The Influence of Familiarization and Conceptual Rewording in Facilitating the Students’ Performance in Solving Types of Mathematics Word Problems

Majid Haghverdi
Department of Mathematics, Islamic Azad University, Arak Branch, Arak, Iran

ABSTRACT

This paper aims to show how the Familiarization rewording, Conceptual rewording and their combination influence on facilitating the process of solving word problems. In this study, eighty students studying at fourth grade were randomly assigned to one of the mentioned conditions plus a control group. All the participants answered to a pretest (Standard test). Then all of them, except for the control group which received Standard test answered one of the three tests (FR, CR, and FR+CR). The results indicated that types of rewordings facilitated students’ performance in solving the given word problems. Further, the findings revealed that rewordings had influenced students ability in solving three types of word problems (Compare, Change and Combine) similarly.

KEYWORDS: Problem-Solving; Mathematics word problems; Familiarization Rewording; Conceptual Rewording.

INTRODUCTION

Mathematics word problems mostly deal with applying mathematical concepts in real world situations. In fact, such problems help students to use their mathematics knowledge in solving their daily problems.

On the other hand, many researchers indicated that students of various grades are able to use successfully calculation algorithms whereas they are not able to solve word problems which need the same algorithms (Mayer and Hegarty, 1996).

One of the most widely classifications of these types is proposed by Riley, Greeno and Haller (1983), who distinguished between Change, Combine and Compare problems. Change problems involve an action which results in an increase or decrease in quantity. A Change problem with an unknown change quantity is “Bong has 6 mangoes. He gave Kim some mangoes. Now Bong 2 mangoes left. How many mangoes did he give Kim?” Combine problem and Compare problems result to static relations between quantities. Combine problems involve sets and subsets. For example, a Combine problem “Lisa has 3 pencils, Rosa has 4 pencils. How many pencils do they have altogether?” Compare word problems are built with a relational statement that brings together the two sets to be compared. “Tino caught 6 crabs. Nato caught 2 crabs. How many more crabs did Tino catch than Nato?” is an example of a Compare problem (Bautista et al, 2009).

The results of many studies implicate language in the solving of word problems (Bautista and et al. 2007). Reusser (1990) in his situation problem solver (SPS) model starts from a linguistic approach, postulating that the understanding of problems requires the interaction of linguistic knowledge, familiarity with the real world situations. Understanding of the familiar situation in a task limits or facilitates process of the problem solving (Reusser, 1990).

Students’ difficulties in problem solving stem from insufficient comprehension outlined in the task. A series of studies support the importance of the situational context for solving word problems (Davis-Dorsey and et al., 1991; Stern, Lehmendorfer, 1992; Reusser, 1989). More familiar context and context related to the solver facilitate the establishment of the relations between the problem and its existing structures.

Researchers have discovered several ways to avoid errors and to enhance comprehension, such as rewording the problem. The influence of rewording types on solving word problem is one of results in the most widely research which has been reported (Cummins, 1991).

Hudson (1983) may be the first who showed that changing the textual of word problems influenced the difficulty of word problems. He presented kindergarten and children at first grade five pictures of birds and two pictures of worms by rewording in two forms (a) and (b) with similar structures.

a) Here are 5 birds and here are 3 worms. How many birds are more the worms?
b) Here are 5 birds and 3 worms. Suppose that every bird tries to catch a worm. How many birds won't catch a worm?

It is clear that problem (a) is the Standard form and the second one (b) has been changed through rewording. He found that whereas 96% of the 6-year-old children could solve the following problem (b), only 25% could solve the problem (a).

Studies on the reforming of word problems for improving students' understanding can be classified into two groups: Studies that focused on changing described context in problems (Davis-Dorsey et al., 1991; Cummins et al., 1988; Stub and Reusser, 1992; Stern and Lehrndorfer, 1992) and other focused on changing the textual of problem-rewording (De Corte et al., 1985; Cummins, 1991; Davis-Dorsey et al., 1991; Eric, 2005; Vicente and et al., 2007).

**Familiarization rewording**

In studies related to changing of the problem context, situation embedded in the problem was varied. Context of a word problem is the non-mathematical content problems, such as the story stated in a problem.

Some researches on changing of context have pointed out the influence of data personalization, for example, entering student names and other personal information in the problem text (Wright, 1986, Davis-Dorsey et al. 1991, Hart 1996, Wiest 2002). Some other have considered introducing various topics interesting for students such as animals, sport and so on, as effective in problems (Wiest, 2001). As well, studies have shown that concrete versus abstract and factual versus hypothetical contexts for mathematics word problems can have an impact on performance in high school students. Caldwell and Goldin (1987) considered differing levels of abstraction seem to influence problem-solving performance.

There are many different ways for changing of context of word problems; but getting more familiarized with the described situation for problem solver is more common (Hembery, 1992).

The described situation in a problem is counted as a familiar context when the student has a mental schema of the situation, or personally has his/her own experiences about it. Otherwise is counted as an unfamiliar context. Huang (2004) and Cankoy and Ozder (2011) in his study also defined familiar situation in problem for students, as situations which happen in their everyday experiences and school activities.

In several studies, it has been shown that word problems become easier when they are embedded in a familiar context (De Cort et al., 1985; Davies-Dorsy, 1991), for example, if a subject's own name or names of their pets are used. The familiar names may cause children to pay more attention and, moreover, it is easier to remember a familiar name than an unfamiliar one (Stern and Lehrndorfer, 1992).

In following problem, you would find an unfamiliar situation context in has been given to the students.

"The temperature in Ardebil is 10 degrees lower than Tehran. The temperature in Arak is 4 degrees more than Ardebil. The temperature in Tehran is 15 C. What is the temperature in Arak?"

By changing unfamiliar situation of described in the problem to a familiar one, familiarization rewording is carried out and the problem is restated in following form:

"Ali's score in mathematics exam is 10 grades lower than the literature exam, and in geography exam it is 4 grades better than the mathematics' one. Ali gets score 15 in the literature exam. What is his score in the geography exam?"

**CONCEPTUAL REWORDING**

Empirical studies have showed that small changes in the wording of problem texts have a remarkable effect on students' problem solving performance.

In rewording of word problems, the semantic relations between the sets implied in the problem are stated more explicitly and more transparently. Also, the text of the problem is modified without changing its semantic structure which is called Conceptual rewording (Vicente and et al, 2007). In this rewording some statements maybe added to the problem or by changing the statement arrangement (Eric, 2005); and this rewording makes more explicit, the mathematics/semantic relations between the given and unknown sets (Vicente and et al., 2007). The following word problem is suitable example for Conceptual rewording:

The temperature in Ardebil is 10 degrees lower than Tehran. The temperature in Arak is 4 degrees more than Ardebil. The temperature in Tehran is 15 C. What is the temperature in Arak?"

This problem is became changed into Conceptual rewording form as following:

"The temperature in Tehran is 15 C. The temperature in Tehran is 10 degrees more than Ardebil. The weather in Tehran is warmer than Ardabil. The temperature in Ardabil is 4 degrees lower than Arak. What is the temperature in Arak?"

As it can be seen, in Conceptual rewording by changing the statements arrangement and adding some words to problem text with "italic" form are clarified semantic relations.

Although the influence of the rewording and the level of problems difficulty especially in Change, Compare and Combine problems have been studied by many investigators (Lewis and Mayer 1987, Cummins 1991). But lack of studies in the field of the effects of rewording in types of problems is completely felt. So based on the reviewed literatures, the following question and hypotheses have been developed:
"Which types of rewordinings (FR, CR and FR+CR) has more effect on facilitating solving types of word problems both generally and separately?"

**HYPOTHESES**

1. The use of the types of rewording (FR, CR, and FR+CR) influenced on facilitating solving word problems generally.
2. Applying FR, CR and FR+CR influenced on facilitating solving compare word problems
3. Applying FR, CR and FR+CR influenced on facilitating solving change word problems
4. Applying FR, CR and FR+CR influenced on facilitating solving combine word problems.
5. There no difference in applying 3 types of rewording in facilitating solving word problems.
6. Applying the types of rewording have the same influence in facilitating solving compare, change and combine word problems.

**METHODOLOGY**

Independent variable is rewording which it had three level/groups. In examining influence of types of rewording on facilitating problem solving, dependent variable is the score obtained in problem solving test. This score is resulted from the scores obtained in compare, change and combine word problems.

**Subjects** were 80 students at the fourth grade from a school of Arak city. They were divided into four groups. In each group 20 students were randomly assigned. The average age of the subjects was 10 years approximately.

**Research Tools** were four mathematics paper-and-pencil tests that were created by the researchers. Tests FR, CR, FR+CR have been related to three independent variables, Familiarization rewording, Conceptual rewording and their combination. Likewise, test Standard has been used for Standard test and pre-test for all groups and also for post-test of control group as well.

The Standard test consisted of six one-step arithmetic word problems (2 Compare problems, 2 Change problems and 2 Combine problems). For this test, the problems have been selected such that they have two characteristics. First, the situation of these problems was unfamiliar for subjects. Second, text of problems lacked extra description of relations and concepts.

In the beginning, all of word problems of mathematics textbook from four-grade makes a list to prepare text Standard and 6 unfamiliar situations for students were selected from among these problems with using Delphi Method (according to Fish and Busby, 2005; Adler & Ziglio, 1996; Skulmosk & Hartman, 2002) by 5 mathematics head teachers (who were introduced by Arak Education Organization).

As mentioned in introduction, problems with unfamiliar situations make their situation comprehension more difficult for students to understand.

Test FR has been created based on independent variable rewording of the problems. This test is like to test Standard and it is made only with varying described situation in problem to a familiar situation. This test was prepared by same 5 persons. They had agreement on 6 situations with Delphi method by reviewing familiar situations in textbooks.

Test CR was prepared based on changing statements arrangements in problems and reformation on some of their statements in order to make explicit the semantic relations between the given and unknown sets which mentioned in the introduction and also confirmed by the same teachers. We used problem rewording, using the procedure developed by De Corte et al. (1985) and Davis-Dorsey and et al. (1991) to make the problem representation more explicit and thus easier to translate through the clear identification of key elements and their interrelations. Test FR+CR was a combination of both them, and it created by applying two types of changes (refer to appendix).

**RESULTS**

After performing the Standard, FR, CR and FR+CR tests the students' responses were coded; two points for correct final response and one point for appropriate process of solution, but without correct final response and otherwise zero point was given.

The mean of the scores obtained by students are shown in Table 1. After getting the results of the tests, data was gathered and analyzed using one-way ANOVA design regarding research hypotheses.

With regard to Table (1) and results obtained, for hypotheses it is deduced:

1. Applying FR, CR and FR+CR influenced on facilitating solving word problems generally (F(3,76)= 42.54, p<0.05).
2. Applying FR, CR and FR+CR influenced on facilitating solving compare word problems.

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1. Arak is a city of Markazi province of Iran.
(F(3,76)= 10.55, p<0.05).
3. Applying FR, CR and FR+CR influenced on facilitating solving change word problems (F(3,76)= 11.78, p<0.05).
4. Applying FR, CR and FR+CR influenced on facilitating solving combine word problems (F(3,76)= 11.78, p<0.05).

Therefore hypotheses (1), (2), (3) and (4) are confirmed.

**Table 1. Means students’ scores**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Contextual</td>
</tr>
<tr>
<td>Compare</td>
<td>1.40</td>
<td>1.50</td>
</tr>
<tr>
<td>Change</td>
<td>1.45</td>
<td>1.55</td>
</tr>
<tr>
<td>Combine</td>
<td>1.60</td>
<td>1.75</td>
</tr>
<tr>
<td>Total Score</td>
<td>4.45</td>
<td>4.80</td>
</tr>
</tbody>
</table>

In addition, Post-hoc Tukey’s analysis was performed for the comparison of the mean of tests that their results are shown in Table 2.

Based on results obtained from Post-hoc Tukey implying that types of rewording of problems were effective on facilitating the solving mathematics word problems. The results presented in Table 2 show that the mean difference of pairs AD, BD, CD, A₁D₁, A₂D₂,A₃D₃ in regarding to comparison the kinds of reforming of word problems is significant (p<0.05). But pairs of AB, AC, BC and also A₁B₁, A₁C₁, B₁C₁, A₂B₂, A₂C₂, B₂C₂, A₃B₃, A₃C₃, B₃C₃ in regarding to types of problems (Compare, Change and Combine) are not significant (p>0.05). The results from Tukey post-test indicate the significant cases are related to difference of each type of the rewordings problems with control group. the following results are deduced:

**Table 2. Post-hoc Tukey: Comparison of the mean tests for students' scores**

<table>
<thead>
<tr>
<th>Compared pairs</th>
<th>AB</th>
<th>AC</th>
<th>AD</th>
<th>BC</th>
<th>BD</th>
<th>CD</th>
<th>A₁B₁</th>
<th>A₁C₁</th>
<th>A₂D₁</th>
<th>B₁C₁</th>
<th>A₂B₂</th>
<th>A₂C₂</th>
<th>A₂D₂</th>
<th>B₂C₂</th>
<th>A₃B₃</th>
<th>A₃C₂</th>
<th>A₃D₃</th>
<th>B₃C₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean difference</td>
<td>0.5</td>
<td>0.45</td>
<td>4.35</td>
<td>.95</td>
<td>3.85</td>
<td>4.8</td>
<td>0.2</td>
<td>0.2</td>
<td>1.5</td>
<td>0.4</td>
<td>0.0</td>
<td>0.25</td>
<td>1.35</td>
<td>0.25</td>
<td>0.3</td>
<td>0.0</td>
<td>1.5</td>
<td>0.3</td>
</tr>
<tr>
<td>P</td>
<td>0.72</td>
<td>0.78</td>
<td>0.00</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.76</td>
<td>0.76</td>
<td>0.00</td>
<td>0.21</td>
<td>1.00</td>
<td>0.73</td>
<td>0.00</td>
<td>0.73</td>
<td>0.60</td>
<td>1.00</td>
<td>0.00</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Note: types of rewordings, A= familiarization rewording; B= Conceptual Rewording; C= familiarization-conceptual rewording; Type of Problems, 1= Change problem; 2=Compare problem; 3=Combine problem

5. There no difference in applying 3 types of rewording in facilitating solving word problems.
6. Applying the types of rewording have the same influence in facilitating solving compare, change and combine word problems.

Therefore hypotheses (5) and (6) are rejected.

**DISCUSSION**

This study examined influence of types of rewording of problems (FR, CR, and FR+CR) in facilitating solving Compare, Change and Combine word problems.

The results obtained showed that there are positive effects for types of rewording (FR, CR, FR+CR) facilitating solving mathematics word problems.

These results are in line with Eric’s study (2005), who concluded that types of rewording has a positive effect on students’ performance in solving word problems, especially when the rewording was based on "Chronological order of events", "Personalization", "Chunking” and "Repositioning the givens". Findings of this research also support previous studies such as, Hudson (1983), De Corte (1985), Davis-Dorsey (1991), Cummins (1991), Vicente and et al. (2007, 2008).

Although Vicente and et al. (2007) concluded that using of structural changes in problem have more effect on problem solving performance than situational changes, but results of our study showed that the familiarizing the context of problem have effect in process of understanding and solving word problems.

In many research studies it was noticed that familiar contexts enhance word problem solving by increasing the meaningfulness of contexts and motivating students to solve the problems (Cordova & Lepper 1996, Lopez & Sullivan 1992, Ku & Sullivan 2002, Conko & Ozder 2011).

These results revealed the need to familiarize the described situation via entering information of personal and interested for students (Davis Dorsey 1992, Wiest 2002) and through adding the abstract and factual situations (Caldwell and Goldin, 1987) for them as well. Because according to Reusser's SPS model, familiar
contexts facilitate comprehension of situation in a given task which is the most important step in problem solving. Furthermore, Hart (1996) claimed that through personalizing word problems, students' background knowledge could be tapped such that helping them to bridge between existing and new knowledge.

In summing up, the results of research show that FR and CR helping grade 4 students, equally, in facilitating solving word problems and there is no any preference between these two types of rewording.

Other finding of this study was examining the influence of types of the rewording in three types of Compare, Change and Combine problems. By comparing the performance of subjects with control group it is showed that there was not significant difference between them after rewording problems in types of problems. Further, the findings revealed that rewordings had influenced students ability in solving three types of word problems (Compare, Change and Combine) similarly. Thus changing of the problem context or making more explicit semantic relation of problems had not significant effect on their performance in types of word problems.

In this respect, familiarization rewording and conceptual rewording can help overcome some of the difficulties that students experience in learning to solve word problems. Therefore the teachers can use all types of rewording to help the students in solving word problems.

Limitations
Concerning, the necessity of the using contextual reforming of problems, this article aims at studying the effect of familiarization rewording in process of understanding and comprehending the problems. While this study intends not to emphasize merely on solving problems with familiar contexts in that case it make the students be able only to solve the limited number of problems. So the results of this study suggest that the authors of the textbooks should pay more attention to the effect of contextual factors in solving word problems by selecting the logical appropriate contexts for each every grade.

Therefore cognition and control these factors will help the students to improve their problem solving skills. In this study number of familiar and unfamiliar situation were selected out of the problems in mathematics textbooks based on the suggestion of the mathematics head teachers according to Delphi method. These selections originated typically in the Iranian mathematics textbooks. So, it could somewhat possible to extend this method other countries mathematics textbooks.

In spite of, picking the problem original wording from Farsi language, I did my best to shift them into the most appropriate possible English. I wonder if I am successful in that enough.

REFERENCES


Appendix

Standard test

1. 17 people are working at part A in a factory. 65 people are working at a factory. How many people are working at part B? (Combine 4)
2. There are 650 kilogram cotton in stock 1. There are 1600 kilogram of cotton in stock 1 and 2. There are some cotton in stock 2. How much cotton is in stock 2? (Combine 4)
3. The temperature in Ardebil is 10 degrees lower than Tehran. The temperature in Tehran is 15 C. What is the temperature in Ardabil?" (Compare 4)
4. Hamid is 7 years younger than Ali. Hamid is 25 years old. How old is Ali? (Compare 4)
5. A water engine supplied 120 cubic meters water yesterday. The farm needs 300 cubic meters water. How much water should be supplied? (Change 2)
6. The temperature of a liquid reached to 4 ° C by a cooling system. If the temperature of liquid has been 28 ° C at first. How many degree the temperature has been changed? (Change 2)

Test (FR)

1. 17 children are drinking apple juice. 65 children are drinking juice. How many children are drinking grape juice? (Combine 4)
2. There are 6 pencils in desk 1. There are some pencils in stock 2. There are 16 pencils in desk 1 and 2. How many pencils are in stock 2? (Combine 4)
3. Ali’s score in mathematics is 10 grades lower than the literature. Ali’s score in literature is 15. What is his score in mathematics? (Compare 4)
4. Hamid has 7 pens less than Ali. Hamid has 25 pens. How many pens does Ali have? (Compare 4)
5. There are 120 cakes in a carton. There are 300 students in school. How many cakes are needed in this school? (Change 2)
6. An elevator is in 4 floors now. If the elevator has been at 28 floor at first. How many floors travels the elevator traveled? (Change 2)

Test (CR)

1. 65 people are working at part A and part B in a factory. 17 people are working at part A. The rest of the people are working at part B. How many people are working at part B? (Combine 4)
2. There are some cotton in the stocks of factory. There are 600 kilogram of cotton in stock 1. And also there are some cotton in stock 2. There are 1600 kg of cotton in both stock altogether. How much cotton is in stock 2? (Combine 4)
3. The temperature in Tehran is 15 C. The temperature in Ardabil is 10 degrees lower than Tehran. The weather in Tehran is warmer than Ardabil. What is the temperature in Ardabil? (Compare 4)
4. Hamid is 25 years old. Ali is older than Hamid. Hmid is 7 years younger than Ali. How old is Ali? (Compare 4)
5. A farm needs totally 300 cubic meters water. Yesterday an engine supplied 120 cubic meters water. How much more water should be supplied? (Change 2)
6. The liquid temperature is 28 ° C at first. A cooling system decreased the temperature of liquid and it reached to 4 ° C. How many degree the temperature has been changed? (Change 2)

Test (FR+CR)

1. 65 children are drinking juice. 17 children are drinking apple juice. The rest of the children are drinking grape juice. How many children are drinking grape juice? (Combine 4)
2. There are some pencils in two desks in room. There are 6 pencils in desk 1. There are some pencils in desk 2. There are 16 pencils in both desks altogether. How many pencils is in desk 2? (Combine 4)
3. Ali’s score in literature is 15. His score in mathematics is lower than the literature by 10 grades. What is Ali’s score in mathematics? (Compare 4)
4. Hamid has 25 marbles. Ali’s marbles is more than Hamid’s by 7. How many marbles does Ali have? (Compare 4)
5. There are 300 students in school. There were 120 cakes in carton. How many cakes are needed? (Change 2)
6. An elevator is at 28 floor at first. Then it moves to -4 floors. How many floor the elevator traveled? (Change 2)