Recognition of Students’ Difficulties in Solving Mathematical Word Problems from the Viewpoint of Teachers

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ABSTRACT

This study attempted to detect students’ difficulties in solving mathematical word problems from their teachers' perspectives. Participants were 52 mathematics teachers of Arak middle schools whom were chosen randomly. The results showed that the students' difficulties were mostly sprung from their disabilities in representation and understanding of word problems, making a plan and defining the related vocabularies. The findings revealed that, the causes of the student difficulties were text difficulties, unfamiliar contexts in problems and using inappropriate strategies. Finally teachers suggested to help students in teaching them to look for a pattern, draw a picture and rewording the problems.

KEYWORDS: Student’s Mathematical Word Problems, Unfamiliar Contexts, Text Difficulties.

INTRODUCTION

A common view among most of the researchers, mathematics teachers, students and parents is that, doing mathematics is considered as the heart of mathematics (Cankoy & Ozder 2011, Cockcroft 1982, Kaur 1997, NCTM 2000, Schoenfeld 1985). An important component of mathematics training is solving word problem. Real-world problems that require mathematics for solution typically do not come to use as equations ready to be solved but rather as word or pictorial representations that must be interpreted symbolically, manipulated, and solved. It is for this reason that word problems are introduced in the earliest stages of mathematics instruction (Cummins, 1991). Verschafel et al. (2000) defined word problems as verbal descriptions of problem situation wherein one or more question are raised, the answer to which can be obtained by the application of mathematical operations to numerical data available in problem statement.

The mathematics word problems among mathematic problems mostly deal with relating the real world situations to mathematical concepts. In fact, such problems help students to use their mathematics knowledge in solving their daily problems. The mathematics word problems are known as instruments which develop the students’ ability and talent in solving math problems (De Coeet et al., 1989). On the other hand, results obtained from numerous researches indicate that most of the students in various academic grades are facing with many difficulties in their trying to solve such problems. These students are able to use successfully calculation algorithms whereas they are not able to solve word problems which need the same algorithms (Mayer and Hegarty, 1996). Geary (1994) says "children make errors when solving word problems than solving comparable number problems". The reason for such inability is the fact that solving such problems demands mathematical computations along with other kinds of knowledge including linguistic knowledge, which are required for understanding the problems (Cummins et al, 1988).

The presence of a high percentage of word problems in mathematics textbooks led the authors to conduct a more comprehensive search of the literature on word problems and problem solving. We found that these problems have been alternately referred to in the literature as story problems, word problems, word problems and problem solving situations and that helping students read and understand these word problems has been a reoccurring topic in professional literature for the last century.

Numerous researchers reported that teachers have many difficulties when solving arithmetic word problems. Weber (1966) wrote about the difficulty students and teachers had with arithmetic word problems, labeling them "demon problem". Interestingly, it appears that the mathematics community has addressed the topic of word problems under the larger concept of problem solving since the 1980s. One call for classroom attention to problem solving strategies came from the National Council of Teachers of Mathematics (2000), which contended in its widely-read and cited Principles and Standards of School Mathematics, "Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. The standards also suggested that teachers give students opportunities for the application of problem solving strategies across all mathematics content areas. In NCTM's (2006) latest publication,
Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics, problem solving continues to be a key theme. While recent literature on best practices for problem solving in mathematics is prolific (Ednes and Potter 2008, Sanchez et al 2002, Cifarelli and Sheets 2009), no current information was found on how teachers addressed mathematical word problems instructing their students.

There are many factors which contribute to word problems. In several studies, it has been shown that word problems become easier when they are embedded in a familiar context (De Corte et al, 1985, Davies-Dorsey 1991). The familiar contexts may cause children to pay more attention and, moreover, it is easier to remember a familiar situation than an unfamiliar one (Stern and Lehrndorfer, 1992).

Although the influence of different factors in solving mathematics word problems have been studied, But, until now, not any research has been done in order to examine teachers’ viewpoints of student difficulties in solving mathematics word problems.

So based on the reviewed literatures, the purpose of this study was to identify what teachers reported as their students difficulties in solving mathematics word problems and causes of those difficulties. This study also investigated teachers' perspectives on why the ability to solve word problems is important for their students and what classroom practices and specific strategies they use in their attempts to foster student problem solving success.

METHOD

Participants were 52 mathematics teachers (18 female & 34 male) from Arak middle schools whom were selected randomly. Data were gathered through the use of an interview guide. Johnson and Christensen (2011) described an interview guide as a common protocol that an interviewer follows while interviewing subjects. The inclusion of an interview protocol and a series of open-ended questions allow the interviewer to obtain both qualitative and quantitative data. The authors constructed an interview guide for use in teacher interviews that consisted of an interview protocol and open-ended questions which addressed the purpose of the study.

The completed interview guide was examined by two professors of education and two teachers not involved in the study, who checked the guide and questions for clarity and completeness. The final interview guide contained the protocol and 8 open-ended questions, which were then field-tested on three graduate students. The interview questions were organized in appendix.

Two of the authors independently conducted a content analysis of the transcriptions of the teacher interviews to determine the commonalities and trends. For this study, we began with an a priori approach to content analysis. The first step in the content analysis was the creation of an a priori checklist that contained 8 categories based on the open-ended questions and the purpose of the study. One category existed for each of the 8 questions in the interview guide. This is consistent with Stemler’s (2001) recommendations for conducting a content analysis. Two of the authors then did an initial content analysis using the a priori categories on seven of the transcriptions. This was done to determine whether the a priori categories were appropriate. The coding process consisted of each of the two authors independently reading the seven transcripts and identifying units of meaning and designating them into appropriate categories. The authors met and compared coded information on the seven transcriptions analyzed. The authors found a high degree of agreement on the designated information from the transcripts and some disagreement on placement in categories.

RESULTS

Teacher’s responses in interviews were tabulated and compared. Findings from the analysis of interviews with Grade 6-8 teachers are showed in Table (1) when asked in Question 2 to describe any difficulties that their students have when working mathematical word problems.

Table (1): Teacher Reported Student Difficulty

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>Total</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representation and understanding the problem</td>
<td>15</td>
<td>18</td>
<td>12</td>
<td>45</td>
<td>51%</td>
</tr>
<tr>
<td>Making a plan</td>
<td>13</td>
<td>8</td>
<td>2</td>
<td>27</td>
<td>31%</td>
</tr>
<tr>
<td>Vocabulary</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>10%</td>
</tr>
<tr>
<td>Background knowledge</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3%</td>
</tr>
<tr>
<td>Higher level thinking</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2%</td>
</tr>
<tr>
<td>Determining reasonableness</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2%</td>
</tr>
<tr>
<td>Computation</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1%</td>
</tr>
</tbody>
</table>

The results of the content analysis demonstrated that almost half of the teacher’s responses (51%) indicated that solving mathematics word problems is difficult for students because students struggle with representation and understanding the problems. Two other difficulties cited in teachers’ responses involved students’ inability to make a plan to solve the problem (31%) and a lack of vocabulary knowledge (10%).
Presented in Table (2) are the findings from the analysis of teachers’ responses when they were in Questions 3 and 4 of the interview guide about their perspectives of the causes of student difficulties when solving word problems. As Table (2) shows, responses that addressed the causes of student difficulties fell into four areas: text difficulties (39%), unfamiliar contexts in problems (26%), using inappropriate strategies (17%) and language factors (12%) were the least teacher-reported causes of student difficulties.

Table (3) contains the finding from the analysis of teachers’ responses about the classroom practices the teachers used for teaching word problems. The data analyzed came from interviewee responses to Question 5, of interview guide concerning what practices the teachers used when teaching word problems and which practice they would consider as their best method.

The results from these open-ended questions demonstrated that teachers identified using six different classroom practices for problem solving instruction, with the responses varying only slightly across grade levels. Having the students solve the word problems independently (41%) was the most frequently cited practice. Cooperative grouping (23%) and explaining of the situations (17%) were also reported as classroom practices for mathematics word problem instruction. The findings from the content analysis of teachers’ responses when they were asked, in Questions 6, 7 and 8 of the interview guide, what strategies they taught their students to use when solving word problems are contained in Table (4).

The most popular strategy reported by teachers was teaching the students "looking for a pattern" (51%). Additional strategies frequently cited by teachers were: drawing a picture (29%) and rewording (10%). The average number of strategies across the grade levels that the teachers reported teaching to their students was three.

**DISCUSSION**

The main goal of this study was recognition of students’ difficulties in solving mathematical word problems from their teachers’ perspectives. This study also investigated what classroom practice and specific strategies teachers stated they used in their attempts to foster student problem solving success. The findings that addressed the first aim concerning teachers’ perspectives in the difficulties their students have when solving mathematics word problems revealed that almost half of teachers indicated that their students struggled with representation and understanding the problems. This was consistent with Braselton and Decker’s (1994) findings that students’ ability to read and comprehend the mathematics text is necessary before they can apply mathematical skills. They concluded that reading in mathematics class is a complex mixture of words, numbers, letters, symbols and sometimes graphics.
Mayer and Hegarty [18] recognized that the most significant element in the process of word problem solving was the stage of understanding of the problem. Foong (1994) in her studies of pre-service teachers found that unsuccessful problem solvers tended to attend to obvious details, translating statement by statement without having a global representation of the problem.

Responses that addressed the causes of students’ difficulties fell into three areas: text difficulties, unfamiliar contexts, using of inappropriate strategies. The first most-cited cause of the students’ difficulties reported by teachers was the text of the word problems. Text difficulties referred to the words used in problems from the textbook or other curricular materials that the students had to solve. A second aspect of text difficulty was that the problems the students needed to solve were complex and frequently involved more than one step. This made the problems harder for the students to read and solve.

Many researchers such as Batista (2009) and Eric (2005) in their researches stated that complexities in the context of the problem caused enormous problems for students. Numerous teachers’ responses noted that the problems were not "real world" and not relative to their students’ experiences. Other finding of this study showed that word problems become easier for students when they are embedded in a familiar context (De Corte et al., 1985, Davis-Dorsey 1991). Hembree (1992) in a meta-analysis of 44 studies explored six pairs of problem context and concluded that familiar contexts strongly influenced students’ problem solving performances in a positive way. This is consistent with the results of the present study. In many research studies it was noticed that familiar contexts enhance word problem solving by increasing the meaningfulness of contexts and motivating students to solve the problems (Cordova & Lepper 1996, Lopez & Sullivan 1992, Ku & Sullivan 2002). Therefore, it can be concluded that familiarity of a word problem reduce problem difficulty and enhance problem solving.

Teachers’ responses indicated that the students had a weak foundation because the previous teachers did teach inappropriate strategies. In this research, most of teachers believed that students in elementary course have learnt the strategy of finding key words but they haven’t been instructed to understand and recognize these problems. Thus, in guidance course, students attempt to solve the word problems solely by this strategy.

Findings for the second aim, that consisted what teachers reported as the classroom practices they used for teaching students to solve mathematics word problems, revealed the teachers identified six different classroom practices they used for problem solving instruction, with the responses varying only slightly across grade levels. Having the students solve the word problems independently was the most frequently cited practice. This independent work was often mentioned as following teacher modeling of problem solving strategies, but teachers did not state the number of examples demonstrated or the regularity of prior modeling.

Other practices reported were cooperative grouping and use of manipulative. While neither cooperative grouping nor using manipulative can be thought of as new ideas, apparently the majority of teachers interviewed are not incorporating using them into teaching word problems. Additionally, a surprisingly small number of the teachers stated that they had the students practice with real life problems. These findings suggest that majority of middle teachers are not those elucidated in professional literature on reading mathematics using real life contexts (Bates and Wiest, 2004), and personalized word problems (Davis-Dorsey, 1991). It seems that the familiarity of the context used was also an important factor. In many research studies it was noticed that familiar contexts enhance word problem solving by increasing the meaningfulness of contexts and motivating students to solve the problems (Cordova & Lepper 1996, Ku & Sullivan 2002). Therefore, it can be concluded that familiarity of a word problem reduce problem difficulty and enhance problem solving.

The third aim focused on what specific problem solving strategies teachers reported teaching to their students to solve mathematical word problems. While no single strategy was taught to the students by a majority of the teachers, the most popular strategy reported by teachers was teaching the students to identify key words in the text, by circling, underlining or highlighting this information. These findings on teachers’ instruction of problem solving strategies are a concern, as numerous-based studies have shown the explicit strategy instruction benefits student problem solving ability (Hembree, 1992). In addition, there is a body of literature concerning the teaching of reading strategies in math class though the data acquired in this study found that teacher use of these strategies was scare.

Making changes in the phrasing of the problem context is another strategy. In many researchers, it has been shown that making changes the phrasing of the problem context has a remarkable impact on solving word problems by students. (Hudson 1983, Vicente 2007) The teachers interviewed painted of being largely left to develop and find their own practices and strategies. More the teachers’ responses to where they had learned the strategies they used indicated that they learned strategies informally, from other teachers or as a result of personal experience, than from any other source.

To summarize, the following recommendations can be offered for researchers, teachers, pre-service teachers and curriculum experts in light of the findings and current practice:
In order to reinforce guidance-students’ skills in problem solving, book compilers are suggested to be more meticulous in designing and choosing appropriate contents because unfamiliar contents and language complexities in the problem context makes students unable to recognize the problem. Teachers are also
suggested to instruct different strategies such as "seeking for similar model", "designing a specific form for the problem" and "making changes in the phrasing of the problem" to their students so that they learn how to recognize the problem and then how to design a method for solving it.

REFERENCES


**Appendix:**

**Interview Questions:**

1- In your view, what kind of knowledge is required for students to solve word problems?

2- In your view, what’s the most significant problem solving difficulty for students?

3- In your view, why word problems are difficult?

4- Which of the student’s drawbacks make them unable to solve word problems?

5- What kind of practice you use to reinforce student’s skills in solving word problems?

6- What kind of strategy have you already learnt?

7- Which of the strategies you teach to your students?

8- Which of the strategies is not your favorite strategy for solving word problems?