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Proposing a Model to Predict Efficiency and Related Risk by Using Stochastic Data Envelopment Analysis Technique and the Imperialist Competitive Algorithm

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ABSTRACT

This paper proposes a new model which provides some insights on using Stochastic Data Envelopment Analysis (SDEA) technique in the future performance appraisal for similar units of an organization. This research contributes to advance current knowledge by proposing new model in which, regardless of having benefits of DEA, resolve some head problems of the context. For instance: 1) the efficiency estimate impossibility, 2) liability to measuring of acceptable risk taking for managers in regards to achieving each units predetermined efficiency, and 3) the unreal distribution of weights to inputs and outputs of the DEA model. We used from Imperialist Competitive Algorithm (ICA) refereeing to have a nonlinear and complexity model. ICA is one of the newest evolutionary optimization algorithms. Finally, in order to reach a better understanding of the proposed model by considering ICA, it was applied to predict efficiencies for a number of Iranian Bank branches. The high correlation between real efficiencies (were obtained with real outputs and DEA model) and predicted efficiencies (were obtained with proposed model) for all of branches in finish of the predicted financial period, which represented the validity of the proposed model.

KEYWORDS: Stochastic Data Envelopment Analysis (SDEA), Efficiency, Risk, Imperialist Competitive Algorithm (ICA).

1. INTRODUCTION

Evaluating and comparing the performance of similar units of an organization is an important part of that organization management duty. One of the most important tools of relative performance comparing of these units is a quantitative, precise and powerful approach called Data Envelopment Analysis (DEA). This technique is considered not only in performance evaluation but also in management helping; more precise recognition of it's under control units. This method has also some major shortcomings that the most important of them are impossibility of predicting efficiency, lack of determining acceptable risk level for the managers in the direction of achieving the predicted efficiencies in each unit and also unreal weight distribution to the inputs and outputs. For preparing the possibility of predicting efficiency and the level of its dependent risk we can benefit from a mathematical model which is based on Stochastic Data Envelopment Analysis SDEA by entering stochastic effects, environmental factors like economic condition on the inputs and outputs of the under control units. Also for resolving the problem of the lack of unreal distribution of weights to inputs and outputs of the DEA model we can use the expert's suggestions in limiting the weights of the model inputs and outputs.

Stochastic constraints programming is a very important and useful method in stochastic programming that Charnes & Cooper (1959) entered the chance constrained programming in research operation literature for the first time. They along with Rouds (1978) stated the discussion of data envelopment analysis for calculating efficiency. Sengupta et al. (1982) stated the stochastic DEA models. In other words, these researchers combined the models of data envelopment analysis with chance constraint programming (CCP) and they used the obtained stochastic models for estimating efficiency and considering the measuring errors of input variables. The conducted researches about the weights of inputs and outputs in DEA are limited and the most important of them are the articles of Dayson & Thanassoulis (1988), Charnes et al. (1989), Roll and Golany (1993) and Jahanshahloo et al.

Land et al. (1993) have proposed a model which is known as LLT. In this model the considered both constraints of the envelopment form of CCR model as the stochastic variables. After proposing LLT model, Cooper et al. (1996) proposed a new model by applying Saimon Satisfactory model. This new model is a combination of the concept of satisfactory decision making with CCDEA models or data envelopment analysis with stochastic constraints. Jackson (2001) estimated the efficiency in the free market by using data envelopment analysis. Cooper et al. (2002) proceeded to analysis of technical efficiency by using stochastic constraints programming approach. Saati et al. (2003) presented a method for obtaining a common set of fuzzy inputs and outputs weights. They first suggest their model for deterministic data and then developed it for fuzzy

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data. Houang et al. (2005) proposed the combined model of SDEA and chance constraint programming. Cooper et al. (2006) stated the last proposed model in the SDEA ground. In this paper they proposed the output based BCC random model and improved it by applying the stated concepts in BCC model and the hypothesis of being random the inputs and outputs and normal distribution for them.

Needing to solve applied problems is an unavoidable and yet difficult affair. So a lot of research algorithms have been created with different philosophies. Evolutionary algorithms is a group of randomize optimization algorithms in which the evolutionary rules of the nature are used for optimization. These algorithms are usually used for solving the parameters optimization problems that other formal methods cannot solve them. Recently a new algorithm called Imperialist Competitive Algorithm ICA is presented by Atashpaz-Gargari & Lucas (2007) in the ground of evolutionary calculations which inspired not only from a natural event but also from a humanistic- social event. This algorithm has continuous nature and has proved its efficiency in different works. In this paper, the strong imperialist competitive algorithm is used for solving the presented model for predicting efficiency in a future financial period in each similar units of an organization and determining the amount of acceptable risk in achieving the predicted efficiency for them and the results have been compared with real efficiency of the DEA model for those units.

Louzano & Villa (2007) presented the two stage Array procedure. In the first stage the corresponding weights were obtained by input and output parameters by using Analytic Hierarchy Process AHP with considering the decision makers interests and suggestions and in the second stage, multi objective DEA was presented according to determined weights and the Genetic Algorithm was used for its solving. Wong et al. (2007) compared the multi- objective DEA with genetic algorithms and simulation annealing and finally Shahraaieni (2010) proposed the scenario analysis and performance scenario in data envelopment analysis by using genetic algorithm. Udhayakumar et al. (2011) proposed stochastic simulation based on genetic algorithm for chance constrained data envelopment analysis problems. Lu & Yu (2012) proposed data envelopment analysis for evaluating the efficiency of genetic algorithms on solving the vehicle routing problem with soft time windows.

The rest of this paper is organized as follows: Section 2 presents a stochastic DEA model analysis. Formulation proposed stochastic DEA model is presented in section 3. In the section 4 the Imperialist Competitive Algorithm is described in detail. An application derived from this empirical study and practical example is documented in section 5. Discussion and future extensions are summarized in the last section (section 6).

2. Stochastic DEA analysis

Data Envelopment Analysis (DEA) assumes that there are n DMUs (j=1, 2,…, n) whose whole set is denoted by j. The performance of each DMU is characterized by its production process of m inputs (Xij for i=1,2,…,m) to yield s outputs (Yrj for r=1,2,…,s). It is also assumed that all DMUs have input and output vectors and all the components of these vectors are positive.

DEA model:

$$\begin{aligned} Max \quad & E_k = \sum_{r=1}^{s} u_r\, y_{rk} \\ st: \quad & \sum_{i=1}^{m} v_i\, x_{ik} = 1 \\ & \frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{i=1}^{m} v_i\, x_{ij}} \le 1 \quad j = 1 \ \cdots \ n \\ & u_r, v_i \ge o \end{aligned} \qquad (1)$$

SDEA model:

$$\begin{aligned} Max \quad & E\,(E_k = \sum_{r=1}^{s} u_r\, \hat{y}_{rk}) \\ st: \quad & \sum_{i=1}^{m} v_i\, x_{ik} = 1 \\ & P_r\,[\frac{\sum_{r=1}^{s} u_r\, \hat{y}_{rj}}{\sum_{i=1}^{m} v_i\, x_{ij}} \le \beta_j\,] \ge 1 - \alpha_j \quad j = 1 \ \cdots \ n \\ & u_r, v_i \ge o \end{aligned} \qquad (2)$$

Where the above two models are designed to measure the performance (DEA efficiency) of the specific k-th DMU (in j) as Ek. The symbols (Vi and Ur) represent weight multipliers related to the i-th input and the r-th output, respectively. In Eq. (2), Pr stands for a probability and the superscript "^" represent that $\hat{y}_{rj}$ is a stochastic variable.

It is important to mention that this study is interested in future planning where the quantity of inputs can be controlled as decision variables, whilst being unable to control outputs, because these quantities depend upon external factors such as an economic condition. Hence, the inputs are considered as deterministic variables and the outputs are considered as stochastic variables. To describe the analytical structure of our SDEA model, it is compared with a traditional DEA model, often referred to as "DEA ratio form". Mathematically, the two models have the following formulations:

Model (1) is formulated under the condition that each DMU is evaluated by a ratio of its total weighted outputs to total weighted inputs. The original DEA model determine the ratio of all DMUs to be less than or equal to unity. Consequently, it belongs to an efficiency range between 0 and 100%. Meanwhile, Eq. (2) formulates the ratio to be less than or equal to βj (a prescribed value in the range between 0 and 100%) that represents an expected efficiency level of the j-th DMU. Cooper et al. (2006) consider the expected efficiency score as an "aspiration level" that is usually requested by an outside authority and/or a budgetary limitation. Since βj is set to be unity in Eq. (1), the deterministic model (1) can be considered as a special case of the SDEA model (2).

The other symbol αj stands for the probability that output/input ratio becomes more than βj with a choice of weight multipliers. Thus, αj is considered as a risk criterion representing utility of a manager. On the other hand, 1-αj shows the probability of attaining the requirement. Like βj, the risk criterion (αj) is also a described value that is measured in the range between 0 and 1. When αj =0 in Eq. (2), it is certainly required that the output/input ratio becomes less than or equal to βj. Conversely, αj=1 omits the requirement under any selection of weight multipliers. The objective of Eq. (1) is formulated by $\sum_{r=1}^{s} u_r y_{rk}$ while that of Eq. (2) is expressed by $E(\sum_{r=1}^{s} u_r \hat{y}_{rk})$, where the symbol "E" stands for an expected value of the sum of weighted $\hat{y}_{rk}$.

3. Formulation Proposed Stochastic DEA Model

In this study, the constraints and objective of Eq. (2) are reformulated by CCP proposed by Cooper (2002). (Research by Cooper et al. (2006) shows how to incorporate the CCP technique into the DEA ratio form. In the SDEA models of these papers, both inputs and outputs are stochastic variables. Hence, our formulation presented in this study can be considered as a special case of their SDEA).

The constraints of Eq. (2), including the stochastic process, can be rewritten as follows:

$$P_r\left\{\sum_{r=1}^{s} u_r\, \hat{y}_{rj} \le \beta_j\, (\sum_{i=1}^{m} v_i\, x_{ij})\right\} \ge 1-\alpha_j \qquad (3)$$

Eq. (3) is equivalent to:

$$P_r\left\{\frac{\sum_{r=1}^{s} u_r\, (\hat{y}_{rj} - \bar{y}_{rj})}{\sqrt{V_j}} \le \frac{\beta_j \sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, \bar{y}_{rj}}{\sqrt{V_j}}\right\} \ge 1-\alpha_j \qquad (4)$$

where $\bar{y}_{rj}$ is the expected value of $\hat{y}_{rj}$ and:

$$V_j = (u_1 \quad u_2 \quad \ldots \quad u_s) \times \begin{pmatrix} v(\hat{y}_{1j}) & \operatorname{cov}(\hat{y}_{1j}, \hat{y}_{2j}) & \ldots & \operatorname{cov}(\hat{y}_{1j}, \hat{y}_{sj}) \\ \operatorname{cov}(\hat{y}_{2j}, \hat{y}_{1j}) & v(\hat{y}_{2j}) & \ldots & \operatorname{cov}(\hat{y}_{2j}, \hat{y}_{sj}) \\ \vdots & & & \vdots \\ \operatorname{cov}(\hat{y}_{sj}, \hat{y}_{1j}) & \ldots & \ldots & v(\hat{y}_{sj}) \end{pmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_s \end{pmatrix} \qquad (5)$$

Here, Vj indicates the variance-covariance matrix of the j-th DMU in which the symbol "v" stands for a variance and the symbol "cov" refers to a covariance operator. To reformulate Eq. (4) by CCP, this study introduces the following new variable ($\hat{Z}_j$) :

$$\hat{Z}_j = \frac{\sum_{r=1}^{s} u_r\, (\hat{y}_{rj} - \bar{y}_{rj})}{\sqrt{V_j}} \qquad j=1,\ldots,n \qquad (6)$$

which follows the standard normal distribution with zero mean and unit variance. Substitution of Eq. (6) in Eq. (4) produces:

$$P_r\left\{\hat{Z}_j \le \frac{\beta_j \sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, \bar{y}_{rj}}{\sqrt{V_j}}\right\} \ge 1-\alpha_j \qquad (7)$$

Since $\hat{Z}_j$ follows the standard normal distribution, the invariability of Eq. (7) is executed as follows:

$$\frac{\beta_j \sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, \bar{y}_{rj}}{\sqrt{V_j}} \ge Z^{-1}(1-\alpha_j) \ , \quad j=1,\ldots,n \qquad (8)$$

Here, Z stands for a cumulative distribution function of the normal distribution and Z^{-1} indicates its inverse function. The SDEA model (2) is obtained by replacing Eq. (3) by Eq. (8) and its resulting formulation becomes:

$$\begin{aligned}
&MaxE_K = E(\sum_{r=1}^{s} u_r \hat{y}_{rk}) \\
&st: \sum_{i=1}^{m} v_i x_{ik} = 1 \\
&\beta_j \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \geq \sqrt{V_j} Z^{-1}(1-\alpha_j) \;,\; j = 1,.., n \\
&u_r \geq o,\; r = 1,..., s \;, v_i \geq o \;, i = 1,..., m
\end{aligned} \tag{9}$$

This research assumes that a stochastic variable ($\hat{y}_{rj}$) of each output is expressed by:

$$\hat{y}_{rj} = \bar{y}_{rj} \pm b_{rj} \varsigma \;\;, r = 1,\ldots s \;\;,\; j = 1,\ldots n$$

where $\bar{y}_{rj}$ is an expected value of $\hat{y}_{rj}$ and brj is it's standard deviation. (Section 5 of this paper describes how to determine the average and the standard deviation from his/her prediction of a decision maker(s). Cooper et al. (2006) proposed the assumption along with a practical rationale.) It is also assumed that a single random variable (ς) follows a normal distribution $N(0,\sigma^2)$. Under such an assumption, Vj becomes:

$$V_j = (u_1 \quad u_2 \quad \ldots \quad u_s) \times \begin{pmatrix} b_{1j}^2\sigma^2 & b_{1j}b_{2j}\sigma^2 & \ldots & b_{1j}b_{sj}\sigma^2 \\ b_{2j}b_{1j}\sigma^2 & b_{2j}^2\sigma^2 & \ldots & b_{2j}b_{sj}\sigma^2 \\ \vdots & & & \vdots \\ b_{sj}b_{1j}\sigma^2 & \ldots & \ldots & b_{sj}^2\sigma^2 \end{pmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_s \end{pmatrix} \tag{10}$$

Incorporation of Eq. (10) into Eq. (9) provides:

$$\begin{aligned}
&Max\, E(\sum_{r=1}^{s} u_r \hat{y}_{rk}) \\
&st: \sum_{i=1}^{m} v_i x_{ik} = 1 \\
&\beta_j \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \geq (\sum_{r=1}^{s} u_r b_{rj}\sigma) Z^{-1}(1-\alpha_j) \;,\; j = 1,.., n \\
&u_r \geq o,\; r = 1,..., s \;, v_i \geq o \;, i = 1,..., m
\end{aligned} \tag{11}$$

Next, paying attention to $\hat{y}_{rj} = \bar{y}_{rj} \pm b_{rj}\varsigma$, we reformulate the objective of Eq. (11) as follows:

$$Max\, E(\sum_{r=1}^{s} u_r \hat{y}_{rk}) = E(\sum_{r=1}^{s} u_r(\bar{y}_{rj} \pm b_{rj}\varsigma)) = E(\sum_{r=1}^{s} u_r \bar{y}_{rk} \pm \sum_{r=1}^{s} u_r b_{rk}\varsigma) = \sum_{r=1}^{s} u_r \bar{y}_{rk} \tag{12}$$

It is assumed that the random variable (ς) follows a normal distribution N(0,1) in Eq. (12). Under such an assumption (so,σ=1), and because of E(ς)=0, consequently the SDEA model can be written in the following model:

$$\begin{aligned}
&Max \quad E_k = \sum_{r=1}^{s} U_r \bar{y}_{rk} \\
&st: \sum_{i=1}^{m} v_i x_{ik} = 1 \\
&\sum_{i=1}^{m} v_i(\beta_j x_{ij}) - \sum_{r=1}^{s} u_r \{ \bar{y}_{rj} + b_{rj} Z^{-1}(1-\alpha_j) \} \geq o \;,\; j = 1,.., n \\
&u_r \geq o,\; r = 1,..., s \;, v_i \geq o \;, i = 1,..., m
\end{aligned} \tag{13}$$

3.1. Inputs/Outputs Weights range limitation:

In DEA, input/output weights variation ranges are permitted. It may causes reachable weights from solving the model are different with managers' acceptable weights. In this paper we use assurance region for the weights that proposed by Thomson et al. consequently the final suggested SDEA model is as the following model:

$$\begin{aligned} &Max \quad E_k = \sum_{r=1}^{s} U_r \bar{y}_{rk} \\ &st: \sum_{i=1}^{m} v_i \; x_{ik} \;= 1 \\ &\sum_{i=1}^{m} v_i (\beta_j \, x_{ij}) - \sum_{r=1}^{s} u_r \{ \bar{y}_{rj} + b_{rj} \, Z^{-1} (1-\alpha_j) \} \geq o \quad , \quad j = 1,..,n \\ &U_r \leq \frac{upper \quad wheight_r}{lower \quad wheight_{r+1}} U_{r+1} \\ &U_r \geq \frac{lower \quad wheight_r}{upper \quad wheight_{r+1}} U_{r+1} \qquad r = 1,...,s \;\; , v_i \geq o \; , i = 1,...,m \end{aligned} \tag{14}$$

Where "upper wheight$_r$" and "lower weight$_r$" are the upper and lower bounds of the r-th unit. These weights are based Decision Maker's suggestion. The inputs (x_{ij}) of each DMU are adjusted by a prescribed aspiration level (β_j). The outputs (y_{ij}) of the j-th DMU) are expected values (y_{ij}) in Eq. (14). The goal of this model is the maximize efficiency of the k-th unit as E_k, With determination the risk level of the k-th unit (α_j) to reach E_k. Our model is the NP_hard problem and we used from ICA algorithm to solve it refereeing to have a nonlinear and complexity model.

4. Imperialist Competitive Algorithm

According to Atashpaz-Gargari and Lucas (2007) Imperialist Competitive Algorithm (ICA) is a new socio-politically motivated global search strategy that has recently been introduced for dealing with different optimization tasks. Along with above mentioned study so many researchers believe that this evolutionary optimization strategy has shown great performance in both convergence rate and better global optima achievement (Atashpaz-Gargari & Lucas, 2007; Rajabioun, Hashemzadeh, Atashpaz-Gargari, Mesgari, & Rajaei Salmasi, 2008b; Biabangard-Oskouyi, Atashpaz-Gargari, Soltani, & Lucas, 2009; Sepehri Rad & Lucas, 2008; Atashpaz-Gargari, Hashemzadeh, Rajabioun, & Lucas 2008; Rajabioun, Atashpaz-Gargari, & Lucas, 2008a). Nevertheless, its effectiveness, limitations and applicability in various domains are currently the subject of extensive study. In order to find the optimal priorities for each user in recommender systems, Sepehri Rad & Lucas (2008) apply ICA in "Prioritized user-profile" approach to recommender systems, trying to offer more personalized recommendation by assigning different priority importance to each feature of the user-profile in different users. This finding is also reported by Rajabioun et al (2008).

The considered ICA algorithm is shown in fig. 1. Like to other evolutionary algorithms, this algorithm starts with an initial population. Each member of the population is called a country. Some of the best countries (in optimization terminology, countries with the least cost) are selected to be the imperialist states and the rest form the colonies of these imperialists. Based on this algorithm, all colonies of initial countries are divided among the mentioned imperialists based on their power. The power of each country, the counterpart of fit, the power of each country, the counterpart of fitness value in the GA, is inversely proportional to its cost. The imperialist states together with their colonies form some empires. After forming initial empires, the colonies in each of them start moving toward their relevant imperialist country. This movement is a simple framework of assimilation policy that was pursued by some of the imperialist states. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. This fact is modeled by describing the total power of an empire as the power of imperialist country plus a percentage of mean power of its colonies.

4.1. Building of initial empires

The objective of optimization is to discover an optimal solution in terms of the variables of the problem. An array of variable values is formed to be optimized. In the GA terminology, this array is known "chromosome", but in ICA the term "country" is used for it. In an Nvar dimensional optimization problem, a country is a 1×Nvar array. This array is defined as following:

$$country = [\, p_1, p_2, p_3, ..., p_{N_{var}} \,] \tag{15}$$

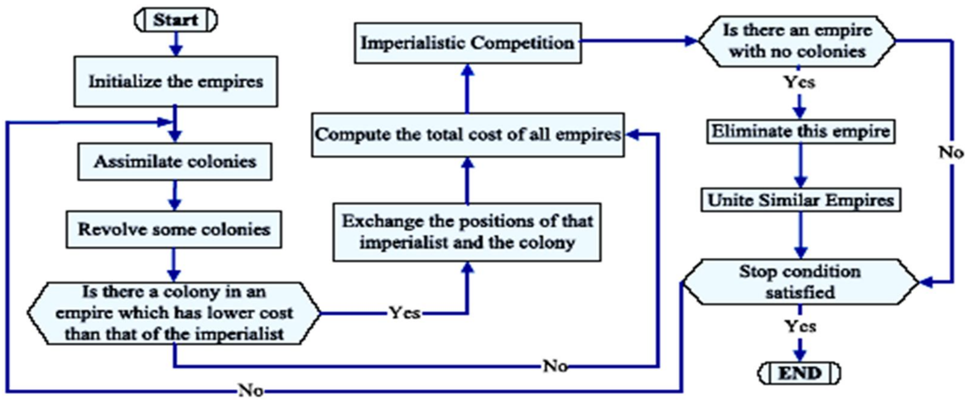

Figure 1. Flow chart of the Imperialist Competitive Algorithm

Where, $p_i s$ are the variables to be optimized. The variable values in the country are represented as floating point numbers. Each variable in the country can be interpreted as a socio-political characteristic of a country. From this point of view, all the algorithm does is to search for the best country which is the country with the best combination of socio-political characteristics such as culture, language, economical policy, and even religion. From optimization point of view this leads to finding the optimal solution of the problem, the solution with the least cost value. The cost of a country is found by evaluation of the cost function f at variables ($p_1, p_2, p_3, ..., p_{N_{var}}$). So we have:

$$Cost = f(country) = f(p_1, p_2, p_3, ..., p_{V_{Var}}) \tag{16}$$

the first step of the optimization algorithm, initial countries of the size Ncountry are produced. Nimp of the most powerful countries are selected to form the empires. The remaining Ncol of the initial countries will be the colonies belonging to an empire. To shape the initial empires, the colonies are separated among imperialists based on their power. That is, the initial number of colonies of an empire should be directly proportionate to its power. To proportionally divide the colonies among imperialists, the normalized expenditure of an imperialist is defined by:

$$C_n = c_n - \max_i \{c_i\} \tag{17}$$

Where c_n is the cost of the nth imperialist and C_n is its normalized cost. Having the normalized cost of all imperialists, the normalized power of each imperialist is defined by:

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \tag{18}$$

The initial colonies are divided among empires based on their power. Then the initial number of colonies of the nth empire will be:

$$N.C_n = round\{P_n . N_{col}\} \tag{19}$$

Where N.Cn is the initial number of colonies of the nth empire and Ncol is the total number of initial colonies. To separate the colonies, N.Cn of the colonies are randomly selected and given to the nth imperialist. These colonies along with the nth imperialist form the n-th empire.

4.2. Assimilation: movement of colonies toward the imperialist

Pursuing assimilation policy, the imperialist states tried to absorb their colonies and make them a part of themselves. In detail, the imperialist states made their colonies to move toward themselves along different socio-political axis like culture, language and religion. In the ICA, this process is modeled by moving all colonies toward the imperialist along different optimization axis. This movement has been illustrated in Fig. 4. Considering a 2-dimensional optimization problem, in this figure the colony is absorbed by the imperialist. Then colony will get closer to the imperialist in these axes. Continuation of assimilation will cause all colonies to be fully assimilated into the imperialist. In the ICA, the assimilation policy is modeled by moving all colonies toward the imperialist. Figure 2 shows this movement where a colony moves toward the imperialist by x units. The new position of colony is shown in a darker color. The direction of the movement is the vector from the colony to the imperialist. In this figure x is a random variable with uniform (or any proper) distribution. Then:

$$x \sim \mathrm{U}(0, \beta \times \mathrm{d}) \quad (20)$$

Where, β is a number greater than 1 and d is the distance between the colony and the imperialist state. β>1 leads to the colonies to get closer to the imperialist state from both sides. Assimilating the colonies by the imperialist states did not result in the direct movement of the colonies toward the imperialist. In other word, the direction of movement is not necessarily the vector from colony to the imperialist. To model this fact and to increase the capability of searching more regions around the imperialist, a random amount of deviation is added to the direction of movement. Fig. 3 illustrates the new direction. In this figure θ is a parameter with uniform (or any proper) distribution. Then:

$$\theta \sim \mathrm{U}(-\gamma, \gamma) \quad (21)$$

Where, γ is a parameter that adjusts the deviation from the original direction. In spite of that the values of β and γ are based on random choice, that is in most implementations a value of about 2 for β and about π/4 (Rad) for γ results in good convergence of countries to the global minimum.

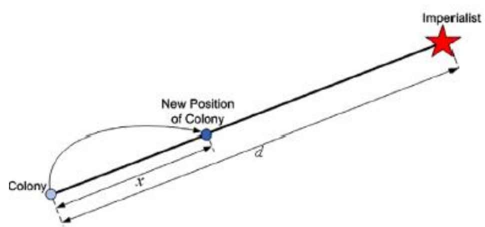

Figure 2. Movement of colonies to ward their relevant imperialist.

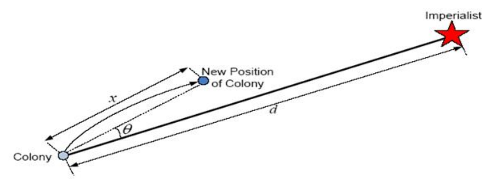

Figure 3. Movement of colonies toward their relevant imperialist in a randomly deviated

4.3. Revolution; a sudden change in socio-political characteristics of a country

Revolution is a basically change in power or organizational structures that occurs in a relatively short period of time. In the terminology of ICA, revolution causes a country to suddenly change its socio-political characteristics. That is, instead of being assimilated by an imperialist, the colony randomly changes its position in the socio-political axis. The revolution increases the exploration of the algorithm and prevents the early convergence of countries to local minimums. The revolution rate in the algorithm presents the percentage of colonies in each colony which will randomly change their position. A very high value of revolution decreases the exploitation power of algorithm and can reduce its convergence rate.

4.4. Exchanging positions of the imperialist and a colony

While moving toward the imperialist, a colony might reach to a place with lower cost than the imperialist. In this case, the imperialist and the colony change their positions. Then the algorithm will be continued by the imperialist in the new position and the colonies will be assimilated by the imperialist in its new position. As illustrated in Figure. 4a the position exchange between a colony and the imperialist is depicted. In this figure the best colony of the empire is shown in a darker color. This colony has a lower cost than the imperialist. Fig. 4b presents the empire after exchanging the position of the imperialist and the colony.

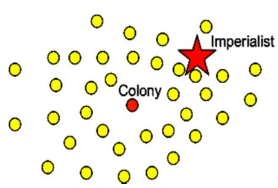

Figure 4a. Exchanging the positions of a colony and the imperialist.

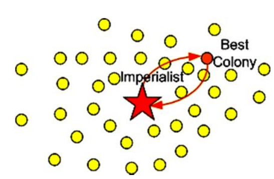

Figure 4b. The entire Empire after position exchange.

4.5. Integrating similar empires

In the movement of colonies and imperialists toward the global minimum of the problem some imperialists might move to similar positions. If the distance between two imperialists becomes less than threshold distance, they both will shape a new empire that is a combination of these empires. All the colonies of two empires become the colonies of the new empire and the new imperialist will be in the position of one of the two imperialists.

4.6. Entire power of an empire

The total power of an empire is mainly affected by the power of imperialist country. However the power of the colonies of an empire has an impact, albeit negligible, on the total power of that empire. This fact is modeled by defining the total cost of an empire by:

$$T.C._{n} = Cost(imperialst_{n}) + \xi mean\{Cost(\text{Colonies of empire}_{\text{n}})\} \quad (22)$$

Where T.C.n is the total cost of the n-th empire and ξ is a positive small number. A little value for ξ causes the total power of the empire to be determined by just the imperialist and increasing it will increase the role of the colonies in determining the total power of an empire. The value of 0.1 for ξ has shown good results in most of the implementations.

4.7. Imperialistic competition

All empires try to take the possession of colonies of other empires and control them. The imperialistic competition slowly provides decrease in the power of weaker empires and an increase in the power of more powerful ones. The imperialistic competition is modeled by just picking some (usually one) of the weakest colonies of the weakest empire and making a competition among all empires to possess these (this) colonies. Fig. 5 illustrates an orientation of the modeled imperialistic competition. Based on their total power, in this competition, each of empires will have a likelihood of taking possession of the mentioned colonies. In other words, these colonies will not definitely be possessed by the most powerful empires, but these empires will be more likely to possess them.

To start the competition, initially a colony of the weakest empire is selected and then the possession probability of each empire is found. The possession probability Pp is proportionate to the total power of the empire. The normalized total cost of an empire is simply grasped by:

$$N.T.C._{n} = \max_{i}\{T.C._{i}\} - T.C._{n} \quad (23)$$

Where T.C.n and N.T.C.n are the total cost and the normalized total cost of n-th empire, respectively. Having the normalized total cost, the possession probability of each empire is given by:

$$P_{P_n} = \left| \frac{N.T.C._{n}}{\sum_{i=1}^{N_{imp}} N.T.C._{i}} \right| \quad (24)$$

To divide the mentioned colonies among empires vector P is designed as following:

$$\mathbf{P} = \left[p_{p_1}, p_{p_2}, p_{p_3}, \ldots, p_{p_{N_{imp}}} \right] \quad (25)$$

Then, vector R with the same size as P whose elements are uniformly distributed by random numbers is created:

$$\mathbf{R} = \left[r_1, r_2, r_3, \ldots, r_{N_{imp}} \right] \;,\; r_1, r_2, r_3, \ldots, r_{N_{imp}} \sim \mathrm{U}[0,1] \quad (26)$$

Then vector D is formed by subtracting R from P:

$$\mathbf{D} = \mathbf{P} - \mathbf{R} = \left[D_1, D_2, D_3, \ldots, D_{N_{imp}} \right]$$
$$= \left[p_{p_1} - r_1, p_{p_2} - r_2, p_{p_3} - r_3, \ldots, p_{p_{N_{imp}}} - r_{N_{imp}} \right] \quad (27)$$

Referring to vector D the mentioned colony (colonies) is handed to an empire whose relevant index in D is maximized. The process of selecting an empire is similar to the roulette wheel process which is utilized in choosing parents in GA. But this way of selection is much faster than the traditional roulette wheel. Because it is not required to calculate the cumulative distribution function and the selection is based on only the values of probabilities. Thus, the process of selecting the empires can solely substitute the roulette wheel in GA and increase its execution speed. The continuation of the mentioned steps will hopefully cause the countries to converge in to the global minimum of the cost function. Different criteria can be used to stop the algorithm. One idea is to use a number of maximum iteration of the algorithm, called maximum decades, to stop the algorithm. Or the end of imperialistic competition, when there is only one empire, can be considered as the stop criterion of the ICA. On the other hand, the algorithm can be stopped when its best solution in different decades cannot be improved for some consecutive decades.

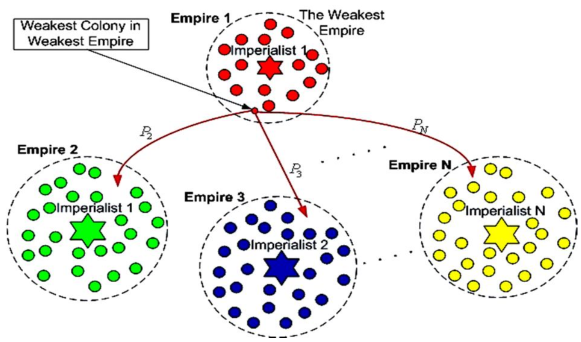

Figure 5. Imperialistic competition

5. An application of proposed SDEA model

5.1. Estimation of Output

To determine $\bar{y}_{rj}$ and b_{rj} of $\hat{y}_{rj}$ this study utilizes three different kinds of output estimate. A decision maker(s), who is involved in future planning, is asked to forecast the following three estimates on each output of the j-th DMU: 1) the most likely estimate (MLrj), 2) the optimistic estimate (OPrj), and 3) the pessimistic estimate (PErj). The ML is the most realistic estimate of $\hat{y}_{rj}$. From a statistical viewpoint, it is considered as the mode (the highest point) of the probability distribution for each output. The OP is aimed to be the unlikely but possible output quantity if everything goes well. It can be seen as an estimate of the upper bound of the probability distribution. The PE is intended to be the unlikely but possible output quantity if everything goes wrong. It is an estimate of the lower bound of the probability distribution.

Assuming that the data follows the beta probability distribution, this study converts the three estimates into the expected value and variance of each out- put. The expected value of its distribution is approximately:

$$\bar{y}_{rj} = (OP_{rj} + 4\,ML_{rj} + PE_{rj})/6 \qquad (28)$$

The variance becomes:

$$b^2_{rj} = (OP_{rj} - PE_{rj})^2/36 \qquad (29)$$

Where ML_{rj} is a mode and $((OP_{rj} + PE_{rj})/2)$ shows a midrange between OP_{rj} and PE_{rj} the expected value can be seen as a weighted arithmetic mean of the mode and the midrange. The mode has two-thirds of the entire weight. It is important to note that the above type of estimation is widely used in PERT/CPM (Program Evaluation and Review Technique/Critical Path Method). PERT/CPM is a management science technique for planning activity times and scheduling, while this study uses the technique to estimate the expected value and variance of each output. Using the proposed approach, future uncertainty regarding each output, which may fluctuate due to many economic factors, can be incorporated into our DEA formulation (Sabzehparvar, 2002).

5.2. Input/output weights determination

Assuming $D = \{D_1, D_2, ..., D_r\}; r \geq 2$ be a group of r decision makers expressing r reciprocal judgment matrixes $\{R^{(K)}_{n\times n}; k = 1, ..., r\}$ corresponding to pair wise comparisons for a set of n criterions $A = \{A_1, A_2, ..., A_n\}$, where $R^{(K)}_{n\times n} = \left(r_{ij}^{(K)}\right)$ is a positive squared matrix which validates $r_{ii}^{(K)} = 1, \quad r_{ji}^{(K)} = \frac{1}{r_{ij}^{(K)}} \succ 0$ for $i \neq j$.

The judgments $r_{ij}^{(K)}$ represent the relative importance to the decision maker Dk of Ai compared to Aj. The comparison matrix given by the k-th decision maker is denoted as follows:

$$R_{n\times n}^{(K)} = \begin{pmatrix} 1 & \cdots & r_{ink} \\ \vdots & r_{ijk} & \vdots \\ r_{n1k} & \cdots & 1 \end{pmatrix} \tag{30}$$

With normalize the comparison matrix and using row geometric mean method, the weights of criterions are obtained (Saati et al., 2003).

5.3. Practical example

Consider a bank which has 10 branches and the bank supervisor is going to predict the efficiency of its branch and also the risk that every branch manager should accept for reaching the predicted efficiency from the respect of the allocated budget, for the future financial year. It should be said that in this system the branches inputs (personnel expenses, official expenses and the costs of the place renting (suppose the branches places are being leased)), are programmed at the end of each year for the future financial year in the framework of budgeting system. But about the branches output (the amount of granted facilities and the flow of inter-bank services), predicting is conducted based on PERT/CPM technique by using the output values of each branch in the last financial years. The branches inputs and the estimated outputs are defined in table 1 by the supervisor as in the form of the optimistic estimate (OP), the most likely estimate (ML) and pessimistic estimate (PE). The supervisor is going to do the reforming measures for improving the efficiency of his under control set by estimating the efficiency and its dependent risk for each branch and for the future financial year. The expected efficiency of the supervisor is considered 1 for all the decision making units.

Table 1. The budgeted inputs and output estimates in year 2008

Branch code	**Inputs**			**Outputs**					
	Official expenses	**personal expenses**	**Rent costs**	**granted facilities ($\hat{y}_1$)**			**inter-bank services ($\hat{y}_2$)**		
				OP	**ML**	**PE**	**OP**	**ML**	**PE**
1	24	46	298	5800	5027	4800	410	362	300
2	25	41	295	5920	4972	4910	430	356	310
3	32	40	300	5750	5019	4952	421	353	320
4	33	44	305	5610	5083	4823	412	354	313
5	27	46	296	5520	5088	4899	418	367	304
6	21	42	297	5742	5010	4962	429	347	310
7	19	38	301	5825	5017	4898	432	346	317
8	22	39	292	5912	4970	4992	409	353	326
9	24	45	294	5852	4994	4901	399	352	309
10	20	41	306	5712	5031	4925	415	349	311

5.3.1. Computational results

First, we estimate the expected value and related standard deviation of outputs for each branch of bank with PERT/CPM technique. Results are obtained as shown in table 2.

Table 2. Mean and standard deviation of output estimates

Bank branch	$\bar{y}_{1j}$	$\bar{y}_{2j}$	b_{1j}	b_{2j}
1	5118	360	12.9	4.28
2	5120	361	12.9	4.47
3	5130	359	11.5	4.10
4	5128	357	11.4	4.06
5	5129	365	10.1	4.35
6	5124	355	11.4	4.45
7	5132	356	12.4	4.37
8	5131	358	12.3	3.71
9	5122	353	12.53	3.87
10	5127	354	11.4	4.16

Since the proposed SDEA model based on the normal distribution assumption for outputs, we use Normal probability plot with Stat Graphics plus 2.1 software for outputs of branches that results are shown in Fig. 6.

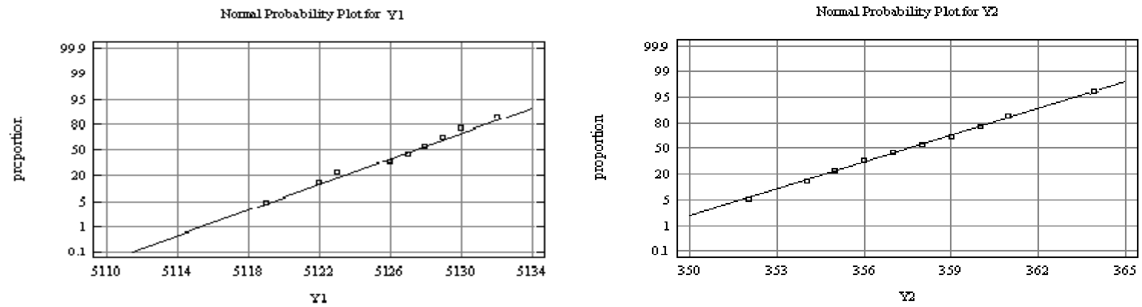

Figure 6. Normal plot for outputs (y_1 and y_2)

In this example, data sets of bank's outputs be positive and similar be negative for inputs inherently. Consequently, inputs and outputs values were normalized. Also in order to know the subjective preferences of bank experts about the upper/lower weights of each output, the questionnaires were distributed among four individual of deputies of the bank supervising and finally the preferred weight of each output was obtained by using eq. 30 as following:

Table 3. outputs weights

Outputs	Lower weight	upper weight
granted facilities	0.504	0.616
inter-bank services	0.315	0.385

The proposed SDEA model in section 5, involves variables Ej, ur ,vi and Z1-αj. The goal of the optimization of this model is estimate efficiency (Ej) and related risk level (αj) for each branch of bank. Since Ek represent predicted efficiency and αk represent predicted related risk level for k-th branch of bank, so we composed proposed model for each branch and solved with ICA. That is noted in this practical example, top manager of bank consider 10%≤αj≤50% for each branch. For solving the composed models with ICA, first, we defined initial solution as a following country:

$$country=[v_1,v_2,v_3,z_1,...,z_{10},u_1,u_2]$$

where the symbols (vi and ur) represent weight multipliers related to the i-th input and the r-th output, respectively. Also Zj, (j=1,2,…,10) determine the risk level of the j-th unit (αj) to reach Ek. The goal of ICA algorithm is finding the best country in order to maximize objective function. We set the parameters β=2, γ=π/4 and ξ=0.05 in algorithm. The results of 10 trails on proposed SDEA model for branch 1of bank presented in Table3. These results have obtained according to different number of Imperialist and colonies that indicated in table 4.

Table 4. outputs predicted and related risk level with ICA algorithm for branch 1 of Bank

Initial Solution	Number of Trails	Empire's Percentage	Efficiency (E_1)	$Z^{-1}(1-\alpha_1)$	Risk (α_1)
20	400	20%	0.879	0.55	29.1%
40	400	40%	0.882	0.56	28.8%
60	400	20%	0.894	0.57	28.4%
80	400	40%	0.901	0.50	30.8%
100	400	20%	0.907	0.52	30.1%
120	400	40%	0.897	0.51	30.5%
140	400	20%	0.904	0.52	30.1%
160	400	40%	0.919	0.58	28.1%
180	400	20%	0.938	0.56	28.8%
200	400	40%	0.959	0.53	29.8%

In this model we stop after 400 generations for each problem because the percent of improvement in term of objective function was very low against in previous generations. The ICA for proposed SDEA model is coded in Matlab 7.4.0 software. All the problems are solved on an Intel (R) core 2 duo CPU 2.00 GHz computer with 2 GB RAM. Table 5 represents the average of optimum value of 10 trails for each branch of bank which are obtained from proposed SDEA model. Also convergence graph for branch 1 is presented in Fig 7.

Table 5. Mean of outputs predicted and related risk level with ICA algorithm for branches of Bank

Branch	Efficiency (E_j)	$Z^{-1}(1-\alpha_j)$	Risk (α_j)
1	0.908	0.54	29.5%
2	0.821	0.46	28.3%
3	0.855	0.51	30.5%
4	0.917	0.50	25.8%
5	0.935	0.52	27.1%
6	0.831	0.51	24.5%
7	0.870	0.52	26.2%
8	0.813	0.58	23.2%
9	0.839	0.56	25.6%
10	0.890	0.53	21.3%

In order to verify the our model performance, the real efficiencies were obtained with DEA model and real outputs (eq.1) for all of branches in finish the predicted financial period and results of these were compared with results of predicted efficiencies were obtained with proposed SDEA model, that table 5 represented it. The correlation rate between real efficiencies and predicted efficiencies is calculated with Stat Graphics plus 2.1 software. The high correlation rate (**0.9557**) has obtained represents the validity of proposed SDEA model.

Table 6. comparison between real and predicted efficiencies

Branch	granted facilities $(\hat{y}_1)$	inter-bank services $(\hat{y}_2)$	Real efficiency	Predicted efficiency
1	5132	371	0.995	0.908
2	5134	372	0.971	0.821
3	5144	370	0.986	0.855
4	5142	368	0.996	0.917
5	5143	376	1	0.935
6	5138	366	0.972	0.831
7	5146	367	0.987	0.870
8	5145	369	0.959	0.813
9	5136	364	0.976	0.839
10	5141	365	0.989	0.890
Correlation rate between real efficiencies and predicted efficiencies = 0.9557				

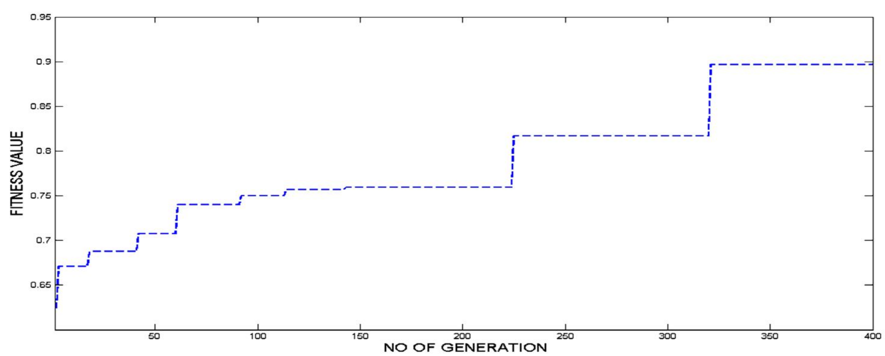

Fig 7 *Efficiency function for branch 1 of Bank*

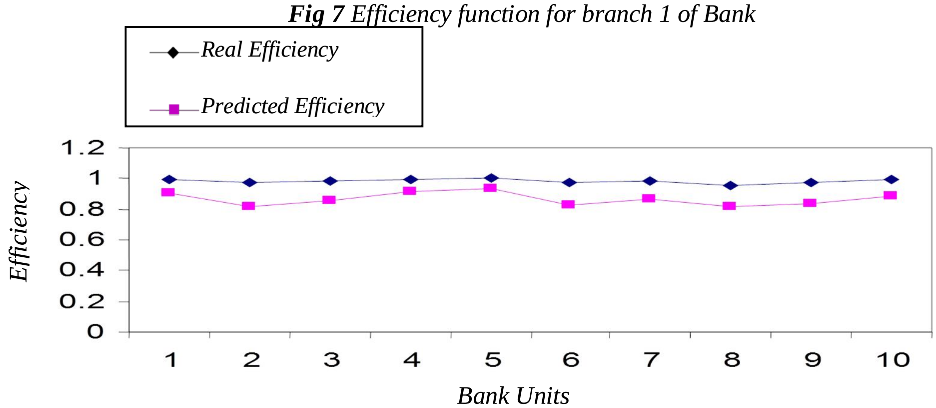

Figure 8. Real and Predicted Efficiencies Comparison Chart

6. Discussion and Conclusion

This paper proposed a new type model of Stochastic Data Envelopment Analysis (SDEA), that incorporates future information on outputs into its analytical framework. To document its practicality, the proposed SDEA model was applied to predict Efficiencies and related Risks level for kind of Iranian bank. Imperialist Competitive Algorithm (ICA) the newest evolutionary optimization algorithms) was used to solve the proposed SDEA model refereeing to have a nonlinear and complexity model. In order to verify the proposed SDEA model performance, the real efficiencies were obtained with real outputs and DEA model for all of branches in finish of the predicted financial period and these results were compared with the results of predicted efficiencies were obtained with proposed SDEA model. The high correlation (0.9557) was obtained in this examination which represents the acceptable validity of the proposed SDEA model.

In this study, a normal distribution is assumed to express the distribution of a stochastic variable. It is a straight forward matter to conduct a statistical test in the framework of SDEA analysis and the normal distribution. It is recommended to examine whether other distributions can be used for future analysis and apply them for forming the new models. Also we used ICA algorithm to solving models, but it is possible to use others evolutionary optimization such as GA, SA, TS, ACO algorithms that it can future research tasks.

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