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On Reliability Criterion Approach for Three Hypotheses Optimal Testing of Three Independent Markov Chains

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ABSTRACT

The problem of three hypotheses logarithmically asymptotically optimal (LAO) testing for a model consisting of three simple homogeneous stationary Markov chains is studied. Three independent Markov chains is considered. This problem was introduced by Ahlswede and Haroutunian. It is supposed that three probability distributions are known and each Markov chain is independently of others follows to one of them. The matrix of asymptotic interdependencies (Reliability Matrix) in Optimal testing of this model is studied.
KEYWORDS: Independent Objects, Error Reliability, Reliability, Markov chain.

1.INTRODUCTION

Recently Ahlswede and Haroutunian in [1] formulated an ensemble of new problems on multiple hypotheses testing for many objects and on identification of hypotheses. The problem of many (M > 2) hypotheses testing on distributions of a finite state Markov chain is studied in [11] via large deviations techniques (LDT). In this paper we consider the case of three Markov chains which independently follow to one from three given probability distributions. In section 2 we recall main definitions and results of [7] and [11] for many hypotheses testing and in section 3 present the problem of hypotheses testing for three independent objects via Markov chains.

2. LAO Hypotheses Testing for one Markov Chains

Let the finite set $\chi = \{1,2,\ldots, I\}$ be the state space of Markov chain and $X = (x_0, x_1, \ldots, x_N)$, $x_n \in \chi$, $n = 1,2,\ldots, N$, $X \in \chi^{N+1}$, $N = 0,1,2,\ldots$, be a vector of observed states of three simple homogeneous stationary Markov chain with finite number I of states. The three hypotheses H_l concern the matrix of the transition probabilities
$P_1 = \{P_1(j|i)\}$, $P_2 = \{P_2(j|i)\}$, $P_3 = \{P_3(j|i)\}$, $i, j = 1, 2, \ldots, I$.
The stationary of the chain provides existence for each $l = 1,2,3$, of the unique stationary distribution
$Q_l = \{Q_l(i),\ i = 1,2,.,\ldots, I\}$, such that

$$\sum_{i=1}^{I} Q_l(i)\, P_l(j|i) = Q_l(j), \qquad \sum_{i=1}^{I} Q_l(i) = 1, \quad i, j = 1, 2, \ldots, I, \quad l = 1,2,3.$$

The joint distributions of pair (i, j) $\in I^2$ are

$$Q_l O P_l = \{Q_l(i)\, P_l(j\,|i),\ i, j = 1, 2, \ldots, I\}, \qquad l = 1,2,3.$$

We denote by $D\,(QOP||Q_l OP_l)$ the Kullback – Leibler divergence

$$D\,(QOP||Q_l OP_l) = \sum_{i,j} Q(i)\, P(j|i)\, \frac{\log Q(i)\, P(j|i)}{\log Q_l(i)\, P_l(j|i)} = D(Q||Q_l) + D(QOP||QOP_l),$$

of a joint distribution

$$QOP\,(i,j) = \{Q(i)\, P(j|i),\ i, j = 1, 2, \ldots, I\}.$$

From joint to distribution $Q_l OP_l$, where the divergence for marginal distributions is

$$D\,(Q||Q_l) = \sum_i Q(i)\, \frac{\log Q(i)}{\log Q_l(i)}, \quad l = 1,2,3, \qquad i = 1, 2, \ldots, I.$$

The probability of vector $X \in \chi^{N+1}$ of the Markov chain with transition probability P_l and stationary distribution Q_l, is the following

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$$\mathrm{Q}_l\mathrm{OP}_l^{\mathrm{N}}(\mathrm{X}) = \mathrm{Q}_l(x_0)\prod_{n=1}^{N}\mathrm{P}_l\left(x_{\mathrm{n}}|x_{\mathrm{n-1}}\right),\ l = 1,2,\dots,\mathrm{I},$$

$$\mathrm{Q}_l\mathrm{OP}_l^{\mathrm{N}}(\mathrm{A}) = \bigcup_{\mathrm{X}\in A}\mathrm{Q}_l\mathrm{OP}_l^{\mathrm{N}}(\mathrm{X}), \qquad \mathrm{A}\subset\chi^{\mathrm{N+1}}.$$

The second order type of Markov vector X is the square matrix of I^2 relative frequencies $\{\mathrm{N}(\mathrm{i},\mathrm{j})\,N^{-1},\ i,\mathrm{j} = 1,2,\dots,\mathrm{I}\}$ of the simultaneous appearance in X of the states i and j on the pairs of neighbor places. It is clear that $\sum_{(\mathrm{i},\mathrm{j})\in\chi^2}\mathrm{N}(\mathrm{i},\mathrm{j}) = \mathrm{N}$. Denote by $\mathcal{T}_{QOP}^{N}$ the set of vector X from χ^{N+1} which have the second order type such that for some joint probability distributions QOP $\mathrm{N}(\mathrm{i},\mathrm{j}) = \mathrm{N}\,\mathrm{Q}(\mathrm{i})\,\mathrm{P}(\mathrm{j}\,|\,\mathrm{i}),\ i,\mathrm{j} = 1,2,\dots,\mathrm{I}.$

The set of joint PD QOP on I^2 is denoted by QOP. Non-randomized test $\Phi_N(\mathrm{X})$ one of the hypotheses H_l, l=1,2,3, on the basis of the trajectory $\mathrm{X}=(x_0,x_1,\dots,x_N)$ of the $\mathrm{N}+1$ observations.

We denote $\alpha_{l|m}^{(N)}(\Phi_N)$ the probability to accept the hypotheses H_l under the condition that H_m, $\mathrm{m}\neq l$, is true. For m= l we denote $\alpha_{m|m}^{(N)}(\Phi_N)$ the probability to reject the hypotheses H_m. It is clear that

$$\alpha_{m|m}^{(N)}(\Phi_N) = \sum_{\mathrm{m}\neq l}\alpha_{l|\mathrm{m}}^{(N)}(\Phi_N), \qquad m = 1,2,3. \tag{1}$$

We study error probability exponents of the sequence of tests Φ, which we call reliabilities:

$$\mathrm{E}_{l|\mathrm{m}}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log\alpha^{(\mathrm{N})}{}_{l|\mathrm{m}}(\Phi_N), \qquad \mathrm{m}, l = 1,2,3. \tag{2}$$

It is easy to show using (1) and (2) it follows that:

$$E_{m|m}(\Phi) = \min_{\mathrm{m}\neq l}\mathrm{E}_{l|m}(\Phi). \tag{3}$$

The test Φ_N is defined by a partition of the space χ^{N+1} on sets $\mathcal{G}_l^{\mathrm{N}} = \{\mathrm{X}:\ \varphi^{\mathrm{N}}(\mathrm{X}) = l\}$, $l = 1,2,3$, and $\alpha_{l|\mathrm{m}}^{(N)}(\Phi_N) = \mathrm{Q_m OP_m}(\mathcal{G}_l^{\mathrm{N}})$, $\mathrm{m}, l = 1,2,3.$

Let P be a matrix of transition probabilities of some stationary Markov chain and Q be the corresponding stationary PD. For given positive values of 2 diagonal elements $\mathrm{E}_{1|1}$ and $\mathrm{E}_{2|2}$, we consider the decision rule Φ^* by the sets of distributions

$$\mathrm{R}_l = \{\mathrm{QOP}:\ \mathrm{D}(\mathrm{QOP}\,\|\,QOP_l)\leq E_{l|l}\},\ l = 1,2, \tag{4}$$

$$\mathrm{R}_3 = \{\mathrm{QOP}:\ \mathrm{D}(\mathrm{QOP}\,\|\,QOP_l) > E_{l|l}, \qquad l = 1,2\}.$$

And the function:

$$E_{l|l}^*(E_{l|l}) = E_{l|l},\ l = 1,2, \tag{5}$$

$$E_{l|m}^*(E_{l|l}) = \inf_{\mathrm{QOP}\in\mathrm{R}_l}\mathrm{D}(\mathrm{QOP}\,\|\,\mathrm{QOP}_m), \quad m = 1,2,3,\ m\neq l,\ l = 1,2,$$

$$\mathrm{E}_{3|3}^*(E_{1|1},E_{2|2}) = \min_{l=1,2}\mathrm{E}_{l|3}^*.$$

Theorem 1:

Let $\chi = \{1, 2,\dots, \mathrm{I}\}$ be a finite set of the states of the stationary Markov chain possessing an irreducible transition matrix P and A be a nonempty and open subset or convex subset of joint distributions QOP and Q_m is stationary distribution for P_m, then for the type QOP(x) of vectors X^1, X^2 and X^3 from $\mathrm{Q_m OP_m}$ on χ:

$$\overline{\lim_{N\to\infty}} -\frac{1}{N}\log\mathrm{Q_m OP_m^N}\{\mathrm{X}:\ \mathrm{QOP}(\mathrm{x})\in\mathrm{A}\} = \inf_{\mathrm{QOP}\in\mathrm{A}}\mathrm{D}(\mathrm{QOP}\,\|\,\mathrm{QOP_m}).$$

Theorem2:

Let χ be a finite set, for a family of distinct distributions P_1, P_2 and P_3 the following two statements hold.

If the positive finite numbers $E_{1|1}$ and $E_{2|2}$ satisfy conditions:

$$0 < E_{1|1} < min\ [\ D(Q_2OP_2 \| Q_2OP_1)\ ], \quad (6)$$
$$0 < E_{2|2} < min\ [\ E^*_{2|1}(E_{1|1}),\ D(Q_3OP_3 \| Q_3OP_2)\ ].$$

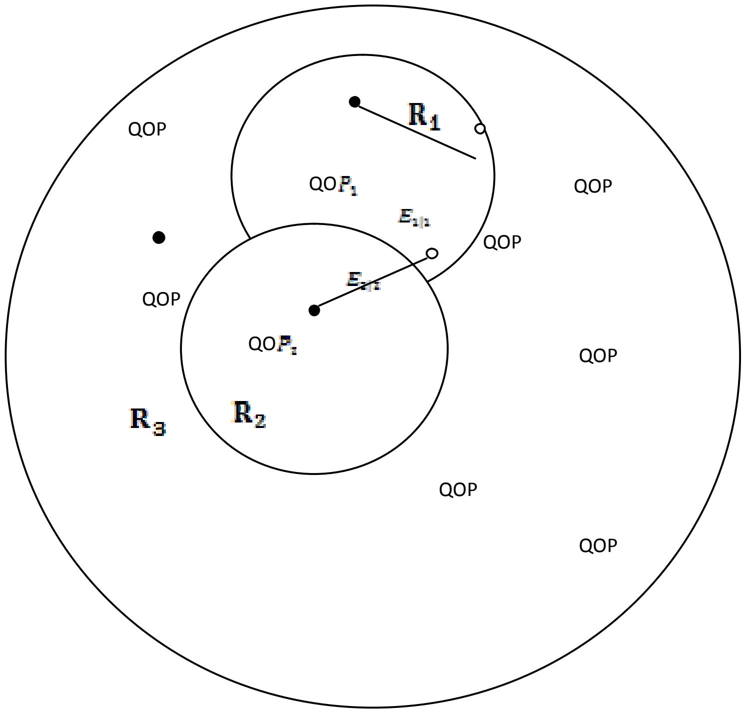
Then:

a) There exists a LAO sequence of testsΦ^*, the reliability matrix of which E (Φ^*) is defined in (5) and all elements of it are positive,

b) Even if one of conditions (6) is violated, then the reliability matrix of an arbitrary test necessarily has an element equal to zero, (the corresponding error probability does not tend exponentially to zero).

Remark (1):

From the definition (3), (5) and (6) it follows that: $E_{m|m} < E^*_{M|m}$ and also, $E^*_{m|m} = E^*_{M|m}$, m = 1,2.



3. LAO Hypotheses Testing for Three Markov Chains

In this section we expand the concept of section 2 for three independent homogenizes stationary finite Markov chain. Let the finite set χ ={1,…,I} be the state space of Markov chain and $X^1 = (x_0^1, x_1^1, \dots, x_N^1)$, $X^2 = (x_0^2, x_1^2, \dots, x_N^2)$ and $X^3 = (x_0^3, x_1^3, \dots, x_N^3)$, $x_n^1, x_n^2, x_n^3 \in \chi$, $n = 1,2,\dots,N$, $X^1, X^2, X^3 \in \chi^{N+1}$, N = 0, 1, 2, …, be vectors of observed states of three simple homogeneous stationary Markov chain with finite number I of states. The probability of the vector $X^1, X^2, X^3 \in \chi^{N+1}$ of the Markov chain with transition probabilities P_l and one of stationary distribution Q_l is

$$Q_l OP_l^N(X^i) = Q_l(x_0^i) \prod_{n=1}^{N} P_l\left(x_n^i, x_{n-1}^i\right), \quad l = 1,2,3\ , \quad i = 1,2,3\ ,$$

$$Q_l OP_l^N(A) = \bigcup_{X^i \in A} Q_l OP_l^N(X^i), \qquad l = 1,2,3\ , \quad i = 1,2,3\ , \qquad A \subset \chi^{N+1}.$$

We have three hypotheses H_1, H_2, H_3 and call the procedure of making decision on the base of N+1 observations the test, which we denote by Φ_N.The test Φ_N for this model can be composed by
$\Phi_N = (\varphi_N^1, \varphi_N^2, \varphi_N^3)$.

So the space χ^{N+1} will be divided into three parts,

$$\mathcal{G}_{l,i}^N = \{X^i : \varphi_N^i(X^i) = l\}, \quad l = 1,2,3, \quad i=1, 2, 3.$$

We denote by $\alpha^{(N)}_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi_N)$ the probability of the erroneous acceptance by the test $\Phi_N$of the three of the hypotheses $(H_{l_1}, H_{l_2}, H_{l_3})$ provided that the hypotheses $(H_{m_1}, H_{m_2}, H_{m_3})$ is true,

$$\alpha^{(N)}_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi_N) = \mathrm{Q}_{\mathrm{m}_1}\mathrm{OP}^{\mathrm{N}}_{\mathrm{m}_1}(\mathcal{G}^{\mathrm{N}}_{l_1,1})\,.\,\mathrm{Q}_{m_2}\mathrm{OP}^{\mathrm{N}}_{m_2}(\mathcal{G}^{\mathrm{N}}_{l_2,2}).\,\mathrm{Q}_{m_3}\mathrm{OP}^{\mathrm{N}}_{m_3}(\mathcal{G}^{\mathrm{N}}_{l_3,3}),\quad (\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3) \neq (l_1,l_2,l_3),\ m_i, l_i = 1,2,3.$$

The probability to reject a true three of hypotheses $(\mathrm{H}_{\mathrm{m}_1}, \mathrm{H}_{\mathrm{m}_2}, \mathrm{H}_{\mathrm{m}_3})$ by analogy with (1) is the following

$$\alpha^{(N)}_{m_1,m_2,m_3|m_1,m_2,m_3}(\Phi_N) = \sum_{(\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3)\neq(l_1,l_2,l_3)} \alpha^{(N)}_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi_N), \tag{7}$$

We also study corresponding limits $E_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi_N)$ of error probability exponents of the sequence of tests Φ, which we call reliabilities:

$$\mathrm{E}_{l_1,l_2,l_3|\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N} log\, \alpha^{(\mathrm{N})}{}_{l_1,l_2,l_3|\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N),\quad \mathrm{m}_i, l_i = 1,2,3,\qquad i = 1,2,3, \tag{8}$$

It is easy to show using (7) and (8) it follows that

$$E_{m_1,m_2,m_3|m_1,m_2,m_3}(\Phi) = \min_{(\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3)\neq(l_1,l_2,l_3)} \mathrm{E}_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi), \tag{9}$$

The matrix E (Φ) = $\{E_{l_1,l_2,l_3|m_1,m_2,m_3}(\Phi)\}$ is called the reliability matrix of the sequence Φ of tests.
The sequence of tests Φ^* is called logarithmically asymptotically optimal (LAO) for the model with three Markov chains if for given positive values of six diagonal elements of the reliability matrix E (Φ^*) all other elements of it are maximal.

Lemma 1:
If elements $\mathrm{E}_{l|\mathrm{m}}(\varphi^i)$, m, l = 1,2,3 , i=1,2,3 , are strictly positive, then the following equalities hold for $\Phi = (\varphi^1, \varphi^2, \varphi^3)$:

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_1|\mathrm{m}_1}(\varphi^1) + \mathrm{E}_{l_2|\mathrm{m}_2}(\varphi^2) + \mathrm{E}_{l_3|\mathrm{m}_3}(\varphi^3)\quad \text{if}\quad \mathrm{m}_i \neq l_i,\ i = 1,2,3, \tag{10.a}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_1|\mathrm{m}_1}(\varphi^1) + \mathrm{E}_{l_2|\mathrm{m}_2}(\varphi^2)\quad \text{if}\quad \mathrm{m}_1 \neq l_1,\ \mathrm{m}_2 \neq l_2,\quad i = 1,2,3, \tag{10.b}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_3|\mathrm{m}_3}(\varphi^3) + \mathrm{E}_{l_2|\mathrm{m}_2}(\varphi^2)\quad \text{if}\quad \mathrm{m}_2 \neq l_2,\ \mathrm{m}_3 \neq l_3,\quad i = 1,2,3, \tag{10.c}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_1|\mathrm{m}_1}(\varphi^1) + \mathrm{E}_{l_3|\mathrm{m}_3}(\varphi^3)\quad \text{if}\quad \mathrm{m}_1 \neq l_1,\ \mathrm{m}_3 \neq l_3,\quad i = 1,2,3, \tag{10.d}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_1|\mathrm{m}_1}(\varphi^1)\quad \text{if}\quad \mathrm{m}_1 \neq l_1,\ \mathrm{m}_2 = l_2,\quad \mathrm{m}_3 = l_3,\quad i = 1,2,3, \tag{10.e}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_2|\mathrm{m}_2}(\varphi^2)\quad \text{if}\quad \mathrm{m}_2 \neq l_2,\ \mathrm{m}_3 = l_3,\quad \mathrm{m}_1 = l_1,\quad i = 1,2,3, \tag{10.f}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi) = \mathrm{E}_{l_3|\mathrm{m}_3}(\varphi^3)\quad \text{if}\quad \mathrm{m}_3 \neq l_3,\ \mathrm{m}_2 = l_2,\quad \mathrm{m}_1 = l_1,\quad i = 1,2,3, \tag{10.g}$$

Proof:
It follows from the independence of the Markov chains that

$$\alpha_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \alpha_{l_1|m_1}(\varphi^1_N).\ \alpha_{l_2|m_2}(\varphi^2_N).\ \alpha_{l_3|m_3}(\varphi^3_N),\qquad m_i \neq l_i, \tag{11.a}$$

$$\alpha_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_3|m_3}(\varphi^3_N)\right)\alpha_{l_2|m_2}(\varphi^2_N).\,\alpha_{l_1|m_1}(\varphi^1_N),\ m_3 = l_3,\ m_1 \neq l_1,\ m_2 \neq l_2, \tag{11.b}$$

$$\alpha_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_1|m_1}(\varphi^1_N)\right)\alpha_{l_2|m_2}(\varphi^2_N).\,\alpha_{l_3|m_3}(\varphi^3_N),\ m_1 = l_1,\ m_2 \neq l_2,\ m_3 \neq l_3, \tag{11.c}$$

$$\alpha_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_2|m_2}(\varphi^2_N)\right).\,\alpha_{l_1|m_1}(\varphi^1_N).\,\alpha_{l_3|m_3}(\varphi^3_N), m_2 = l_2,\ m_2 \neq l_2,\ m_3 \neq l_3, \tag{11.d}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_3|m_3}(\varphi^3_N)\right)\left(1-\alpha_{l_2|m_2}(\varphi^2_N)\right)\alpha_{l_1|m_1}(\varphi^1_N)\ \text{if}\ \mathrm{m}_1 \neq l_1, \mathrm{m}_2 = l_2, \mathrm{m}_3 = l_3, \tag{11.e}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_3|m_3}(\varphi^3_N)\right)\left(1-\alpha_{l_1|m_1}(\varphi^1_N)\right)\alpha_{l_2|m_2}(\varphi^2_N)\ \text{if}\ \mathrm{m}_2 \neq l_2, \mathrm{m}_1 = l_1, \mathrm{m}_3 = l_3, \tag{11.f}$$

$$\mathrm{E}_{l_1,l_2,l_3\,|\,\mathrm{m}_1,\mathrm{m}_2,\mathrm{m}_3}(\Phi_N) = \left(1-\alpha_{l_2|m_2}(\varphi^2_N)\right)\left(1-\alpha_{l_1|m_1}(\varphi^1_N)\right)\alpha_{l_3|m_3}(\varphi^3_N)\text{if}\ \mathrm{m}_2 \neq l_2, \mathrm{m}_1 = l_1, \mathrm{m}_3 = l_3, \tag{11.g}$$

According to the definitions (7) and (8) and from equalities (11) we obtain relations (10).
Our aim is to find LAO test from the compound tests $\{\Phi = (\varphi^1, \varphi^2, \varphi^3)\}$ when strictly positive elements $\mathrm{E}_{1,1,1\,|\,3,1,1}$, $\mathrm{E}_{1,1,1\,|\,1,3,1}$, $\mathrm{E}_{1,1,1\,|\,1,1,3}$, $\mathrm{E}_{2,2,2\,|\,3,2,2}$, $\mathrm{E}_{2,2,2\,|\,2,3,2}$ and $\mathrm{E}_{2,2,2\,|\,2,2,3}$ of the reliability matrix are given.
The elements $\mathrm{E}_{1,1,1\,|\,3,1,1}$, $\mathrm{E}_{1,1,1\,|\,1,3,1}$, $\mathrm{E}_{1,1,1\,|\,1,1,3}$, $\mathrm{E}_{2,2,2\,|\,3,2,2}$, $\mathrm{E}_{2,2,2\,|\,2,3,2}$ and $\mathrm{E}_{2,2,2\,|\,2,2,3}$ of the test for Markov chains can be positive Only in the following two subsets of tests $\{\Phi = (\varphi^1, \varphi^2, \varphi^3)\}$:
$\mathcal{A}$={ $\Phi = (\varphi^1, \varphi^2, \varphi^3)$: $\mathrm{E}_{\mathrm{m}\,|\,\mathrm{m}}(\varphi^i) > 0,\ m = 1,2\ \ i = 1,2,3\}$,
$\mathcal{B}$ = {$\Phi = (\varphi^1, \varphi^2, \varphi^3)$: one m' from [1,2] exist such that $\mathrm{E}_{\mathrm{m}'\,|\,\mathrm{m}'}(\varphi^i) = 0,\ i = 1,2,3$, and for other $\mathrm{m} < 3, \mathrm{E}_{\mathrm{m}\,|\,\mathrm{m}}(\varphi^i) > 0, i = \overline{1,3}$ },
Let us define the following family of sets for given positive elements $\mathrm{E}_{1,1,1\,|\,3,1,1}$, $\mathrm{E}_{1,1,1\,|\,1,3,1}$, $\mathrm{E}_{1,1,1\,|\,1,1,3}$, $\mathrm{E}_{2,2,2\,|\,2,2,3}$, $\mathrm{E}_{2,2,2\,|\,2,3,2}$ and $\mathrm{E}_{2,2,2\,|\,3,2,2}$:

$$\mathrm{R}^{(1)}_1 = \{\mathrm{QOP}: \mathrm{D}(\mathrm{QOP}\,\|\,\mathrm{QOP}_2) \leq \mathrm{E}_{1,1,1\,|\,3,1,1}\},\quad \mathrm{R}^{(1)}_2 = \left\{\mathrm{QOP}: \mathrm{D}(\mathrm{QOP}\,\|\,\mathrm{QOP}_2) \leq \mathrm{E}_{2,2,2\,|\,3,2,2}\right\}, \tag{12.a}$$

$R_1^{(2)} = \{QOP: D(QOP \| QOP_1) \leq E_{1,1,1|1,3,1}\}$, $R_2^{(2)} = \{QOP: D(QOP \| QOP_2) \leq E_{2,2,2|2,3,2}\}$, (12.b)
$R_1^{(3)} = \{QOP: D(QOP \| QOP_1) \leq E_{1,1,1|1,1,3}\}$, $R_2^{(3)} = \{QOP: D(QOP \| QOP_2) \leq E_{2,2,2|2,2,3}\}$, (12.c)
$R_3^{(1)} = \{QOP: D(QOP \| QOP_1) > E_{1,1,1|3,1,1}\}$, $R_3^{(1)} = \{QOP: D(QOP \| QOP_2) > E_{2,2,2|3,2,2}\}$, (12.d)
$R_3^{(2)} = \{QOP: D(QOP \| QOP_1) > E_{1,1,1|1,3,1}\}$, $R_3^{(2)} = \{QOP: D(QOP \| QOP_2) > E_{2,2,2|2,3,2}\}$, (12.e)
$R_3^{(3)} = \{QOP: D(QOP \| QOP_1) > E_{1,1,1|1,1,3}\}$, $R_3^{(3)} = \{QOP: D(QOP \| QOP_2) > E_{2,2,2|2,2,3}\}$. (12.f)

And consider the following values:

$E^*_{l_1,l_2,l_3|m_1,m_2,m_3} = \inf_{Q:\, Q\in R^i_{l_i}} D(QOP \| QOP_{m_i})$, $m_k = l_k$, $m_i \neq l_i, i \neq k$, $i,k = 1,2,3$, (13.a)

$E^*_{l_1,l_2,m_3|m_1,m_2,m_3} = \inf_{Q:\, Q\in R^2_{l_2}} D(QOP||QOP_{m_2}) + \inf_{Q:\, Q\in R^1_{l_1}} D(QOP||QOP_{m_1})$ $m_3 = l_3$, $m_1 \neq l_1$, $m_2 \neq l_2$, (13.b)

$E^*_{l_1,l_2,m_3|m_1,m_2,m_3} = \inf_{Q:\, Q\in R^3_{l_3}} D(QOP||QOP_{m_3}) + \inf_{Q:\, Q\in R^1_{l_1}} D(QOP||QOP_{m_1})$ $m_2 = l_2$, $m_1 \neq l_1$, $m_3 \neq l_3$, (13.c)

$E^*_{l_1,l_2,m_3|m_1,m_2,m_3} = \inf_{Q:\, Q\in R^2_{l_2}} D(QOP||QOP_{m_2}) + \inf_{Q:\, Q\in R^3_{l_3}} D(QOP||QOP_{m_3})$ $m_1 = l_1$, $m_3 \neq l_3$, $m_2 \neq l_2$, (13.d)

$E^*_{l_1,l_2,l_3|m_1,m_2,m_3} = E^*_{m_1,m_2,l_3|m_1,m_2,m_3} + E^*_{m_1,l_2,m_3|m_1,m_2,m_3} + E^*_{l_1,m_2,m_3|m_1,m_2,m_3}$, $m_i \neq l_i$, $i = \overline{1,3}$, (13.e)

$E^*_{m_1,m_2,m_3|m_1,m_2,m_3} = \min_{(l_1,l_2,l_3)\neq(m_1,m_2,m_3)} E^*_{l_1,l_2,l_3|m_1,m_2,m_3}$, (13.f)

The optimal values of the reliabilities of the LAO tests sequence will be the following:

$E^*_{1,1,1|3,1,1} = E_{1,1,1|3,1,1}$, $E^*_{1,1,1|1,3,1} = E_{1,1,1|1,3,1}$, $E^*_{1,1,1|1,1,3} = E_{1,1,1|1,1,3}$, (14)
$E^*_{2,2,2|3,2,2} = E_{2,2,2|3,2,2}$, $E^*_{2,2,2|2,3,2} = E_{2,2,2|2,3,2}$, $E^*_{2,2,2|2,2,3} = E_{2,2,2|2,2,3}$,

Theorem3:

If all distributions QOP_m, $m = 1,2,3$, are different and consequently $D(QOP_l \| QOP_m) > 0$, $l \neq m$,
Then the following three statements are valid:
a) when given strictly positive elements $E_{1,1,1|3,1,1}$, $E_{2,2,2|3,2,2}$, $E_{1,1,1|1,3,1}$, $E_{2,2,2|2,3,2}$, $E_{1,1,1|1,13}$, $E_{2,2,2|2,2,3}$, meet the following conditions:

$E_{1,1,1|3,1,1} < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (15.a)
$E_{1,1,1|1,3,1} < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (15.b)
$E_{1,1,1|1,1,3} < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (15.c)
$E_{2,2,2|3,2,2} < min\, [E^*_{2,2,2|1,2,2}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (15.d)
$E_{2,2,2|2,3,2} < min\, [E^*_{2,2,2|2,1,2}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (15.e)
$E_{2,2,2|2,2,3} < min\, [E^*_{2,2,2|2,2,1}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (15.f)

Then:
a) There exists a LAO test sequence $\Phi^* \in \boldsymbol{\mathcal{A}}$, the reliability matrix of which E (Φ^*) is defined in (13) and all elements of it are positive,
b) When even one of the inequalities (15) is violated, then there exists at least one element of the matrix E (Φ^*) aqual to 0,
c) The reliability matrix E (Φ^*) of the tests Φ^* from the families $\boldsymbol{\mathcal{B}}$ necessarily contains equal to zero.
Proof:
a) we can write for three Markov chains as follows:

$0 < E_{1|3}(\varphi^1) < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (16.a)
$0 < E_{1|3}(\varphi^2) < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (16.b)
$0 < E_{1|3}(\varphi^3) < min\, [\inf_{Q_3} D(Q_3OP_3 \| Q_3OP_1), \inf_{Q_2} D(Q_2OP_2 \| Q_2OP_1)]$, (16.c)
$0 < E_{2|3}(\varphi^1) < min\, [E^{*1}_{2|1}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (16.d)
$0 < E_{2|3}(\varphi^2) < min\, [E^{*2}_{2|1}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (16.e)
$0 < E_{2|3}(\varphi^3) < min\, [E^{*3}_{2|1}, \inf_{Q_3} D(Q_3OP_3 \| Q_3OP_2)]$, (16.f)

We shal prove, for example, the inequality (16.e) which are the consequence of the inequality (15.e). consider the tests $\Phi = (\varphi^1, \varphi^2, \varphi^3) \in \mathcal{A}$ such that $E^*_{2,2,2|2,3,2}(\varphi) = E^*_{2,2,2|2,3,2}(\varphi)$ and $E^*_{2,2,2|2,1,2}(\varphi) = E^*_{2,2,2|2,1,2}(\varphi)$.

The corresponding error probabilities $\alpha_{2,2,2|2,3,2}(\varphi_N)$ and $\alpha_{2,2,2|2,1,2}(\varphi_N)$ are given as products defined by (5.c). According to (2) and (5) we obtain that:

$$E_{2,2,2|2,1,2}(\varphi) = E^{*2}_{2|1} + \overline{\lim_{N\to\infty}} log\ (1-\alpha^{(N)}{}_{2|2}(\Phi^2_N)) + \overline{\lim_{N\to\infty}} log\ (1-\alpha^{(N)}{}_{3|3}(\Phi^3_N)), \qquad (17.a)$$

$$E_{2,2,2|2,3,2}(\varphi) = E^{*2}_{2|3} + \overline{\lim_{N\to\infty}} log\ (1-\alpha^{(N)}{}_{2|2}(\Phi^2_N)) + \overline{\lim_{N\to\infty}} log\ (1-\alpha^{(N)}{}_{3|3}(\Phi^3_N)), \qquad (17.b)$$

According to (16.b), (16.e) and (17) we obtain that:
$E^{*2}_{2|1} = E_{2|1}(\varphi^2)$,
$E^{*2}_{2|3}(\varphi) = E_{2|3}(\varphi^2)$,
There (16.e) is consequence of (15). According to Theorem (1) there exist LAO sequences of tests φ^{1*}, φ^{2*} and φ^{3*} for the first, the second and the three such that the elements of the matrices E (φ^{1*}), E (φ^{2*}) and E (φ^{3*}) are determined according to (13). We consider the sequence of tests φ^*, which is composed of the three of sequences of tests φ^{1*}, φ^{2*} and φ^{3*}, also we will show that φ^* is LAO and other elements of the matrix E(φ^*) are determined according to (13).
From (15), (16) and remark (1) it follows that the requirements of Lemma are fulfilled. With using Lemma we can deduce that the reliability matrix E (φ^*) can be obtained from matrices E (φ^{1*}), E (φ^{2*}) and E (φ^{3*}) as in (10).
Now we show that the compound test Φ^* for three Markov chains is LAO, that it is optimal. Suppose that for given $E_{1,1,1|3,1,1}$, $E_{1,1,1|1,3,1}$, $E_{1,1,1|1,1,3}$, $E_{2,2,2|3,2,2}$, $E_{2,2,2|2,3,2}$ and $E_{2,2,2|2,2,3}$, there exist a test $\Phi' \in \mathcal{A}$ with matrix E(Φ'), such that it has at least one element exceeding the respective element of the matrix E(Φ^*).
This contradicts to the fact, that LAO tests have been used for the Markov chains X^1, X^1 and X^3.
b) When one of the inequalities (15) is violated, we see, some of elements in the matrix E (Φ^*) must be equal to zero.
c) When $\Phi \in \boldsymbol{\mathcal{B}}$, then from (15.e) and remark (1) it follows that the elements $E_{m',m',m'|3,3,3} = 0$.

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