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Motion of a Small Spherical Particle in a Fluid Vortex

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ABSTRACT

In this paper, the differential equations of the rotation of a small sphere suspended in a fluid vortex are written in the radial and tangential directions. Two types of vortices are considered; the forced vortex and the free vortex. Solutions are obtained numerically for different values of the non-dimensional parameter α^2 giving the relative magnitude of the fluid drag to the mass of the particle. Solutions for the radial displacement R, the radial velocity U and the angular velocity $\bar{\omega}$ are given in tables versus time and are represented in graphs. Comparison is made between the solutions of the two kinds of vortices.
KEY WORDS: Vortex, drag, rotation, angular velocity, centrifugal force, tornado.

INTRODUCTION

Vortices occur in fluid flows due to pressure variations and shearing flows. In oceans and seas ships avoid entering zones of vortices. Also, aero-planes frequently enter atmospheric vortices causing strong vibrations and great risks. In scientific laboratories; centrifugation and Couette viscometer techniques are encountered; both depend on vortex motion. In industry, separation of different dispersed species is performed by centrifugation. Centrifugal filters are also known.

The fluid dynamical models of vortices are the forced and the free vortex. Rotation in the forced vortex is analogous to rigid body rotation where the radial displacement of fluid particles is absent. In the tangential motion the fluid velocity is given by Ωr[1]; here Ω is the angular velocity of the vortex and r is the radial position from the center of rotation. In the free vortex; radial velocity is absent and during free rotation the angular momentum of any fluid particle is conserved. The tangential velocity in free vortex is given by $\frac{c}{r}$[2], c is constant and r is the radial position also. Aside from this difference the equations- as will be shown later- are the same.

To list some of the references on the subject matter, first we refer to the paper by Domon and Watanabe[3] on mass transport by a vortex ring. Uchiyama and Yagami[4]gave a study of vortex simulation for the interaction between a vortex ring and solid particles.They[5] also gave numerical simulation for the collision between a vortex ring and solid particles. Uchiyama and Naruse[6]gavethree-dimensional vortex simulation for particulate jet generated by free falling particles. Earlier Crowe et al.[7] described particle dispersion by coherent structures in free shear flows. Finally,Chein and Chung[8] studied effects of vortex pairing on particle dispersion in turbulent shear flows.

Formulation of the problem

Consider the polar coordinates$r-\theta$ with pole at the origin $r=0$. The mass of fluid is rotating in the counterclockwise direction around the origin. The stream lines are concentric circles with common center at the origin. For forced vortex, the tangential velocity is given by $u_\theta = \Omega r$ whereas for free vortex $u_\theta = \frac{c}{r}$, Ω and c are constants. In both cases the radial components of velocity of fluid are zero.

At the instant $t = 0$ a small spherical particle is released in the flow field with initial radius r_0; zero angular velocity and $u_0$as radial component of velocity. The fluid will drive the particle to rotation, exerting drag force [9]

$$F_{Drag} = c_D A \frac{1}{2} \rho u^2$$

$u$is the relative velocity between the particle and the flow; ρ is the mass density of fluid; A is the projected area of sphere and c_D is the drag coefficient and is a function of the Reynolds's number $\frac{\rho u d}{\mu}$. $d$is the diameter and μ is the fluid viscosity.

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The equations of motion of the particle can be written as $m\underline{a} = \underline{F}$; m is the mass of the particle; $\underline{a}$ is the acceleration and $\underline{F}$ is the exerted external force on particle. In the radial and tangential directions respectively; the equations are

$$m(\ddot{r} - r\dot{\theta}^2) = -c_D A \frac{1}{2} \rho \dot{r}^2 \tag{1}$$

$$m(r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta}) = c_D A \frac{1}{2} \rho (r\dot{\theta} - u_{\theta fluid})^2 \tag{2}$$

In equation (2), $u_{\theta fluid} = \Omega r$ for the forced vortex and $u_{\theta fluid} = \frac{c}{r}$ for free vortex.
The quantities to the left are the radial and tangential components of acceleration in polar coordinates.

Now, the two equations are non-dimensionalized by using r_0 for distances and $\Omega t = \tau$ for non-dimensional time with $\overline{\omega} = \frac{\omega}{\Omega}$, $r_0 R = r$, then $u_0 U = u$.
Non-dimensional equations are

$$\dot{R} = \frac{u_0}{r_0\,\Omega} U \tag{3}$$

$$\frac{u_0}{r_0\,\Omega} \dot{U} - R\overline{\omega}^2 = -\alpha^2 \left(\frac{u_0}{r_0\,\Omega}\right)^2 U^2 \tag{4}$$

$$R\dot{\overline{\omega}} + \frac{u_0}{r_0\,\Omega}\, 2U\overline{\omega} = \alpha^2 R^2 \begin{cases} (\overline{\omega} - 1)^2 \text{Forced vortex} \\ \left(\overline{\omega} - \frac{1}{R^2}\right)^2 \text{Free vortex} \end{cases} \tag{5}$$

The parameter $\alpha^2 = \frac{c_D A \frac{1}{2} \rho r_0}{m}$ (non-dimensional) represents the drag to inertia ratio. No analytical solution is possible and only numerical solution is possible.

Numerical solution and results

We seek a solution of $R(\tau)$, $\overline{\omega}(\tau)$ and $U(\tau)$ of the equations (3), (4) and (5) subject to the initial conditions indicated twovalues of $\alpha^2 = 0.1$ and 1 are considered. Solution is carried out using Runge-Kutta of forth order's method[10], the step size is 0.1, time is increased to reach 4 for $\alpha^2 = 0.1$and to the value 0.8 for $\alpha^2 = 1.0$ for forced and free vortex. The results of this calculation are summed up in the following tables and graphs as indicated in the captions. The accuracy of this calculation is obtained by using smaller time $\tau = 0.05$ for the value of R at $\tau = 4$, the maximum error according to Atkinson et al.[10] is found to be of O(10^{-6}).

Table1:Solution for forced vortex at $\alpha^2 = 0.1$

τ	R	U	$\overline{\omega}$
.000	1.0000	1.0000	.0000
.400	1.3922	.9618	.0380
.800	1.7701	.9284	.0813
1.200	2.1362	.9044	.1372
1.600	2.4962	.9003	.2114
2.000	2.8622	.9400	.3109
2.400	3.2607	1.0750	.4450
2.800	3.7490	1.4154	.6250
3.200	4.4506	2.1969	.8652
3.600	5.6342	3.9633	1.2007
4.000	7.9642	8.4865	1.8216

Table 2: Solution for forced vortex at$\alpha^2 = 1.0$

τ	R	U	$\overline{\omega}$
.000	1.0000	1.0000	.0000
.100	1.0953	.9095	.1071
.300	1.2634	.7843	.4092
.400	1.3403	.7614	.6505
.500	1.4175	.7965	1.0177
.600	1.5041	.9731	1.6405
.700	1.6256	1.5931	2.9046
.800	1.8834	4.4674	6.8416

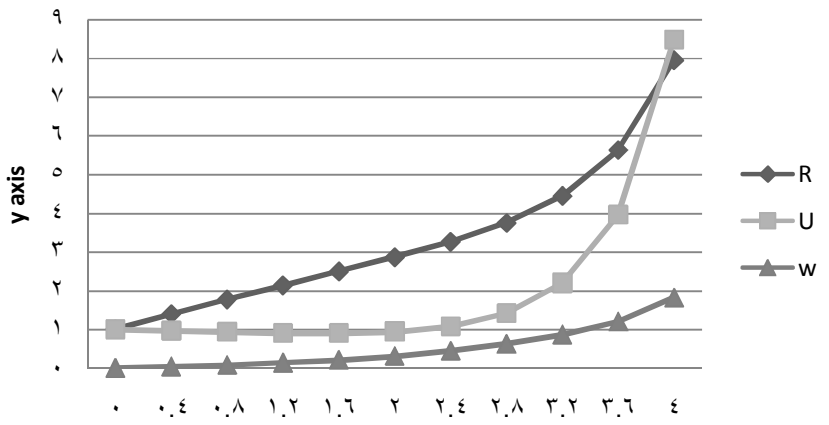
Table 3:Solutions for free vortex at$\alpha^2 = 0.1$

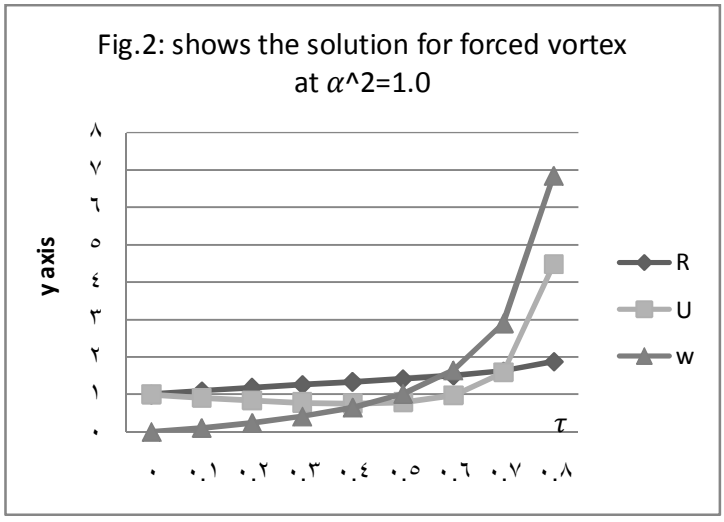
τ	R	U	$\overline{\omega}$
.000	1.0000	1.0000	0000
.400	1.3922	.9616	.0180
.800	1.7697	.9262	.0201
\1.200	2.1335	.8934	.0190
1.600	2.4847	.8629	.0174
2.000	2.8241	.8344	.0158
2.400	3.1525	.8077	.0144
2.800	3.4705	.7827	.0132
3.200	3.7788	.7591	.0122
3.600	4.07C Z80	.7370	.0114
4.000	4.3686	.7161	.0106

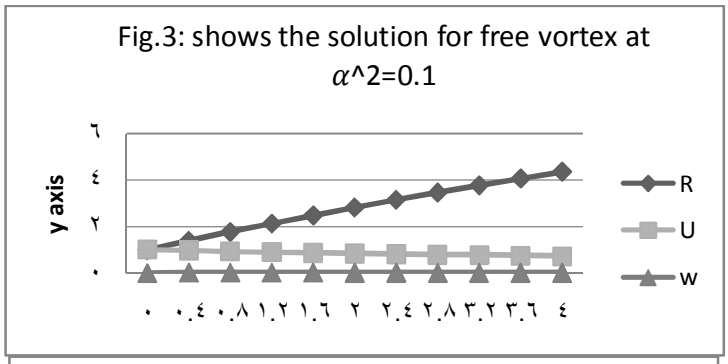
Table 4: solution for free vortex at $\alpha^2 = 1.0$

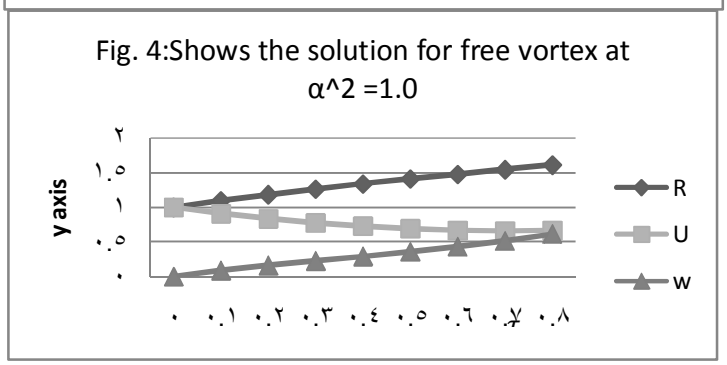
τ	R	U	$\overline{\omega}$
.000	1.0000	1.0000	.0000
.100	1.0953	.9094	.0880
.200	1.1824	.8353	.1605
.300	1.2628	.7752	.2261
.400	1.3379	.7276	.2903
.500	1.4088	.6918	.3571
.600	1.4767	.6681	.4303
.700	1.5428	.6580	.5141
.800	1.6088	.6645	.6140

Fig.1: shows the solution for forced vortex at α^2=0.1









Conclusion

The value of c for the free vortex used in this paper Ωr_0^2.

We indicate that for the radial motion of the particle, the fluid resists the motion, therefore the fluid force on the particle is taken negative. For the rotation positive fluid force is considered, since the fluid assists rotation.

The coupling between U, R and$\overline{\omega}$ in the equations of motions is complicated. Due to this fact it is difficult to discuss any results without complete solutions. Solutions could only be obtained numerically since the equations are nonlinear. However, radial motion starts with deceleration as $\overline{\omega}$ starts with zero value. As $\overline{\omega}$ builds up, the radial accelerationvanishes when $R\overline{\omega}^2 = \alpha^2 \left(\frac{u_0}{r_0\Omega}\right)^2 U^2$. The radius R and $\overline{\omega}$are ever increasing in both forced and free vortices. We observe also that $\overline{\omega}$ for the free vortex increases strongly with reduced R.

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