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Electromagnetic Field Computation Method in Electromagnetic Detection by Using Finite Elements and Boundary Elements Methods

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ABSTRACT

In this paper a method of combining finite elements and boundary elements is proposed to try to solve opened three-dimension eddy field in electromagnetic detection, which subdivide the opened region into closed inner sub-region and opened outer infinite sub-region by defining a suppositional boundary. Then using Finite Elements Method (FEM) and Boundary Elements Method (BEM) in above two sub-region respectively, the A −Φ and A equations restricted by Coulomb criterion are derived. On suppositional boundary, direct coupling method is adopted to combine the FEM equations and BEM equations; therefore the solution of the entire region can be obtained. At last, the three-dimension magnetic field in cable eccentricity detection is computed, and magnetic field distribution and eddy loss are given out. The results show that the magnetic field has a reasonable distribution, and also that the linear relation between the computed results and the value of eccentricity has good coherence with that between the experiment results and the value of eccentricity.

Keywords- Algorithm FEM, BEM, suppositional boundary, opened electromagnetic field.

1. INTRODUCTION

It is difficult to solve the magnetic field in magnetic detection because the field is opened three-dimension domain, and the difficulty is that the computation scale is very large and the infinite boundary cannot be gotten easily, otherwise, this problem is very important to the research and analysis of magnetic detection and the optimization of the configuration, size, material selection and exciting signal frequency of the magnetic sensor.

Now, the method of dealing with this problem is to adopted FEM and a suppositional boundary is proposed to subdivide the infinite region into finite sub-regions, for this the magnetic field is supposed to be ignored out of the suppositional boundary and the value of magnetic field on this boundary is zero. Because of that, this method is called as truncation method. Although the simplicity of this method, some truncation errors will appear because the suppositional boundary has not good coherence with the practical problem [1].

Thanks to the adoption of basis solution meeting radial conditions essentially, BEM makes the contribution to the solution of infinite boundary disappeared and transfers the infinite boundary problem into finite boundary problem, and the integral on the entire domain is transferred into that on finite boundary, so the only boundary of region is need to be discretized, and once the boundary conditions are defined, the quantities of the region can be gotten. [2] Because of characteristics of FEM and BEM, the method of coupling FEM and BEM is a kind of efficiency method.

2. EQUATION OF THREE-DIMENSION EDDY FIELD

Electric displacement can be neglected for time-variation sinusoidal electromagnetic field with middle and low frequency, so Maxwell equations of eddy field are shown as below:

$$\nabla \times H = J_e + J_S \qquad (1)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad (2)$$

$$\nabla . B = 0 \qquad (3)$$

$$\nabla . D = 0 \qquad (4)$$

Where, J_e - eddy current density
and, Js - exciting current density.
media equations are shown as below:

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$$B = \mu H \quad (5)$$
$$J = \sigma E \quad (6)$$
$$D = \omega E \quad (7)$$

Electrical and magnetic continuous conditions on different media interface are shown as below:

$$B_1.n_{12} = B_2.n_{12} \quad (8)$$
$$H_1 \times n_{12} = H_2 \times n_{12} \quad (9)$$

3. DEALING WITH DOMAIN

The electrical and magnetic field region model is shown in figure 1. In order to coupling BEM and FEM, and avoid subdividing the finite element region and boundary element region by natural conditions in eddy region mentioned in papers [3, 4, 5, 6, 7, 8, 9, 10], the coupling conditions are need to be configured and the opened sub-region must contain only one media and cannot have exciting sources, because this will make the equations unsymmetrical ranks too high. So a suppositional boundary Γ_0 is set up in order to subdivide the entire region into a closed sub-region, which contains several kinds of Medias, and an opened sub-region, which contains only one kind of media. Finite element equations are set up in inner sub-region, otherwise boundary element equation in outer sub-region.[11,12,13]

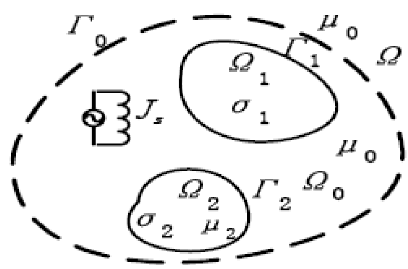

Fig.1 Model of field region

4. EQUATIONS ABOUT A - Φ AND A RESTRICTED BY COULOMB CRITERION

The method of using A −Φ and A is adopted, and that means magnetic vector A and magnetic scalar Φ are used in eddy sub-region Ω1 and only magnetic vector A is used in non-eddy sub-region ($\Omega_0 + \Omega - \Omega 1$). In order to get one and only solution of magnetic vector A, coulomb criterion $\nabla . A = 0$ is introduced, and some equation can be gotten from equations (1) − (7).

4.1 Differential equations in finite element sub-region

In Ω_1

$$\left.\begin{aligned} &\nabla \times \frac{1}{\mu} \nabla \times A + \sigma(j\omega A + \nabla \phi) = 0 \\ &\nabla . \sigma(j\omega A + \nabla \phi) = 0 \end{aligned}\right\} \quad (10)$$

and in Ω_2

$$\nabla \times \frac{1}{\mu} \nabla \times A = J_S \quad (11)$$

and in $\Omega_0 - \Omega_1 - \Omega_2$

$$\nabla \times \frac{1}{\mu} \nabla \times A = 0 \quad (12)$$

For the convenience of writing computation programs, the first equation in (10), (11) and (12) are expressed in united form, so equation (10) and (11) are rewritten like below:

In Ω_0

$$\nabla\times(\frac{1}{\mu}\nabla\times A)-\nabla(\frac{1}{\mu}\nabla.A)+j\,\omega\sigma A+\sigma\nabla\phi-J_S=0 \quad (13)$$

and in Ω_1

$$\nabla.\sigma(j\,\omega A+\nabla\phi)=0 \quad (14)$$

For equation (13), J_s and σ are defined alternatively in different sub-region, and in non-eddy sub-region, σ = 0 and in exciting sources sub-region, $J_s \neq 0$.[14, 15]

4.2 Integral equation in opened sub-region

Green function in three-dimension space is introduced:

$$G=\frac{1}{4\pi r} \quad (15)$$

Where, r - the distance between the considered point and the source point in field.

Equation (15) is weight function, and the method of weighted residual is used in equation (12), so the integral equation in opened region can be gotten.

$$\frac{A}{2}=\iint_S\left[G\,\frac{\partial A}{\partial n}-A\,\frac{\partial A}{\partial n}\right]ds \quad (16)$$

Where, *s* – surface closed by suppositional boundary
and ***n***– unit normal vector.

4.3 Continuous electrical and magnetic conditions on interfaces between different medias

$$A_1=A_2 \quad (17)$$

$$\frac{1}{\mu_1}\nabla.A_1=\frac{1}{\mu_2}\nabla.A_2 \quad (18)$$

$$\frac{1}{\mu_1}\nabla\times A_1\times n_{12}=\frac{1}{\mu_2}\nabla\times A_2\times n_{12} \quad (19)$$

5. ELEMENT EQUATIONS AND BOUNDARY EQUATIONS AND THE COUPLING

5.1 Element equations

By meshing finite sub-region using tetrahedron element and adopting Galerkin discretization method, the element equations can be attained for closed sub-region.

In Ω0

$$\sum_{e_0}\int_{V_e}(\frac{1}{\mu_e}\nabla\times N_l^e.\nabla\times A^e+\frac{1}{\mu_e}\nabla.N_l^e.\nabla.A^e+j\,\omega\sigma_e N_l^e.A^e+j\,\omega\sigma_e N_l^e.\nabla\phi^e-N_l^e.J_s^e)dv=0 \quad (20)$$

and in Ω1

$$\sum_{e_0}\int_{V_e}j\,\omega\sigma_e\nabla N_l^e.(A^e+\nabla\phi^e)dv=0 \quad (21)$$

where, e_0 is the element number, which is related to node *l*, N_l^e is the shape function of element including node *l*, μ_e is the permeability of element and σ_e is the conductivity, V_e is integral region of element.

5.2 Boundary equation and coupling equations

The boundary element integral equation on suppositional boundary Γ_0 is similar to boundary conditions, and BEM gives a discretization algebra equation on suppositional boundary Γ_0 to couple the boundary equations into the element equations for closed inner sub - region Ω_0, and because of this, the solution of entire region ($\Omega_0+\Omega$) can be gotten.

Interface has been discretized into several corresponding area elements before finite element region discretization, and for boundary integral equation, these elements are boundary elements.

Equation (16) is discretized by Galerkin method, and direct coupling method is adopted under the consistency condition and the equilibrium condition $A\big|_{\Omega_0}=A\big|_{\Omega}$ and the equilibrium condition $\left.\frac{\partial A}{\partial n}\right|_{\Omega_0}=\left.\frac{\partial A}{\partial n}\right|_{\Omega}=0$ to get boundary element equations by boundary integral equations (magnetic vector A of boundary is linear interpolation of A at node, at the same

time, normal derivative of A of boundary elements in boundary integral equation can be gotten directly by derivation of interpolation function of A in finite element region.)[16]
In Ω

$$\sum_{e_0} 2\pi \iint_{S_0} N_i^e \sum_l N_l^e A^e) ds - \sum_{e_0} \iint_{S_0} N_i^e \sum_e \left[\iint_S \sum_l \frac{1}{r} (\nabla N_l^e . n) A^e ds \right] ds -$$
$$\sum_{e_0} \iint_{S_0} N_i^e \sum_e \left[\iint_S \sum_l \frac{1}{r} \nabla (N_l^e A^e . n) ds \right] ds - \sum_{e_0} \iint_{S_0} N_i^e \sum_e \left\{ \iint_S \sum_l \frac{1}{r} \nabla . (N_l^e A^e) n ds \right\} ds -$$
$$\sum_{e_0} \iint_{S_0} N_i^e \sum_e \left[\iint_S \frac{r.n}{r^3} \sum_l (N_l^e A^e) ds \right] ds = 0 \qquad (22)$$

where, e_0 is the number of boundary elements, which is related to node *i*, *e* is representative for the number of all boundary elements, N_i^e and N_l^e are shape functions of boundary element at node *i* and finite element at node *l*, s_0 and s is representative for the integral region, and *n* is unit vector, which direction is outer normal of boundary.

6. PRACTICAL COMPUTATION

The probe model in electro eddy detection apparatus of cable eccentricity is shown in figure 2. In the model, M is the support structure of the probe with a outer diameter of 14mm, and N is the coils of probe with 30 turns, and L is the cable core with the diameter of 1mm and the resistivity of 1.724×10^{-8} Ωm. Relative permeability of M is 800 and the insulation thick is 2×10^{-3} m. 1×10^{-3} A current flows in the cable core with the frequency of 3×10^{-3} HZ.

The opened magnetic field was computed by FEM and BEM coupling method in cable eccentricity detection. When cable core has no eccentricity, three dimensions magnetic flux density distribution in probe is shown in figure 3, and figure 4 shows three dimensions magnetic flux density distribution in cable core. Then eddy current density in cable core is shown in figure 5. When the core has eccentricity in the direction of axis Y, eddy loss in core changes with eccentricity distance, and the variation curve is shown in figure 6. At this time, the origin of coordinate system is in the center of cable core having no eccentricity, and in figure 2 direction of coordinate system is shown.

Electro eddy detection is based on electromagnetic induction theory. When a nonmagnetic conductor is in alternate magnetic field, an eddy field induced by eddy current in conductor, which resists the change of main magnetic field, and the eddy magnetic field depends on the value of eddy current. And when the distance between exciting source and nonmagnetic conductor gets longer, the eddy current density will be reduced. So the eddy magnetic field induced by eddy current and the general mixed magnetic field will increase, and magnetic flux and reactance will be added. For this reason, output voltage in detection coils will be added. During electro eddy eccentricity detection, the variation of distance is eccentricity. Because eddy loss in cable core can’t be tested, the output voltage induced by eddy current in detection coils is detected in detection test. Test result of output voltage with eccentricity is shown in figure 7. The linear relation between computation results and eccentricity agrees well with that between test results and eccentricity by comparing figure 6 and figure 7.

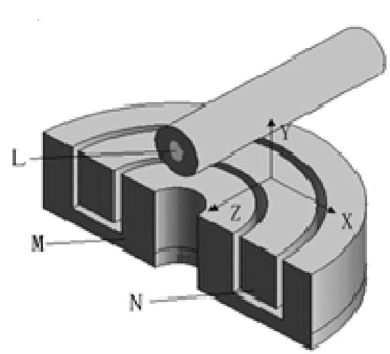

Fig.2 Model of probe

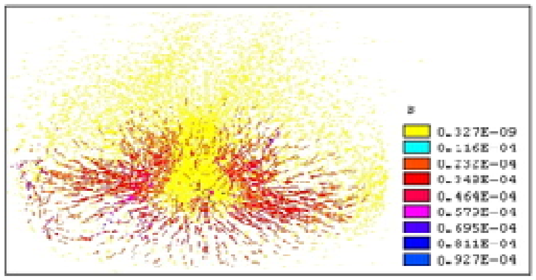

Fig.3 3-D Flux density distribution of probe

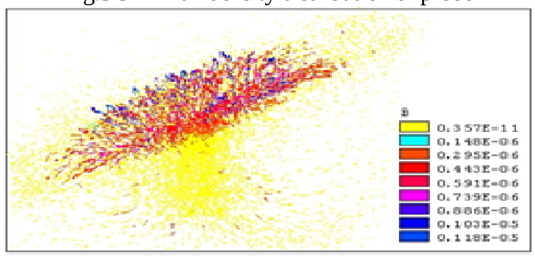

Fig.4 3-D distribution of flux density of cable core

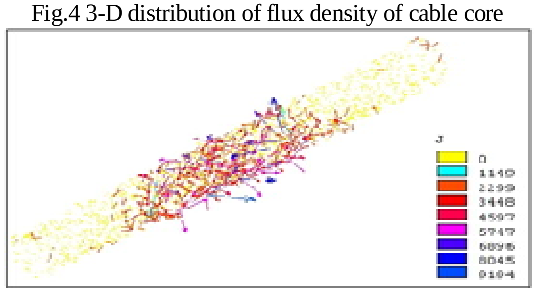

Fig.5 3-D distribution of eddy current density of cable

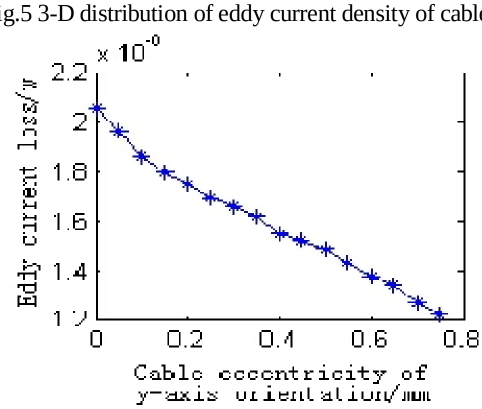

Fig.6 Curve of eddy current loss as cable eccentricity

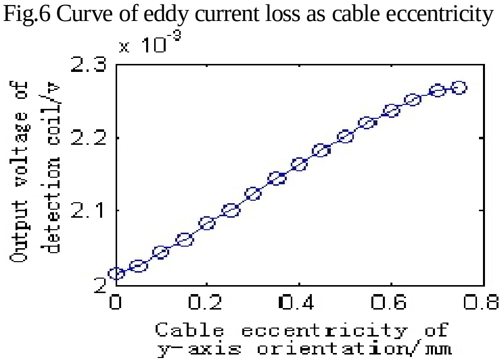

Fig.7 Curve of coil output voltage as cable eccentricity

7. CONCLUSION

In this paper, computation method of three dimensions eddy electromagnetic field with infinite boundary is presented, and suppositional boundary is defined. The infinite region is subdivided into inner sub-region and outer open sub-region, and then finite elements equations and boundary elements equations are respectively set up in these two sub-regions, so the method of coupling FEM and BEM is adopted. This method avoids containing several kinds of medias and sources in opened sub-region and making equations unsymmetrical ranks too high by subdividing the entire region according to natural boundary. Eddy electromagnetic field is computed using this method in cable eccentricity detection. The linear relation between computed results and eccentricity agrees well with that between test results and eccentricity. Therefore, this method is suitable for numerical simulation and computation of magnetic field in electromagnetic sensor, and it can be further developed into a kind of CAM for electromagnetic computation used to optimize the structure and size of electro eddy sensor, selection of material and the frequency of exciting signal.

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