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A New Approach in Analytical Analysis of Eddy Currents in Laminated Core

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ABSTRACT

Conventional methods of eddy-currents modeling are based on solving diffusion equation, which its variable is the magnetic field intensity; and then calculating eddy currents distribution by using the Curl of magnetic field intensity. Using the Curl operator necessitates calculating the derivative of field intensity, which is obtained by some numerical field analysis methods such as Finite Elements Method (FEM). Taking derivative causes many disasters, such as discontinuity in the current density distribution. In this paper, for the first time, diffusion equation is written for the electric field strength as the main variable. Based on the analytical formula derived from the proposed method, the same results are achieved. Simplicity of the proposed method enables us to employ it in numerical analysis such as finite element method.
KEY WORDS: eddy current, laminated core, power loss, skin effect .

INTRODUCTION

Ferromagnetic materials, such as iron, nickel and their alloys are widely used for building cores of magnetic devices [1]. In the design of ac machines and drives, the prediction of power losses in magnetic cores is important [2]. The determination of eddy currents in a given magnetic configuration is always a complicated task. As Depicted in Fig.1, the cores are usually laminated to reduce eddy-current loss, however this loss can still be significant, especially at the higher frequencies produced by pulse-width modulation waveforms in typical ac drives [1].

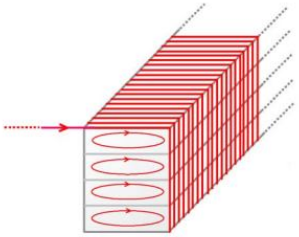

Fig.1: Laminated magnetic core

Design engineers usually predict lamination loss using two methods [2]. One way is to use manufacturer's datasheets. Usually in the datasheet of the core, a cluster of curves can be found, which show the core loss per unit volume under sine-wave excitation versus the maximum flux density for different values of frequency, but such curves are often not available or in practical applications are often different from those under which the datasheets had been obtained. The other method is to use a simple formula for eddy-current loss as a function of lamination thickness, but it is only valid for frequencies of about 60 Hz or less. In the case of most ac drives, significant frequency content in the kilohertz and megahertz regions is required. At such high frequencies, eddy-current loss is much greater than hysteresis loss and, thus, core loss is almost entirely due to eddy currents [2].

The subject of core losses in AC machines and in particular laminations has been a subject of research since the days of Steinmetz [3]. Steinmetz proposed approximate formulas for the harmonic case which are quite similar to those currently used today. His work was empirically derived and supported from a theoretical viewpoint by Bertotti [4]. It is assumed in [3] that the magnetic flux distribution in the sheet is uniform, making the method applicable only in the frequency range where the skin effect is negligible or to the magnetic cores with low conductivities or thin sheets [5]. This latter formulation did not prove accuracy enough which motivated Bertotti to propose an additional term, the excess loss term which accounted for the difference between measurements and theory [6].

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Moreover, in the last decades, a great number of papers have been devoted to obtain approximate eddy-current losses. Conventional methods of modeling eddy-currents are based on solving diffusion equation for magnetic field intensity. The eddy currents distribution is then obtained by taking the Curl of magnetic field intensity. Using the Curl operator necessitates calculating the derivative of field intensity. Taking derivative causes many problems, such as discontinuity in the current density distribution and amplification of the errors [7].

The objective of this paper is to present a new analytical method of eddy current density and eddy current core power loss from Integral form of Faraday's law (Eq.1), that can be used in numerical analysis.

$$\oint E.dl = -\frac{d\phi}{dt} \tag{1}$$

Where *H, J, B, E* are the magnetic field strength, current density, magnetic flux density, and electric field strength respectively.

1.Calculation of eddy current in a single sheet

For a lamination with thickness *d* and height *l* as shown in Fig. 2, and assuming constant magnetic permeability and conductivity, electrical field only has the *z* direction component as a function of *x*.

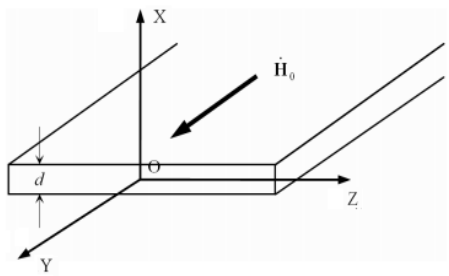

Fig.2: Single lamination and the coordinate system.

Neglecting the displacement current, Maxwell's equations and Ohm's law becomes:

$$\mu\sigma\frac{\partial E_z(x,t)}{\partial t} - \frac{\partial^2 E_z(x,t)}{\partial x^2} = 0 \tag{2}$$

Letting *E(t)* be sinusoidal and assuming the complex exponential form:

$$E_z(x,t) = \mathrm{Re}(E_z(x)e^{j\omega t}) \tag{3}$$

Where ω is the angular frequency. Combining the (Eq.3), (Eq.4) equations, we have :

$$j\omega\mu\sigma E_z(x,t) = \frac{\partial^2 E_z(x,t)}{\partial x^2} \tag{4}$$

Study of Fig.2 will show that, boundary conditions exist for thin sheets with an axially applied field:

$$E_x = E_y = 0 \tag{5}$$

$$\frac{\partial E_z}{\partial y} = \frac{\partial^2 E_z}{\partial y^2} = 0 \tag{6}$$

$$\frac{\partial E_z}{\partial z} = \frac{\partial^2 E_z}{\partial z^2} = 0 \tag{7}$$

$$E_z(x = d/2) = E_0 \tag{8}$$

$$E_z(x = -d/2) = -E_0 \tag{9}$$

Eq. 4 is a standard second order differential equation, which has a general solution and it is convenient to write the solution of differential equation in term of hyperbolic functions.

$$E_z = A\sinh(Tx) + B\cosh(Tx) \tag{10}$$

Where *T* is constant and is equal to:

$$T = (1+j)\sqrt{\pi f \mu\sigma} = \sqrt{j\omega\sigma\mu} \tag{11}$$

Which *A* and *B* being arbitrary constant values. Based on the boundary condition, *E* yields

$$E_z = E_0\frac{\sinh(Tx)}{\sinh(T\frac{d}{2})} \tag{12}$$

There remains only E_0 to be evaluated. Using Faraday's line integral and integrating around the path *ABCD* in Fig.2, yield:

$$\oint E.dl = -\frac{d\phi}{dt} \tag{13}$$

Where ϕ is the total flux within the line *ABCD*. If the height is much greater than the thickness of sheet, the electrical field E_0 in complex notation form is:

$$E_0 = \frac{j\omega\phi}{2l} \tag{14}$$

In the case shown in Fig.2, The magnetic field intensity is described by an ordinary second-order differential equation, which is the Helmholtz equation,

$$\mu\sigma \frac{\partial H_y(x,t)}{\partial t} - \frac{\partial^2 H_y(x,t)}{\partial x^2} = 0 \tag{15}$$

For a sinusoidal applied magnetic strength, Eq.5 becomes.

$$j\omega\mu\sigma H_y(x,t) = \frac{\partial^2 H_y(x,t)}{\partial x^2} \tag{16}$$

Solution of Eq.17 can be found in previous articles and becomes [8]:

$$H_y(x) = H_0 \frac{\cosh(Tx)}{\cosh(T\frac{d}{2})} \tag{17}$$

Where, H_0 is the boundary condition.

$$H_y(\frac{d}{2}) = H_y(-\frac{d}{2}) = H_0 \tag{18}$$

The magnetic field intensity as a function of *x* is shown in Fig.3 for H_0*=1(A/m)*, *σ= 1.42e6* and μ_r*=300* at *f=10, 20* and *50 kHz*. It can be seen that the magnetic field intensity decreases with increasing frequency *f* in the center of the lamination.

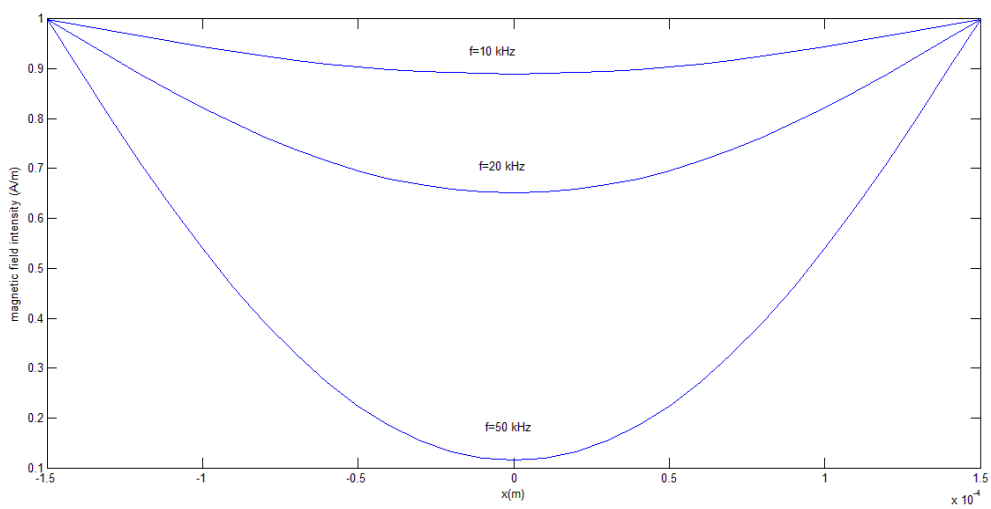

Fig.3: The magnetic field intensity as a function of *x* at *f=10, 20* and *50 kHz*

The magnetic flux flowing through the cross section of a single lamination perpendicular to the *xz* plane is:

$$\phi = \iint_s \mu H_y(x)ds = \int_0^l \int_{-\frac{d}{2}}^{\frac{d}{2}} \mu H_y(x)dxdy = \mu l \int_{-d/2}^{d/2} H_y(x)dx = \frac{\mu l H_0}{\cosh(T\frac{d}{2})} \int_{-d/2}^{d/2} \cosh(Tx)dx$$

$$\phi = \frac{\mu l H_0 \left[\sinh(\frac{Td}{2}) - \sinh(-\frac{Td}{2}) \right]}{T \cosh(\frac{Td}{2})} = \frac{2\mu l H_0}{T} \tanh(\frac{Td}{2}) \tag{19}$$

Substitution of (Eq.20) and (Eq.15) into (Eq.13) and $J = \sigma E$ produces:

$$J_z(x) = \frac{j\omega\sigma\mu H_0}{T} \tanh(\frac{Td}{2}) \frac{\sinh(Tx)}{\sinh(T\frac{d}{2})} = TH_0 \frac{\sinh(Tx)}{\cosh(T\frac{d}{2})} \tag{20}$$

The magnitude of the eddy current density as a function of *x* is shown in Fig.4, for $H_0=1(A/m)$, $\sigma= 1.42e6$ and $\mu_r=300$ at *f=10, 20* and *50 kHz*.

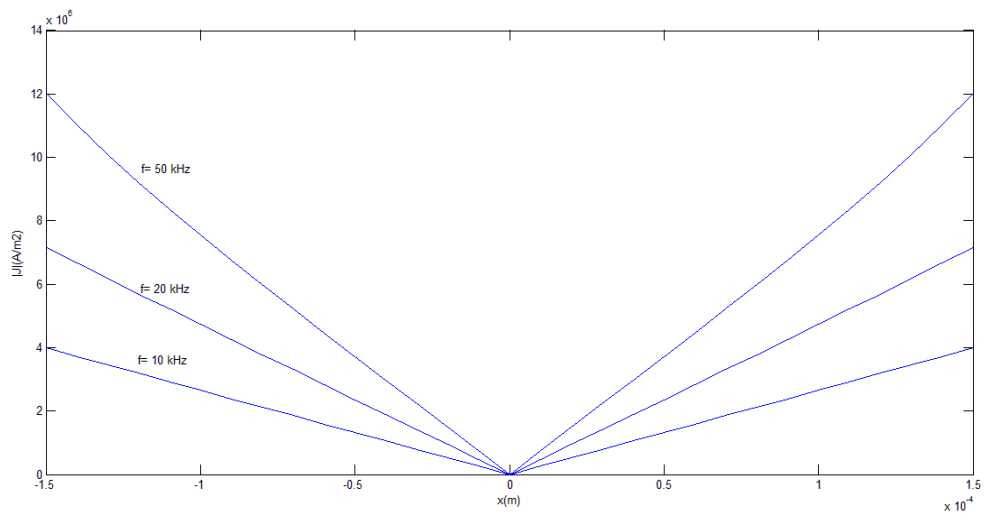

Fig. 4: The magnitude of the eddy current density as a function of *x* at *f=10, 20* and *50 kHz*

2.Eddy current power loss

The eddy current power loss density distribution in a conducting core averaged with respect of time is:

$$P(x) = E.J = \frac{J^2}{\sigma} \qquad (W/m^3) \tag{21}$$

Substitution of (Eq.20) into (Eq.21) produces:

$$P(x) = \frac{1}{\sigma} \left[TH_0 \frac{\sinh(Tx)}{\cosh(T\frac{d}{2})} \right]^2 \tag{22}$$

The average power loss density over the lamination width is the power loss per unit volume, called the specific core loss. Its average value in the sheet is:

$$P_v = \frac{1}{d} \int_{-d/2}^{d/2} P(x)dx = \frac{2}{d} \int_0^{d/2} \frac{1}{\sigma} \left[TH_0 \frac{\sinh(Tx)}{\cosh(T\frac{d}{2})} \right]^2 dx = \frac{TH_0^2}{2\sigma d} \frac{\sinh(dT) - dT}{\cosh^2(\frac{dT}{2})} \tag{23}$$

Next, to demonstrate the validity of Eq.23, the result of Eq.23 is compared with the eddy current loss from the Curl of magnetic field intensity.

RESULTS

Recall that Eq. 23 is obtained from integral form of Maxwell's equations. In the latest research Instead of Eq.23, in [8] Bermúdez proposes Eq.24 that obtained from Curl of magnetic field intensity.

$$P_v = \frac{H_0^2}{\sigma d^2}\gamma\frac{\sinh(\gamma)-\sin(\gamma)}{\cosh(\gamma)+\cos(\gamma)} \qquad (W/m^3) \tag{24}$$

Where $\gamma = d\sqrt{\mu\sigma\pi f}$. Bermúdez in [8] showed that Eq.24 is exact for any frequency. In Fig. 5, we show the losses given by the above analytical formulas versus frequency in a last given lamination. Can be seen, the results of Eq.23 is completely consistent with the results of Eq. 24.

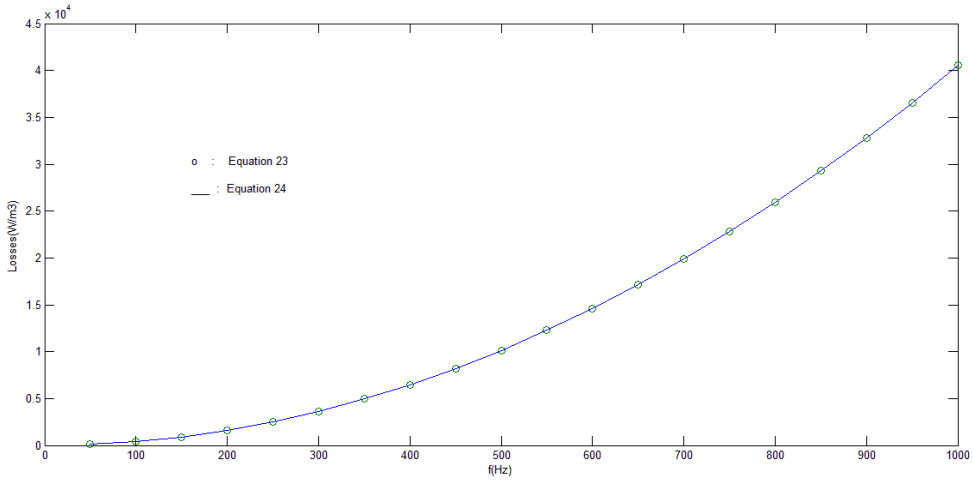

Fig. 5: Average eddy-current loss versus *f(Hz)*

Conclusion

In this paper, a new analytical consideration of eddy current loss is evaluated. For this purpose, Integral form of Faraday's law is employed to calculated eddy current density and eddy current core power loss. Utilization Steinmetz and Bertotti theoretical methods, a comparison between this method and their methods are accomplished. Based on the analytical formula derived from the proposed method, the same results are achieved. Simplicity of the proposed method enables us to employ it in numerical analysis such as finite element method.

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