Technical and Economic Allocation of Combined Cooling, Heating and Power (CCHP) To Powering Sensitive Loads in Power System

Farhad Samaie*1, M. H. Moradi2

1Department of Electrical Engineering, Bahar Branch, Islamic Azad University, Bahar, Iran
2Department of Electrical Engineering, University of Bu Ali Sina, Hamadan, Iran

ABSTRACT

In this paper, we present a hybrid and practical method for allocation of combined cooling heating and power (CCHP) generator at the bus. Firstly, network sensitive buses will be candidate for CCHP installation. At Second stage, utilizing the bus thermal coefficient, the possibility of heat selling around these buses can be calculated using the fuzzy method, then by considering the bus thermal coefficient and electrical power to heat ratio of CCHPs on the market we recommend several CCHPs for this buses. In this section, the financial benefit for investors by selling CCHP heat output is determined (Economic Analysis). In the third stage, the amount of the loss reduction and the voltage improvement due to proposed CCHPs installation using nodal pricing method is observed as financial benefit of distribution company (Technical Analysis). Finally, we obtain the suitable location of CCHPs based on Game Theory and considering the Distribution Company and investors as players. The proposed method is examined in a sample distribution feeder in the city of Hamedan.


INTRODUCTION

With increasing the demand of electrical energy and electrical energy efficiency of small units, these units are more likely to be utilized in the distribution systems and near the consumers. These small units that are connected to the distribution system are called "distributed generation" (DG). the privatization of electricity industry, less environmental pollution, high efficiency and developing methods of electricity generating through the renewable energy are important factors for the development of these generator types.

One of the most important point that should be considered to determining the location and size of distributed generations for supplying electrical energy of sensitive consumer, is the Defense factor.

A study following the 11 September attacks suggested that a system based more on distributed generation plants may be five times less sensitive to systematic attack than a centralized power system [30].

The Blackout in 2003 in North America and reviews the main options to minimize such disruption in the future, was lead to consideration of DG And especially CCHP, to reduce vulnerability of threatening terrorist attack in power systems [31,32,33].

The use of distributed generation units has significant impact on the power systems technical and economic issues [1,2].

A type of these power plants, is electrical and heat co-generation unit (CHP) which supplies the heating or cooling that needed for consumers through its waste heat output and increases the whole power plant efficiency up to 75% and above. Since the gas fuel is available in our country, these power plants are good substitutes for the electricity and heat generation.

The location and capacity determination of distributed generation resources are effective parameters on the technical indicators. Reduction of losses, improvement of the voltage profile and the voltage regulation are considered as significant indicators in the objective functions to optimize the location and capacity of these generators [3,4] and then these defined functions will be optimized by intelligent methods such as GA, PSO and TS and the capacity and location of DG will be determined [5,6].

For placement and capacity determination of "CCHP", in addition to the above technical analysis, the economic analysis is usually considered. In this analysis, the investment criteria is considered to optimize the power, heat, warm water and even cold consumption on the objective function, simultaneously [7,8].
The CCHP installed on the distribution network will change it from passive to active network, and improves the network losses, voltage regulation and profile [10,11]. The Improvement of this technical indicators are considerable by "nodal pricing methods" at the electrical energy price of buses to which CCHP is connected to them. In other words, the CCHP installation is effective at the nodal pricing of buses [9]. In addition to improving the technical indicators that are desirable for distribution companies, CCHP installation will created the opportunity to use the heating and warm water for consumers around the bus, and that is favorable to CHP investors. Allocation and capacity determination of "CCHP" in a way that both technical indicators are improved and while most profits produces are the practical challenges facing researchers, that depends on the strategy and policy of players in this activity, the distribution companies and investors.

The researchers have shown interest in using the "Game Theory" in recent years. Generally, where a group of individuals or firms compete with each other or they cooperate in a team, the Game Theory can be used to model competition between them. Song Yiqun [12] using non-cooperative Game Theory and Nash-Stackelberg equilibrium, a new method for determining the power market is presented. Lance B.cunningham [13] also using Game Theory and Corn out equilibrium, a way to model the transmission line congestion in the electricity market, is presented. Lance B.cunningham [13] cooperative Game Theory has been used, and the consumers of heat and power are considered as members of the coalition to achieve higher profits by reducing investment and increasing the efficiency of co-generating electricity and heating (CCHP).

In this paper a hybrid method has been provided to CCHP allocation on bus. In this method using cooperative Game Theory, investors and distribution companies have been used as the coalition members to achieve higher profits and improved technical indicators of network. The proposed hybrid method has Three stages as follows:

Firstly, network sensitive buses are candidates for CCHP installation. At Second stage, In order to economic analysis, with the investigation of heat consumers around the bus, the bus thermal coefficient that indicates the heat selling possibility of the bus will be extracted by introduced fuzzy function. Then, with regard to heat capacity and electrical energy to heat ratio in the CCHP market, several CCHPs will be specified for the candidate buses, that installation of each CCHP, brings different profit for the investor .

In the third stage, In order to technical analysis, the effect of proposed CCHP installation on the technical indicators of network, same reducing losses and improving voltage profile and regulation by nodal pricing method, in the form of profits for distribution companies is calculated. And since the distribution companies and investors considering as players, the CCHP capacity and its electrical power to heat ratio considering as the players' strategies, the suitable CCHP is determined from the proposed CCHPs by Game Theory approach. This paper is arranged as follows:

Game theory approach is described in section 2; the fuzzy bus thermal coefficient for economic analysis and the nodal pricing method for technical analysis are defined in sections 3 and 4, respectively. The optimization method is described in Section 5 and finally the case study results for the sample feeder in the city of Hamadan are provided.

**Game Theory approach**

In the game theory, a game is a set of rules known to all players that will determine any of their choices and the consequences of every choice. The normal form of game represents the number of players, set strategies, and the payoff functions of each player. Assuming there are n players, a set of players is:

\[ N = \{1, 2, \ldots, n\} \]

The decisions set that player i can get it is named "strategy space of player i " and is shown as follows:

\[ S_i = \{s_{i1}, s_{i2}, \ldots, s_{im}\} \]

Since there are n players, the strategies of all players are:

\[ S = \{S_1, S_2, \ldots, S_n\} \]

Where:

- \( s_{ij} \): The jth strategy of player i.
- \( m_i \): The total number of strategies.
- \( s_{ij} \): The jth strategy of player “i" in the strategy set.

On the other hand, payoff function for player "i" shows the outcome or result (including profit, utility, etc.) that player "i" will achieve at the end of the game. This payoff will depend on the chosen strategies by all players, and is shown as follows:

\[ u_i = u_i (s_{1j}, s_{2j}, \ldots, s_{nj}) \]
That \( s_{ij} \in S_i \), shows \( j^{th} \) strategy of player "i" in the strategy set (Si). Also the combination of all players strategy is called strategy profile, and is shown as follows:

\[ s = (s_{1j}, s_{2j}, \ldots, s_{nj}) \]

Thus the normal form of an n-persons game, represents the player’s strategy space \((S_1, \ldots, S_n)\) and their payoff function \((u_1, \ldots, u_n)\), is shown as follows [22].

\[ G = \{ S_1, \ldots, S_n; u_1, \ldots, u_n \} \]

Osborne, M.J. and Rubinstein [21] have shown that the solution of "Game" is a continuous selection of equilibrium strategies, the Nash equilibrium is used usually. In this equilibrium:

\[ \forall i, \forall s_{-i} \in S_{-i} \quad U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i}) \quad (1) \]

Where :

- \( s_i \) : Nash equilibrium strategy of player i
- \( s'_i \) : None- Nash equilibrium strategy of player i
- \( s_i \) : Other players’ strategy at the Nash equilibrium, That \( s_i \in S_i \) is the Nash equilibrium strategy of player i and \( s'_i \in S_i \) is None -Nash equilibrium strategy of player i.

The Nash equilibrium is a condition achieved by a set of strategies, and the players’ decision to deviate from such state will reduce the profit. Search to find the equilibrium point includes the following steps:

1. Forming a set of possible strategies, except dominant strategies, (the \( s'_i \) strategy of player i), so that fulfills the following condition [21]:

\[ \forall s_{-i} \in S_{-i} \quad U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i}) \quad (2) \]

2. Search to find the equilibrium point.

the Nash equilibrium is determined with regard to the 1. In terms of theory, there will be many equilibrium points, which in [21] some methods are presented for reducing the number of equilibrium points.

3. Considering of the rationality and the possibility of organized coalition for players.

4. Chosen methods to organize coalitions and the distribution of excess profits in the coalition participants.

If there is a possibility of a coalition among the players, the possible strategies of coalition may increase the dimensions of problem significantly.

Finally, the output of this method is semi-optimal path for all companies and their coalitions with regard to competitors’ strategy. In this paper, in order to allocate and determine the capacity of CCHP "The Static Game with complete information" is used. In this method, players are:

- Electric Power Distribution Company State (player A)
- Investors (player B)

The possible strategies:

- The electrical power to heat ratio of different CCHP technologies which are given in Table1 [20].
- Choose the capacity of CCHP, that has been considered 0.5 and 1 MW in this paper.

<table>
<thead>
<tr>
<th>Table 1. characteristics of CCHP technologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>technology</td>
</tr>
<tr>
<td>power to heat ratio</td>
</tr>
</tbody>
</table>

By obtaining the Nash equilibrium point, the suitable location and capacity of the CCHP generator will be achieved for installing in the bus network.

**Economic Analysis Using The Bus Thermal Coefficient**

The power at bus "i" is :

\[ P_{T_i} = P_{e_i} + P_{h_i} \quad (3) \]

And

\[ P_{h_i} = \sum_{j=1}^{n} P_{h_j} \quad (4) \]
Samaie and Moradi, 2012

Where :

\( P_{ei} \): Active power consumption at bus "i".

\( P_{hi} \): The electrical equivalent of heat selling possibility at bus "i".

\( P_i \): The total power.

In the above equations, \( P_{hi} \) is supplied by CCHP source that only connected to bus "i", and if it will be supplied by other buses, heat and cooling loss, eliminate this possibility while \( P_{ei} \) can be supplied by other buses of network. The optimization problem can be divided into two parts:

- Optimization with regard to consumption of \( P_{ei} \) for each bus of network that can be also supplied by generators at other buses.
- Optimization with regard to \( P_{hi} \) the sale of heat (equivalent to electric power) for each bus of network that is supplied by generator at the same bus only.

**Bus Thermal Coefficient (BTC):**

Indicates the possibility of selling steam and warm water to Defense Sensitive buses, and with regard to the consumers around the bus is calculated as follows:

\[ BTC_{i} = \frac{P_{e_{i}}}{1 MW}, \quad BTC_{i} \geq 0.1 \quad (5) \]

Where :

\( P_{e_{i}} \): The possibility of heat selling (equivalent to electric power) to the consumer "j" at bus "i".

\( N \): Total number of consumers around each bus.

\( BTC_i \): Bus thermal coefficient of bus "i".

\( Q_{h_{ij}} \): The heat consumption (equivalent to electric power) of consumer "j" at bus "i".

\( \beta \): Type of consumer.

\( d \): The distance between the heat consumer and power plant.

\( \chi \): Coefficient of CCHP technology that depends on the conditions that heat be generated by CCHP.

\( \psi \): Fuel delivery coefficient.

The thermal coefficient of bus will be achieved by normalization the possibility of heat selling to 1MW. Finally, the buses with higher amount of BTC are eligible for CCHP installation that will be considered in the calculations of objective function optimization. \( P_{hi} \) is the function of effective coefficients phase sharing (minimum) of heat selling and will be expressed by equation (6):

\[ P_{hi} = \sum_{j=1}^{N} P_{h_{ij}} = \sum_{j=1}^{N} Q_{h_{ij}} \times f_{ij} (\beta \cap d \cap \chi \cap \psi) \quad (6) \]

**Calculation of \( \beta \):** According to the National Building Regulations in Iran [23], there are four groups of building types, A to D. This grouping is based on the following three factors:

- Continuing the use of building during the day and the year.
- The temperature difference between the interior and exterior of the building.
- The significance of stabilization of temperature of indoor spaces.

\( \beta \) is determined based on the user type in Table 2. Higher \( \beta \) indicates more possibility of heat selling to the consumer.

**Table 2. Buildings classification according to the National Building Regulations**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Type</th>
<th>User Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospitals, hotels (4 and 5 stars), industries with the heating consumption for the generation process (cement, steel, melted metals, sugar, food, greenhouse)</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>Integrated academic and large schools (with dormitory), skyscrapers, large residential complexes (with central heating systems)</td>
<td>0.75</td>
<td>B</td>
</tr>
<tr>
<td>Stores, factories (heating and sanitary use only), international airport</td>
<td>0.5</td>
<td>C</td>
</tr>
<tr>
<td>Places of business (shopping centers), offices</td>
<td>0.25</td>
<td>D</td>
</tr>
<tr>
<td>Spread consumers that can not using of central heating systems</td>
<td>0</td>
<td>All cases</td>
</tr>
</tbody>
</table>

**Amount of heat consumption (equivalent to electrical power) \( Q_{h_{ij}} \):**

The calculation of the energy needed for different loads (various applications) according to references [15,16], has been done for 1000 m² infrastructure, and this point is considered that, Hamadan city uses from natural
gas of the main pipeline with special heating value of 9434 Kcal/m³ or 1060 Btu / ft³. For example, in multi-unit residential building that use the central heating systems (for 1000 m² infrastructure) A) The warm water consumption : 231.84 (kw) B) The heat consumption for heating : 117.16 (kw) Total heating and warm water consumption of different buildings is shown in Fig.1.

![Bar chart showing total heating and warm water consumption for different buildings](image)

**Fig. 1.** $Q_{hi}$ for different consumers, with infrastructure of 1000m²

**The distance between heating consumer and power plant (d):** The other issue that should be considered at heating distribution is the distance between heating consumer and power plant, so that by increasing the distance, heat selling possibility will be reduced while the transport cost will be increased. In other words, the bus thermal coefficient (fitness) is proportional to the inverse distance:

$$f(d) \approx \frac{k}{d}$$

That, $d$ is the difference between heating consumer and power plant and coefficient $k$ is depends on the heat transferring system that achieves based on the practical results. The possibility of heat and warm water transferring to the different distances is expressed by following fuzzy membership function (Fig.2):

$$f(d) = \begin{cases} 
1 & d < 333 \\
\frac{1050 - d}{717} & 333 \leq d \leq 1050 \\
0 & d \geq 1050 
\end{cases}$$

![Graph showing fuzzy digit corresponding f(d)](image)

**Fig. 2.** The fuzzy digit corresponding f(d)

**Fuzzy membership function:** fuzzy digit $f(d)$ in parametric mode is the regular pair of $(\underline{f(d)}, \overline{f(d)})$ which must satisfy the following requirements:

1. $\overline{f(d)}$ Continuous boundary function from left.
2. $\underline{f(d)}$ Continuous boundary function from right.
3. $\overline{f(d)} \leq \underline{f(d)}$, $0 \leq f(d) \leq 1$
**Determination of Technology Coefficient (x)**: This ratio expresses which technology is used to generate electricity and heat in the CCHP (Table 3). Coefficients $x_1$ to $x_5$ can be determined according to the CHP thermal output. For example, gas turbine technology, which provides heat, warm water, LP and HP steam, has highest coefficient of $x$. In some of the CHP units, a variety of Absorption chillers [27], Adsorption chillers [28], and Desiccant dehumidifiers systems in humid areas [29] can be used and they changed to CCHP units. In these systems the technology coefficient will be raised.

<table>
<thead>
<tr>
<th>Table 3. various CCHP technologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology</td>
</tr>
<tr>
<td>Typical power to heat ratio</td>
</tr>
<tr>
<td>The Power electrical efficiency(HHV)</td>
</tr>
<tr>
<td>Total efficiency(HHV)</td>
</tr>
<tr>
<td>Using of output heat</td>
</tr>
<tr>
<td>$X_{CHP}$</td>
</tr>
<tr>
<td>$X_{CCHP}$</td>
</tr>
</tbody>
</table>

**fuel delivery Coefficient ($\psi$)**: Since the natural gas is used as the main fuel for these power plants and gas lines have three pressures, 1000 PSI for gas transmission, 250 PSI and 60 PSI for gas distribution in the cities; therefore, considering the consumers distance around each bus from the transmission and distribution gas lines (d), and the experimental results obtained from the gas company, the corresponding fuzzy digits ($\psi(d)$) with different gas pressures is shown in Fig.3.

![Fig. 3. fuzzy digit corresponding to $\psi(d)$ for the pressure of (1) 1000PSI, (2) 250PSI and (3) 60PSI](image)

Finally through determining of bus thermal coefficient, the amount of saving the thermal cost of each bus (with regard to government support in this area [19]) will be obtained after CCHP installation as follows:

$$C_{H_i} = BTC_i \times \Delta t_i \times \lambda_H$$  \hspace{1cm} (7)

Where:

- $C_{H_i}$: saving the thermal cost after CCHP installation, \( \frac{5}{365} \$ \)
- $\lambda_H$: The cost of per "MWh" heating, is equal to 7.2 $, since the project of "targeted subsidies" is executed.
- $\Delta t_i$: 8760 hour in a year.

**Technical Analysis Using The Nodal Pricing Method**

The distributed generation resources in the network will change the power flow and losses on two-level of transmission and distribution networks. In many tariffs plants in distribution level, use from the equally share of losses cost for consumers, that discourages the consumers for the CCHP installation [24]. For solving this problem...
we can utilize the "Nodal Pricing Method". The price of electricity in the nodes indicates the marginal price of electricity in the network buses [9], in this paper the characteristics of formulas are defined as follows:

Marginal losses coefficient (MLC) is the active power losses network change \( \frac{\partial P}{\partial P} \) due to change in production or consumption of the active power \( (P_{\text{e}_i}) \) and the reactive power \( (Q_{\text{e}_i}) \) in bus ‘i’ that defined as follows [17]:

\[
\rho_{P, e_i} = \frac{\partial P_{L}}{\partial P_{e_i}} \quad (8)
\]

\[
\rho_{Q, e_i} = \frac{\partial P_{L}}{\partial Q_{e_i}} \quad (9)
\]

Where:
\( \rho_{P, e_i} \): Marginal losses coefficient of active power at the bus "i".
\( \rho_{Q, e_i} \): Marginal losses coefficient of reactive power at the bus "i".

The medium point between generation and transmission levels is called "power supply point" (PSP). If "\( \lambda \)" is the price of active power in PSP in \( \frac{\text{S}}{\text{kW}} \) and if the active and reactive power consumption at bus “i” change as \( P_{\text{e}_i} \) and \( Q_{\text{e}_i} \) respectively and no congestion exists in the distribution network, then we can calculate the nodal pricing for active and reactive power as follows:

\[
N_{a}^{e} = \lambda + \lambda \cdot \rho_{P, e_i} = \lambda (1 + \rho_{P, e_i}) \quad (10)
\]

\[
N_{r}^{e} = \lambda \cdot \rho_{Q, e_i} \quad (11)
\]

The price of electrical bill without CCHP installation on the period \( \Delta t \) will be obtained as follows:

\[
C_{\text{eo-CCHP}}(P_{\text{e}_i}, Q_{\text{e}_i}) = (N_{a}^{e}(P_{\text{e}_i}, Q_{\text{e}_i}) \times P_{\text{e}_i} + N_{r}^{e}(P_{\text{e}_i}, Q_{\text{e}_i}) \times Q_{\text{e}_i}) \Delta t \quad (12)
\]

And the total of it for each feeder is equal to:

\[
C_{\text{eo-CCHP}}^{\text{total}} = \sum_{i=1}^{N} C_{\text{eo-CCHP}}(P_{\text{e}_i}, Q_{\text{e}_i}) + (\lambda \times P_{\text{e}_i}) \Delta t \quad (13)
\]

CCHP installation decreases the distribution losses, and so the nodal pricing will be reduced [26]. The price of electrical bill with CCHP installation on the period \( \Delta t \) at bus “i” will be obtained as follows:

\[
C_{\text{i-CCHP}}(P_{\text{e}_i}, Q_{\text{e}_i}) = \{ (N_{a}^{CCHP}(P_{\text{e}_i}, Q_{\text{e}_i}) \times (P_{\text{e}_i} - P_{\text{CCHP}_{\text{e}_i}}) + N_{r}^{CCHP}(P_{\text{e}_i}, Q_{\text{e}_i}) \times (Q_{\text{e}_i} - Q_{\text{CCHP}_{\text{e}_i}}) \} \Delta t + \{C_{\text{CCHP}} \times P_{\text{CCHP}_{\text{e}_i}} \} \Delta t \quad (14)
\]

And the total of it for each feeder is equal to:

\[
C_{\text{CCHP}}^{\text{total}} = \sum_{i=1}^{N} C_{\text{i-CCHP}}(P_{\text{e}_i}, Q_{\text{e}_i}) + (\lambda \times P_{\text{CCHP}_{\text{e}_i}}) \Delta t \quad (15)
\]

Where:
\( N_{a}^{e} \): Nodal pricing of active power without CCHP
\( N_{a}^{CCHP} \): Nodal pricing of active power with CCHP
\( N_{r}^{e} \): Nodal pricing of reactive power without CCHP
\( N_{r}^{CCHP} \): Nodal pricing of reactive power with CCHP
\( Q_{\text{e}_i} \): Reactive power consumption at bus i
\( P_{\text{CCHP}_{\text{e}_i}} \): Active power supplied by the CCHP at bus i
\( Q_{\text{CCHP}_{\text{e}_i}} \): Reactive power supplied by the CCHP at bus i
\( C_{\text{e}_i} \): Price of electricity supplied by the network without CCHP
\( C_{\text{CCHP}_{\text{e}_i}} \): Price of electricity supplied by the network with CCHP
\( P_{\text{CCHP}_{\text{e}_i}} \): Price of electricity supplied by CCHP.
Samaie and Moradi, 2012

\[ P_{\text{CCHP}} \]: Active power losses by considering CCHP.
\[ P_{\text{CCHP}} \]: Active power losses without CCHP.

The CCHP is intended as a negative load at its bus and to simplify the calculations assume that \( Q_{\text{CCHPi}} \) and \( P_{\text{CCHPi}} \) are zero at all buses except that DG is installed.

\[
P_{\text{CCHPi}} = \begin{cases} 
0, & i \neq i_{\text{best}} \\
 P_{\text{CCHPi}}, & i = i_{\text{best}} 
\end{cases}
\]

And

\[
Q_{\text{CCHPi}} = \begin{cases} 
0, & i \neq i_{\text{best}} \\
 Q_{\text{CCHPi}}, & i = i_{\text{best}} 
\end{cases}
\]

The larger difference \( C_{\text{no-CCHP}}^{\text{total}} - C_{\text{CCHP}}^{\text{total}} \) leads to the distribution company profit increases by DG installation, and its formulation will be as follows:

\[
T = C_{\text{no-CCHP}}^{\text{total}}(a) - (C_{\text{CCHP}}^{\text{total}}(b) + C_{\text{CCHP}}^{\text{total}}(c))
\]

Where:

- \( T \): Benefits of technical indexes improvement (for the distribution company)
- \( C_{\text{no-CCHP}}^{\text{total}}(a) \): Price of electricity supplied by the network without CCHP
- \( C_{\text{CCHP}}^{\text{total}}(b) \): Price of electricity supplied by the network with CCHP
- \( C_{\text{CCHP}}^{\text{total}}(c) \): Price of CCHP electricity.

The voltage rise at the CCHP connection point and its impact on the voltage profile needs to be considered [18].

Also the voltage of each bus should be limited within the minimum and maximum defined permissible range in the distribution network; therefore, CCHP should be installed with the voltage condition in accordance relation (18), so that the bus voltage will be limited within its permitted range.

\[
V_{i_{\text{min}}} \leq V_i \leq V_{i_{\text{max}}}, \quad i = 1, ..., N_n
\]

Where:

- \( V_i \): Voltage at bus "i" 
- \( V_{i_{\text{min}}} \): Minimum permitted voltage at bus "i" 
- \( V_{i_{\text{max}}} \): Maximum permitted voltage at bus "i" 
- \( N_n \): Number of network buses

**Technical, Economic and Defense (TED) Algorithm**

Block diagram of the proposed algorithm for optimal allocation of CCHP is as follows (Fig.4):
Fig. 4. Block diagram of CCHP Placement algorithm
Where:
$\text{CHP}_{\text{best},k}$: The CHP installed at bus $i_{\text{best}}$ that follows the $k$ strategy, ($k : 1,...,k_{\text{max}}$).

**Case Study**

In this part, one of the 20 KV Hamadan distribution feeders with 63 buses has been studied. This feeder is fed by Hamadan 63/20 kv station 2 (Fig.5). Specifications of this feeder are presented in table 4:

<table>
<thead>
<tr>
<th>Specifications of studied feeder</th>
<th>Length (KM)</th>
<th>Peak load of current (A)</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>Price of electricity supplied by the network ($^\text{$/ MWh$}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>80</td>
<td>2.3</td>
<td>50</td>
</tr>
</tbody>
</table>

The system has been simulated for a fixed time in this paper. With regard to the reciprocating engines CCHP type, and based on cost of CCHP in table 5, and assuming 75% efficiency achieved through the placement method in this paper, the cost of electricity supplied by CCHP is equal to $53$ $ for a megawatt hour.

**Table 5. Cost of used CHP**

<table>
<thead>
<tr>
<th>the investment of installation price</th>
<th>maintenance and operation cost</th>
<th>operation time</th>
<th>equipment life</th>
</tr>
</thead>
<tbody>
<tr>
<td>US $ (kw)</td>
<td>US $ (h/year)</td>
<td>h/year</td>
<td>(year)</td>
</tr>
<tr>
<td>900-1500</td>
<td>0.5-2</td>
<td>8760</td>
<td>50</td>
</tr>
</tbody>
</table>

According to consumers information, the large thermal loads of feeder are installed on buses: 1, 5, 16 and 22. Their specifications are given in table 6.

**Table 6. Thermal specifications of major consumers buses**

<table>
<thead>
<tr>
<th>Heat and warm water consumption (Pis) (KW)</th>
<th>consumer infrastructure (m²)</th>
<th>Type of Consumption located around each bus</th>
<th>Bus Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3040</td>
<td>37840</td>
<td>Load 1, (office) $C_1$</td>
<td>1</td>
</tr>
<tr>
<td>5619</td>
<td>27825</td>
<td>Load 2, (university)$C_5$</td>
<td>5</td>
</tr>
<tr>
<td>890</td>
<td>11110</td>
<td>Load 16, (office) $C_{16}$</td>
<td>16</td>
</tr>
<tr>
<td>1000</td>
<td>13300</td>
<td>Load 22, (Residential) $C_{22}$</td>
<td>22</td>
</tr>
</tbody>
</table>

**Fig. 5. The Sensitive consumers (on feeder)**

The buses in which heat selling possibility are available and $BTC_i \geq 0.1$ are suitable for CCHP installation. In these buses the CCHP capacities are calculated using fuzzy method (Table 7).
Table 7. Determination of CHP thermal capacity for candidate buses

<table>
<thead>
<tr>
<th>CHP capacity based on buses thermal capacity (MW)</th>
<th>BTC</th>
<th>Thermal capacity of bus (kw)</th>
<th>Bus number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>$3040 \times (0.25 \cap 0.75) \cap 1 = 760$</td>
<td>1</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
<td>$5619 \times (0.75 \cap 1 \cap 0.5 \cap 0.25) = 1404$</td>
<td>5</td>
</tr>
<tr>
<td>0.22</td>
<td>0.22</td>
<td>$890 \times (0.25 \cap 0.75 \cap 1) = 220$</td>
<td>16</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25</td>
<td>$1000 \times (0.75 \cap 1 \cap 0.5 \cap 0.25) = 250$</td>
<td>22</td>
</tr>
</tbody>
</table>

- Thermal benefit calculation:
In this stage we assume that CCHPs installed on the all proposed buses (1, 5, 16, 12) have 0.5 & 1 MW capacities and the electrical power to heat ratios is 0.7 and 1. Then for each case the heating cost savings is calculated using equation 7 that is shown in table 8.

Table 8. Benefit of the heating consumers in the different game strategies

<table>
<thead>
<tr>
<th>Power / Heat Ratio = 1</th>
<th>Bus number</th>
<th>Electric capacity (MW)</th>
<th>supplied Heating (MW)</th>
<th>Heat cost saving at each bus (investor profit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.7</td>
<td>44150</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>31536</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.22</td>
<td>63072</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>31536</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0.22</td>
<td>0.22</td>
<td>13875</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.22</td>
<td>0.25</td>
<td>15768</td>
</tr>
<tr>
<td>22</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>15768</td>
</tr>
<tr>
<td>Power / Heat Ratio = 0.7</td>
<td>1</td>
<td>1</td>
<td>0.7</td>
<td>44150</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.7</td>
<td>44150</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.71</td>
<td>88300</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.71</td>
<td>0.71</td>
<td>44781</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0.22</td>
<td>0.22</td>
<td>13875</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.22</td>
<td>0.25</td>
<td>13875</td>
</tr>
<tr>
<td>22</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>15768</td>
</tr>
</tbody>
</table>

- Technical indicators benefit calculation:
CHP installation will improve the network technical indicators, and this improvement is considered as benefit for electrical distribution company. At first we doing Load Flow and then using the nodal pricing for candidate buses. These prices are available for the CHP candidate buses before and after installation (for 0.5 MW and 1 MW) in table 9, also it is assumed that CHP works with "unit power factor", this means it will produce the (real) active power only. As it is shown in table 9 the active nodal price of each bus will be reduced essentially, when CHP is present.

Table 9. Nodal pricing of active power obtained by fuzzy bus thermal coefficient for fixed loads without and with CHP

<table>
<thead>
<tr>
<th>Bus number</th>
<th>CCHP capacity based on bus thermal coefficient (MW)</th>
<th>Nodal pricing of active power at buses without CHP (US $ / MWh)</th>
<th>Nodal pricing of active power at buses with CHP (US $ / MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>51.445</td>
<td>50.945</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>51.475</td>
<td>51.175</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>51.015</td>
<td>50.965</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>51.445</td>
<td>51.24</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>51.14</td>
<td>50.99</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>51.41</td>
<td>51.31</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>51.485</td>
<td>51.035</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>51.505</td>
<td>51.4</td>
</tr>
</tbody>
</table>
By CCHPs installation with the capacities mentioned, using formulas 8 to 17, and table 9, the profits of losses reduction for the CHP buses candidates will be calculated. By considering CCHP installed at bus 1 and doing load flow analysis, the new calculated losses, the amount of electrical energy supplied by the CCHP and network will be determined and the cost of CCHP and network electricity will be calculated (columns 5 and 6, Table 10). The CCHP installation benefits is obtained from the equation \(a-(b+c)\) of column 7 in the table 10. The column 7 indicates the benefits of CCHP installation which is desirable for Distribution Company.

### Table 10. Distribution company profit produced by the generator installed at each bus using the nodal pricing method

<table>
<thead>
<tr>
<th>Bus number</th>
<th>cost of network electricity without CCHP(^2) (a) $ year</th>
<th>CCHP capacity (MW)</th>
<th>losses (MW)</th>
<th>cost of network electricity (b) $ year</th>
<th>cost of CCHP electricity (c) $ year</th>
<th>Distribution company profit a-(b+c)(^\dagger) $ year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1007400</td>
<td>1</td>
<td>0.189</td>
<td>341640</td>
<td>464280</td>
<td>201480</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>0.235</td>
<td>754236</td>
<td>232140</td>
<td>21024</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1</td>
<td>0.193</td>
<td>516840</td>
<td>464280</td>
<td>26280</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>0.248</td>
<td>759930</td>
<td>232140</td>
<td>15330</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>1</td>
<td>0.198</td>
<td>519030</td>
<td>464280</td>
<td>24090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>0.262</td>
<td>766062</td>
<td>232140</td>
<td>9198</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>1</td>
<td>0.207</td>
<td>522972</td>
<td>464280</td>
<td>20148</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
<td>0.281</td>
<td>774384</td>
<td>232140</td>
<td>876</td>
</tr>
</tbody>
</table>

1. The total losses of network will be 0.313 MW without CCHP installation.

### - Game theory for Optimal selection

In the proposed method, the distribution company and investors are players A and B respectively, the strategies which these two players can choose, are electrical power to heat ratio (0.7 or 1) and electrical capacity (0.5 MW or 1 MW) of CCHP. By installation of specified CCHPs at the candidate buses through the above strategies, the benefit of consumers and distribution companies (payoff (wining) for each player) will be determined from table 8 and 10 that are shown in Table 11. We can specify the Nash equilibrium point in static game with above complete information from table 11. This point chosen indicates that benefits of both players are maximum and every player attempting to change these selection will lead to detriment of other players and the whole set. According to Table 11, it can be seen that the choice of strategy \(A_3\) (CCHP installed capacity of 1MW and power to heat ratio of 0.7) at bus 5, the Nash equilibrium of this game is obtained that in this point the player gains A and B are respectively 26,280 and 88,300 dollars per year.

### Table 11. The payoff (wining) amount for players with different Strategies

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
<th>(B_1)</th>
<th>(B_2)</th>
<th>(B_{16})</th>
<th>(B_{22})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>(B_{148’,15768})</td>
<td>201480’, 44150</td>
<td>26280 , 63072</td>
<td>24090’,13875</td>
<td>20148’,15768</td>
</tr>
<tr>
<td>(A_2)</td>
<td>(B_{1530’,31536’})</td>
<td>21024 , 31536’</td>
<td>15330 , 31536’</td>
<td>9198, 13875</td>
<td>876 , 15768</td>
</tr>
<tr>
<td>(A_3)</td>
<td>(B_{15768})</td>
<td>20148’, 44150</td>
<td>26280’, 88300’</td>
<td>24090’,13875</td>
<td>20148’,15768</td>
</tr>
<tr>
<td>(A_4)</td>
<td>(B_{14781’})</td>
<td>21024 , 44150</td>
<td>15330 , 44781’</td>
<td>9198 , 13875</td>
<td>876 , 15768</td>
</tr>
</tbody>
</table>

### CONCLUSION

In this paper, based on Technical, Economic and Defense (TED) Analysis, a new method was proposed for the allocation of Combined Cooling, Heating and Power (CCHP) for the bus.

The CCHP installation in the distribution network improves technical indicators such as reduced losses, improved voltage profile and voltage regulation for the distribution company’s profit ability and furthermore creations possibility of heat selling around the bus and profit ability for the investor.

Here, the distribution companies and investors are considered as players and capacity and power to heat ratio as the strategies of the players. Then using the Nash equilibrium, the equilibrium point is determined by two players...
that this point is maximum for each player and changing this point by one of the players causes to decrease another player gain.

The investor’s benefit obtained from the heat selling that generated around the bus and profits of distribution company due to the technical indicators improvement using the nodal price change that has been calculated before and after installation of CCHP.

Finally, the presented method is applied on the sample feeder in the city of HAMADAN and the optimal location of CCHP is determined. The results are included to show the validity and efficiency of the new technique.

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