerful these methods are.

Let us have a comparison of VaR from POT method and VaR proposed by Basel accord. By the assumption of normality until 1989 with one percent lower quantile yields 1.95. Three times of this figure will be 5.86. Whereas VaR obtained in the upper bound in POT method is equal to 4.93 (Table 1). Hence POT method gives the more accurate results.

This analysis is a good starting point and it demonstrated that EVT could play an important role in the field of risk management.

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