

Pilot Design Optimization Using Modified Differential Evolution Algorithm in SISO and MIMO OFDM Systems

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ABSTRACT

In this paper, we propose a transmitter algorithm for optimizing the placement of the pilot tones in SISO and MIMO OFDM systems. Optimum pilot design for OFDM systems has been well studied, however, most existing optimal pilot placement in the literature are special cases. Our approach based on differential evolution optimizer adapts the time-frequency pilot spaces according to the changes that experience the channel's delay spread and doppler spread as well as pilot power allocation. The main attraction of adopting differential evolution algorithm is that it may facilitate optimal pilot designs with affordable computational costs by avoiding the matrix inversion in the MSE of least-square estimation. Different from most existing work, we do not impose any condition on the pilot design; we explicitly consider regular pilot design as well as irregular ones.

KEYWORDS: Differential evolution (DE), OFDM, MIMO, Mobile multipath channel.

INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) has been widely applied in wireless communication systems due to its high data rate transmission and its robustness to multipath channel delay [1, 2]. Additionally, multiple antenna architecture on the transmitter and receiver side, which is called multiple input multiple output (MIMO) is a suitable technique to improve the OFDM channel capacity [3]. For that reason, OFDM modulation is adopted in a number of standards, e.g IEEE 802.11a/g, IEEE 802.16a/d/e [3], DVB-T, etc.

In OFDM systems, channel estimation is usually performed by sending training pilot symbols on sub-carriers known at the receiver and the quality of the estimation depends on the pilot arrangement. Since the channel's response is a slow varying process, the pilot symbols essentially sample this process and therefore need to have a density that is high enough to reconstruct the channel's response at the receiver side [4].

Two classes of methods are available for pilot arrangements: One is based on regular patterns, where pilot symbols are equally-spaced in time and/or frequency domain, whereas the other relies on irregular patterns.

The optimal spacing design of pilot symbols for OFDM systems has been investigated by several studies over the past ten years. In literature, several methods have been designed for regular pilot lattices that satisfy a suitable Nyquist criterion [3, 5, 6]. These regular patterns are not acceptable for systems in which pilot overhead is of primary concern since the uninterrupted distribution of pilot symbols may be excessive [7]. Recently, irregular pilot arrangements were shown to be optimal in the mean-square error (MSE) sense for certain classes of time varying channels [8, 9].

In this work we propose a transmitter method based on Differential Evolution algorithm for OFDM pilot design optimization. It is specifically tailored to irregular pilot arrangements over multipath channels.

Recently, Differential Evolution (DE) algorithm has become popular and has been applied to a variety of engineering applications. The effectiveness of DE in tackling challenging optimization problems have now widely been recognized by the computational intelligence community.

This paper is organized as follows. Section II presents OFDM system model. In Section III, we evaluate the MSE of LS channel estimation. Pilot design optimization is described in Section IV. The system simulation results are presented in Section V.

SYSTEM MODEL

The system under consideration is given in Fig. 1, which shows a MIMO-OFDM system with N_t transmit antennas, N_r receive antennas, and N subcarriers. Generated OFDM signals are transmitted through a number of antennas in order to achieve diversity.

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Fig.1. Block diagram of MIMO-OFDM system [10].

In MIMO OFDM system shown in Fig. 1 (for SISO-OFDM systems we consider $N_t = 1$), we assume that the duration of the cyclic prefix is long enough to avoid inter-symbols interferences (ISI) and we suppose the OFDM symbol that is transmitted from the *pth* antenna at time index *n* is denoted by the $N \ge 1$ vector X^p (n), after removing the cyclic prefix at the *qth* receive antenna, the received $N \ge 1$ vector $Y^q(n)$ at time index *n* can be written with the following equation [10, 15]

$$Y^{q}(n) = \sum_{p=1}^{N_{t}} X^{p}_{diag}(n) \cdot F \cdot h^{p,q} + W^{q}(n)$$
(1)

Where $h^{p,q}$ is an $L \ge 1$ vector representing the length L channel impulse response from the *pth* transmit antenna to the *qth* receive antenna. Note that F denotes the $N \ge N$ unitary DFT matrix; $W^q(n)$ is additive white Gaussian noise, and (.)_{diag} is a diagonal matrix with column vector (.). In this paper we consider a multipath fading channel. Therefore, when the channel is frequency-selective invariant over each received block OFDM symbol the orthogonality between subcarriers can be fully preserved.

I. Pilot tones design for OFDM systems

There are several criteria used for channel estimation, for reasons of complexity, the estimation can be performed by using either linear MMSE criterion (Wiener filtering) [12, 13] and least squares (LS) criterion. However, in high SNR regimes and when the noise level is low, LS criterion offers a good compromising performance / complexity.

A. 1-D case: Time invariant channels (very low Doppler spread)

In this section we consider channel estimation over the frequency domain.

Knowing that $X^p(n) = D^p(n) + B^p(n)$ [10, 15], where $D^q(n)$ is some arbitrary $N \ge 1$ data vector, and $B^q(n)$ is some arbitrary $N \ge 1$ pilot sequence vector. Then, from (1) we can write

$$Y^{q}(n) = \sum_{p=1}^{N_{t}} D^{p}_{diag}(n)Fh^{p,q} + \sum_{p=1}^{N_{t}} B^{p}_{diag}(n)Fh^{p,q} + N^{q}(n)$$
(2)

Therefore, we consider the data model as in [10, 15]

$$Y^{q} = G.h^{q} + A.h^{q} + W^{q}$$
 (3)

where $h^q = \left[h^{q,1^T}, ..., h^{q,N_t^T}\right]^T$ is a channel impulse response vector of N_t . L length, $G = [D^I_{diag}, F, ..., D^{N_t}_{diag}, F]$ is a $N \times I$

Nt.L matrix, $A = [B^{l}_{diag}, F, ..., B^{Nt}_{diag}, F]$ is a $N \times Nt.L$ matrix, and $(..)^{T}$ is the transpose operation. In this section, LS channel estimation scheme is derived. The LS estimate of h^{q} can then be obtained as [15]

$$\hat{h}^{q} = A^{t}Y^{q} = h^{q} + (A^{H}A)^{-1}A^{H}W^{q}$$
(4)

(.)^H is the Hermitian matrix and A^t is the pseudo-inverse of A can be written as $A^{t} = (A^{H}A)^{-1}A^{H}$. It is assumed that pilot sequences are designed as $P \times Nt.L$ matrix A, which has a full column rank $N_{t}.L$ that requires $P \ge N_{t}L$ (P is the number of pilot tones).

From (4), the MSE of the LS channel estimate is given by [10, 15]

$$MSE = \frac{1}{LN_t} E\left\{ \left\| \hat{h}^{q} - h^{q} \right\|^2 \right\}$$
$$= \frac{1}{LN_t} tr \left\{ A^{t} E\left\{ N^{q} N^{q}^{H} \right\} A^{t}^{H} \right\}$$

For zero-mean white noise, we have $E\left\{N^{q}N^{qH}\right\} = \sigma^{2}I_{P}$ (I_{P} is $P \ge P$ identity matrix), the MSE can be defined as

$$MSE = \frac{\sigma^2}{LN_t} tr\left\{ (A^H . A)^{-1} \right\}$$
(5)

The minimum MSE can be achieved if $A^{H} A = P_{p} I_{L,N_{t}}$ where P_{p} is a fixed power dedicated for training [15].

$$MSE_{\min} = \frac{\sigma^2}{P_p}$$
(6)

However, we have the following inequality

$$MSE \ge \frac{\sigma^2}{P_p} \tag{7}$$

2-D case: Time varying channels

The following section illustrates the evaluation of 2-D MMSE over time varying channels. It has been shown in [16] that the MSE of the LS channel estimation can be written as

$$MSE = \frac{\sigma^2}{(Q+1)LN_t} tr\{(A^H.A)^{-1}\}$$
 (8)

Where $Q = 2[f_{\max}T]$, f_{\max} is the maximum Doppler frequency, T is the duration of an OFDM packet and A is a Px *Nt.L.*(Q+1) matrix. The ceiling of a number is shown by [.].

The least square channel estimation, supposing $A^{H}A$ has full rank and $P \ge N_{t}L(Q+1)$ (we should have at least Nt.L.(Q+1) pilot clusters)

1-D case of time invariant channels is obtained from 2-D case by formally setting Q=0.

To satisfy condition (6) in SISO-OFDM systems pilots have to be equally-powered and equally-spaced [17, 18]. In MIMO-OFDM systems, all pilot amplitudes must be the same (all pilots will have the same power) and pilot tones should be disjoint from pilot tones of any other antenna. All antennas will have the same pilot amplitude [19].

Condition (6) may not be necessarily achieved, because the data power distribution is not forced to be uniform if it does not maximize channel capacity or it does not achieve low PAPR to avoid nonlinear distortion of pilots at the transmit power amplifier design. Indeed, non uniform power loading at the transmitter will provide better overall system performance. In this case optimal pilot placement may be an irregular pilot design with non uniform pilot power (condition (6) is no satisfied). In order to minimize MSE channel estimation in Eq. (8) or (5), we propose a transmitter algorithm based on differential evolution algorithm.

II. Pilot design optimization based on differential evolution algorithm

In this section, we describe a transmitter algorithm for pilot design optimization. In order to optimize the positions of pilot tones, we use DE algorithm to find pilot design minimizing the MSE cost function given by Eq. (8) or (5). Our goal is to search for the optimal pilot positions that will minimize the MSE. The optimal pilot design position can be derived from an extensive matching of all possible positions. But the exhaustive search will be extremely time consuming and thus we will employ the optimization process on the cost function defined in Eq. (8) or (5) and we will reduce the computational complexity using Gershgorin theorem. Since the DE algorithm was originally designed to work with continuous variables, the optimization of continuous problems is discussed first in section A and DE for discrete variable is explained later in section B.

A. Classical continuous DE algorithm

The algorithm in its basic form is for continuous function optimization. DE algorithm introduced a novel parallel direct search for the optimization of continuous problems, it's great advantage is fast convergence and the use of few control parameters. Basic DE algorithm [20, 21] is characterized with its initialization, mutation, re-combination and selection operations used to explore the search space in an iterative procedure, until some termination criteria are met. The basic strategy of DE algorithm can be described as follows.

Operation 1. Initialization Operation 2. Evaluation Operation 3. While (termination criteria are not satisfied) Repeat Mutation Recombination Evaluation Selection Until (termination criteria are met)

Fig.2. DE algorithm operations

- Initialization

DE starts with a population of solutions, not with a single solution for the optimization problem. Population P of generation G contains NP solution vectors called individuals of the population and each vector represents potential solution for the optimization problem. Solutions are represented as vectors of size D with each value taken from some domain. First, all parameter vectors in a population are randomly initialized and evaluated using the fitness function.

- Mutation

The mutation operation is a genetic method which allows DE to maintain the diversity of the population from one generation of a population of algorithm to the next. DE generates new parameter vectors by adding the weighted difference between two parameter vectors to a third vector, temporary or trial population of candidate vectors for the subsequent generation.

Temporary or trial population of candidate vectors in generation G is generated as follows:

 $V_{i,G^{+}l} \!\!=\!\! x_{r1,G} \!\!+\!\! \eta(x_{r2,G} \!\!-\!\! x_{r3,G})$

Where x_i is a target vector and *i*, r_1 , r_2 , $r_3 \in \{1, 2, ..., NP\}$. r_1 , r_2 and r_3 are three randomly chosen indices and η is a scaling factor in range [0,1] that controls the amplification of differential variations.

- Crossover

In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end the trial vector is given by:

 $u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1})$ Is formed by $\int V_{ji,G+1} \quad if \quad randb(j) \le CR \quad or \quad j=0$

 $u_{ji,G+1} = \begin{cases} V_{ji,G+1} & if \quad randb(j) \le CR \quad or \quad j = rnbr(i) \\ x_{ji,G+1} & if \quad randb(j) > CR \quad and \quad j \ne rnbr(i) \end{cases}$

j = 1, ..., D.

where *randb(j)* is the jth evaluation of a uniform random number generator with outcome [0, 1], *CR* is the crossover constant [0, 1] which has to be determined by the user, rnbr(i) is randomly chosen index from *1..D* which ensures that $u_{i,G+1}$ gets at least one parameter from $v_{i,G+1}$.

- Selection

The selection operator determines whether the target vector $u_{i,G+1}$ survives to the next generation.

 $x_{i,G+1} = \begin{cases} u_{i,G+1} & if \quad f(u_{i,G+1}) < f(x_{i,G}) \\ x_{i,G} & else \end{cases}$

where f(.) is the fitness function.

- Termination

The ultimate stopping criterion would be that the optimal MSE solution has indeed been found. However, it is impossible in practice to confirm this. Therefore, we stop the optimization procedure, when any of the following criteria are satisfied:

- We stop DE algorithm when we find a solution for the optimization problem. Therefore, the minimum MSE is achieved.

- The pre-defined maximum affordable number of generations G_{max} has been exhausted.

- Fixed number ΔG_{max} generations have been explored without a trial vector being accepted.

Usually stopping criterion is a maximum number of iterations (generations).

Modified DE algorithm for discrete variable optimization

DE algorithm works only with floating-point variables but the data input for pilot design optimization problem are discrete values of pilot positions indexing. Hence the algorithm existing in its current form must be modified.

Several approaches have been used to deal with discrete variable optimization [22]. Most of them round off the variable to the nearest available value before evaluating each trial vector. To keep the population robust, successful trial vectors must enter the population with all of the precision with which they were generated. The differential evolution algorithm, which in its canonical form is only capable of handling continuous variables, must be extended for optimization of integer variables. First, integer values should be used to evaluate the objective function, even though DE itself may still works internally with continuous floating-point values.

Forward and backward transformation techniques have been developed to extend DE algorithm for problems with integer variables [22].



Fig.3. Forward and backward transformations [22]

A backward transformation method for transforming a population of continuous variables obtained after mutation back into integer variables for evaluating the objective function. Both forward and backward transformations are utilized in implementing the DE algorithm used in the present study for the pilot design optimization problem. Fig. 3 shows how to deal with this inherent representational problem in DE. Level 0 deals with integer numbers (which are used in discrete problems). Level 1 of Fig. 3 deals with floating point numbers, which are suited for DE. At this level, the DE operators (mutation, crossover, and selection) take place. To transform the integer at level 0 into floating point numbers at level 1.

Proposed transmitter algorithm for pilot design optimization

Proposed transmitter algorithm seeks the most appropriate time-frequency pilot space that minimizes the mean square error of channel estimate. We use the MSE function as an objective function for the DE algorithm. The matrix inversion in Eq. (5) will increase the computational load of the optimizer; we can reduce this computational complexity using Gershgorin theorem.

• Fitness function of differential evolution algorithm

We will employ the optimization process on the cost function defined in Eq. (5) as

$$MSE = \frac{\sigma^2}{LN_t} tr \left\{ (A^H . A)^{-1} \right\}$$

Let $\lambda_1, \lambda_2, ..., \lambda_L$ the eigenvalues of $A^H A$, then the eigenvalues of $(A^H A)^{-1}$ are $1/\lambda_1, 1/\lambda_2, ..., 1/\lambda_L$.

Since the trace of a matrix is the sum of its eigenvalues, we have $tr\left\{(A^H.A)^{-1}\right\} = \sum_{p=1}^{N_t L} \frac{1}{\lambda_p}$

Knowing that the power of a subcarrier is the same for all the antennas $A^{H}A$ can be expressed as

$$A^{H}A = \begin{pmatrix} P_{p} & x & x & x \\ x & P_{p} & x & x \\ x & x & \dots & x \\ x & x & x & x & P_{p} \end{pmatrix}$$

where P_p is the diagonal element of matrix $A^H A$.

Let a_{ij} $(i=1,...,N_t.L; j=1,...,N_t.L)$ the elements of matrix $A^H.A$ and $R_{max}=max(R_i)$ the maximum radius of the Gershgorin disk, where $R_i = \sum_{j=1}^{N_tL} |a_{ij}|$.

According to Gershgorin theorem [23] we have

$$|P_p - \lambda_i| \leq R_{\max}$$
, $i=1,\ldots, N_t.L$

Therefore, if $P_p > R_{max}$ we have the following inequality

$$tr\{(A^{H}.A)^{-1}\} = \sum_{p=1}^{L} \frac{1}{\lambda_p} \le \frac{N_t L}{P_p - R_{\max}}$$
 (9)

From (5) and (8), the upper bound of MSE that we can use as an objective function for DE is obtained by:

$$MSE \le \frac{\sigma^2}{P_p - R_{\max}} \tag{10}$$

Knowing that the power dedicated for the pilot is fixed, the value of diagonal element P_p does not change during the optimization process. Therefore, we will use R_{max} as the fitness function for differential evolution algorithm.

Proposed transmitter algorithm

Based on the discussion above, the proposed transmitter algorithm is described by the following steps.

1) We first specify the channel parameters, the impulse response length $N_t L$ and Doppler spread as well as maximum number of iterations ($N_t = 1$ in SISO-OFDM schemes).

2) The total number of pilot tones in one OFDM frame is chosen greater or equal to (Q+1). N_t .L. Then, we define pilot power allocation and initial population for DE (initial pilot design). The initial pilot positions are initialized at equally spaced random values. The use of equally spaced pilot tones as initial population is a good choice for fast convergence.

3) In order to get the best positions of the pilots, we use modified DE algorithm mentioned in section B. The pilot positions are improved by the mutation, crossover, and selection operators. The optimization procedure is repeated until we find a solution for the optimization problem defined in Eq. (6) or until the end of maximum number of iterations.



Fig.4. Description of transmitter adaptive pilot arrangement

III. Simulation results

We consider an OFDM system with N = 128 subcarriers of which 8 serve as pilot tones (Np = 8), and an invariant multipath channel model (Q=0) with 4 paths according to Jackes model (L=4). For optimizing the pilot placement, we use the differential evolution parameters of a population size of 10, scale factor of 0.8 and crossover of 0.9. We first apply our algorithm to SISO-OFDM system. Furthermore, we evaluate the performance of our transmitter algorithm in the context of MIMO-OFDM system.

SISO-OFDM scheme

In order to evaluate the performance of the proposed method, we deal with two cases:

1) We simulate an OFDM system with equi-powered pilot tones (we use pilot symbols 1+i, in this case pilot power is the same for all pilot tones).



Fig.5. The optimal placement for equi-powered pilot tones (regular arrangement)

Fig. 5 shows the designed optimal set for pilot tones using DE method Np > L. As it can be seen, equi-spaced pilots are the optimal pilot design as it is shown in [17, 18] with equi-powered pilot tones. The use of equally spaced pilot tones as initial population is a good choice for fast convergence.

2) We simulate an OFDM system with unequi-powered pilot tones, we use as pilot symbols 1+i, 1+i, 0.2+0.2i, 0.3+0.3i, 1+i, 1+i and 1+i. Pilot symbols are randomly chosen.



Fig.6. The optimal placement for unequi-powered pilot tones (irregular arrangement)

Fig. 6 shows that the optimal set for pilot tones is an irregular arrangement. In this case of unequi-powered pilot tones, we have numerically evaluated the MSE for regular and irregular pilot arrangements, it has been found that irregular pilot arrangement evaluate MSE better than equi-spaced pilot arrangement ($MSE_{equi-spaced}=0.1345$, $MSE_{unequi-spaced}=0.1302$). Therefore, regular pilot arrangements are not always optimal.

MIMO-OFDM scheme

We next consider the application of the proposed method to 2 x 1 MIMO-OFDM scheme with N = 128 subcarriers of which 8 serve as pilot tones (Np = 8), and a multipath channel model with 4 paths according to Jackes model (L=4).



Fig.7. The optimal placement for equi-powered pilot tones in 2x1 MIMO-OFDM system



Fig.8. The optimal placement for unequi-powered pilot tones in 2x1 MIMO-OFDM system

Fig. 7 and Fig. 8 shows that the optimal set for equi-powered pilot tones is an equi-spaced arrangement (we use pilot symbols 1+i for all antennas). However, the optimal set for unequi-powered pilot tones is an irregular arrangement as it is normally the case in SISO-OFDM systems (we use 1+i, 1+i, 1+i, 0.2+0.2i, 0.3+0.3i, 1+i, 1+i and 1+i as pilot symbols for all antennas).

Computational complexity

In OFDM systems with N = 128 subcarriers and Np = 8 pilot tones, the exhaustive search for the best pilot placement is $C_{128}^8 \approx 1.4297 \times 10^{12}$, whereas the number of searches in DE method with a population size of 10 is

- $212 \times 10 = 2120$ for equi-powered pilot tones in SISO-OFDM scheme.
- $913 \times 10 = 9130$ for unequi-powered pilot tones in SISO-OFDM scheme.
- $435 \times 10 = 4350$ for equi-powered pilot tones in 2 x 1 MIMO-OFDM scheme.
- $959 \ge 10 = 9590$ for unequi-powered pilot tones in $2 \ge 1$ MIMO-OFDM scheme.

The numerical results given in this section, shows the computational advantage of DE method. Furthermore, in the DE method, we avoid computing matrix inversion of MSE by use of Gershgorin theorem mentioned in section IV-C, This greatly reduces the complexity and cost of our method knowing that N_p^3 multiplications are needed for MSE matrix inversion in [24], with Gershgorin theorem; multiplications are not required to evaluate the fitness function. For the proposed algorithm, $2.N_{pop}.M.N_{iter}$ multiplications are required, where N_{pop} is the population size, M is the additional multiplications per iteration used by DE algorithm to improve the population $(1 \le M \le N_{pop})$ and N_{iter} is the number of iterations (two multiplications are required for forward and backward transformation techniques per population and per iteration).

IV. Conclusion

The proposed DE transmitter method for OFDM pilot design optimization is specifically tailored to irregular pilot arrangements over multipath channels. The algorithm can be implemented in a computationally efficient manner using the upper bound of MSE for the fitness function instead of using MSE directly.

This study has demonstrated the effectiveness of the DE algorithm as a design tool for irregular pilot arrangements in SISO and MIMO-OFDM systems.

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