

## Cellular Manufacturing System Design with Queueing Approach

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### ABSTRACT

Cellular manufacturing system (CMS) is an application of group technology in manufacturing and involves the processing of a collection of parts on dedicated clusters of machines and manufacturing cells. A queueing system consists of one or more servers that provide service of some sort to arriving who arrive to find all servers busy generally join one or more queues (lines) in front of the servers. This paper presents an integer mathematical programming model for the design of cellular manufacturing systems with queueing system parameters. In this system parts are customers and machines are servers we assume processing time for parts on machines and arrival time for parts to cells are stochastic and described by exponential distribution. The proposed model is verified by a numerical example and evaluated effectiveness of the proposed approach.

**KEYWORDS:** cellular manufacturing system, queueing system, waiting time, utilization factor

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### INTRODUCTION

Cellular manufacturing system (CMS) is an application of group technology in manufacturing and involves the processing of a collection of similar parts (part families) on dedicated clusters of machines or manufacturing processes (cells). With increased global competition and shorter product life cycles, there has been a shift to demands for mid-volume and mid-variety product mixes. Job shop and flow lines cannot provide the efficiency and flexibility to adapt to such needs. Cellular manufacturing system (CMS) have emerged to cope with such production requirements and have been implemented with favorable results [1]. CMS is known to offer several major advantages such as reduction of work-in-process (WIP), reduction of lead times, simplified flow of part and centralization of responsibility. Therefore, the adoption of CMS has consistently formed a central element of many manufacturing systems and has received considerable interest from both practitioners and academicians. Design of cellular manufacturing system (CMS) is a complex, multi-criteria and multi-step process. Ballakur [2] showed that this problem, even under fairly restrictive conditions, is NP-complete. The design of CMS has been called cell formation (CF), part family/machine cell (PF/MC) formation, and manufacturing cell design. Given a set of part types, processing requirements, part type demand and available resources (machines, equipment, etc.), the design of CMS the following three key steps:

1. part families are formed according to their processing requirements.
2. machines are grouped into manufacturing cells.
3. part families are assigned to cells.

The design objectives, a number of strategic issues such as machine flexibility, cell layout, machine types, etc., need to be considered as a part of the CMS problem. Further, any cell configuration should satisfy operational goals (constraints) such as desired machine utilization, production volume, number of manufacturing cells, cell sizes, etc. The followings are typical design constraints in the design of CMS.

1. **machine capacity.** It is obvious that, in the design of CMS, one of the basic requirements is that there should be adequate capacity to process all the parts.
2. **cell size.** the size of a cell, as measured by the number of machines in the cell, needs to be controlled for several reasons. First, available space might impose limits on the number of machines in a cell. If a cell is run by operators, the size of the cell should not be so large that it hinders visible control of the cell. Ranges of cell sizes can be specified instead of a single value of cell size. This would allow more flexibility in the design process.
3. **number of cells.** in practice, the number of cells would be set by organizational parameters such as the size of worker teams, span of supervisory authority, and group dynamics [3]. Given a range of cell sizes, the number of cells are determined and the resultant solutions can be compared.

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4. **utilization levels.** two levels of machine utilization are normally used. maximum utilization is specified to ensure that machines are not overloaded. minimum utilization for a new machine ensures that is economically justifiable to include the new machine in a cell [4].

In practice shorter product life-cycles, higher product variety, costs, unpredictable demand, setup time and other inputs to classical CMS have caused CMS to operate under uncertain environments [5,6]. thus, development models for cell formation problem under uncertainty can be suitable area for researchers and not researched well in the literature [7]. In this way, random parameters can be either continuous or described by discrete scenarios. If probability information is known, uncertainty is described using a probability distribution on the parameters, otherwise, continuous parameters are normally limited to lie in some pre-determined intervals [8,9].

Some real-world limitations in cell formation are: capacity of machines, safety and technological necessities, the number of machines in a cell and the number of cells, intercellular and intracellular costs of handling material between machines, machines must be utilized in effect.

Stochastic programming (SP) is applicable in many areas of production planning. There are some approaches such as stochastic programming, queuing theory and robust optimization which can be applied for uncertainty modeling. In this study, it's assumed that random parameters have continuous probability distribution. Queuing theory can be applied to any manufacturing or service systems. [10].

In this study, it's assumed that random parameters have continuous probability distribution. also, queuing theory will be applied to reach desired. A queuing system consists of one or more servers that provide service of some sort to arriving customers. customers who arrive to find all servers busy generally join one or more queues (lines) in front of the servers, hence the name queuing system. to describe a queuing system, an input process and an output process must be specified. the input process is usually called the arrival process, arrivals are called customers. arrival process described by specifying a probability distribution that governs the time between successive arrivals. to describe the output process (often called the service process) of a queuing system, usually specified a probability distribution (the service time distribution) which governs a customer's service time. in the manufacturing framework, customers can be assumed as parts and servers may be machines. the input process shows how parts arrive at a queue in a cell [11].

The arrival rate is the number of arrivals (customers or parts) per unit time. The service rate is the number of customers (parts) completing service per unit time [12].

Thus, measurements of a queue system such as maximization the probability that each server is busy (utilization factor), minimization of the sum of cost offering the service and the cost of waiting in queues and etc. can be optimized and cells will be formed optimally.

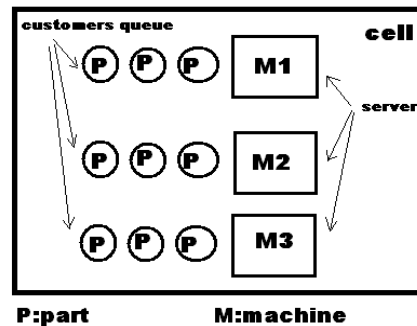


Figure 1: sample of a cell in a CMS problem formed as a queuing system

In this paper, we'll propose a CMS model as a queue theory and effectiveness of this approach will be illustrated.

However queuing theory is not an optimization technique, rather it determines the measures of performance of waiting lines, such as the average waiting time in queue and the productivity of the service facility, which can then be used to design the service installation [13].

## 2. model development :

In this section, we develop a new mathematical model for stochastic cell formation problem. we assume that parts processing time on machines and parts arrival time to cells are uncertain and described by continuous distribution. The aim of this model is to minimize summation of two cost types (1) the idleness cost for machines (2) total cost of customers waiting in the queue. we assume that the interarrival time

between two sequenced parts is described by exponential distribution .( with the rate  $\lambda_i$  for each part).Also processing time for parts follows exponential distribution.(with the rate  $\mu_j$  for each machine).

## 2.1 QUEUEING THEORY:

In this research we assume queueing system with exponential interarrival times and exponential service times modeled as birth-death processes.interarrival times and service times are exponential with parameters  $\lambda$  (arrival rate per unit time) and  $\mu$  (service rate per unit time),respectively.if arrival rate is greater than the service rate,the queue will grow infinitely.we define  $\rho = \frac{\lambda}{\mu}$  where  $\rho$  is probability that a machine is busy(or utilization factor).utilization factor must be less than one .if  $\rho \geq 1$ ,no steady-state distribution exists.[10]For any queueing system in which a steady-state distribution exists, $W_q$  is average time a customer(part)spend in queue (waiting time).[11,13]

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$\text{Since } \rho = \frac{\lambda}{\mu}$$

$$\text{Thus,we write } W_q = \frac{\rho^2}{\lambda(1 - \rho)}$$

In this research we assume M/M/1 queue system for CMS with rate  $\lambda_i$  for each part where parts served by machines.in these conditions,due to operate different parts on each machine and in addition,each part has different arrival rate so,for each machine  $\rho$  is computed using the following property .

**Property:**the minimum of several independent exponential random variables has an exponential distribution with rate  $\lambda = \lambda_1 + \lambda_2 + \dots + \lambda_n$  .

suppose n types of customers,and *ith* type of customer having an exponential interarrival time distribution with parameter  $\lambda_i$  ,arrive at a queue system. using mentioned property,we can see that the interarrival time

for entire queue system has an exponential distribution with parameter  $\sum_{i=1}^N \lambda_i$  .

that each machine j is busy is  $\rho_j = \frac{\sum_{i=1}^N \lambda_i}{\mu_j}$  ,and  $W_q = \frac{\rho_j^2}{\sum_{i=1}^N \lambda_i [1 - \rho_j]}$  , also idleness rate or probability

that each machine j is idle is  $[1 - \rho]$  .

## 2.2 NOTOTION :

### Indices :

$i$  index for part types (  $i = 1, \dots, P$  )

$j$  index for machine types (  $j = 1, \dots, M$  )

$k$  index for manufacturing cells(  $k = 1, \dots, C$  )

### Input parameters :

$P$  number of part types

$M$  Number of machine types

$C$	Number of cells
$a_{ij}$	Equals to 1 if part i require to be processed on machine j ; 0 otherwise
$v_j$	Cost machine j not utilize its capacity
$u_{ij}$	Waiting cost of a customer in queue per unit time
$M_{\max}$	Maximum number of machines permitted in a cell
$M_{\min}$	Minimum number of machines permitted in a cell
$\lambda_i$	Mean arrival rate for part i
$\mu_j$	Number of customers served per unit time by machine j(mean service rate)

**Decision variables :**

$x_{ik}$	Equals to 1,if part i processed in cell k ; 0 otherwise
$y_{jk}$	Equals to 1,if machine j assigned to cell k ; 0 otherwise

**2.3 MATHEMATICAL MODEL :**

min

$$z = \sum_{j=1}^m [1 - \rho_j] \times v_j + \sum_{i=1}^p \sum_{j=1}^m \sum_{k=1}^c W_{q_j} u_{ij} x_{ik} y_{jk} a_{ij} \quad (1)$$

$$\text{s.t: } \sum_{k=1}^c x_{ik} = 1 \quad i = 1, \dots, p \quad (2)$$

$$\sum_{k=1}^c y_{jk} = 1 \quad j = 1, \dots, m \quad (3)$$

$$\rho_j - \sum_{k=1}^c \frac{\sum_{i=1}^p \lambda_i a_{ij} x_{ik} y_{jk}}{\mu_j} = 0 \quad j = 1, \dots, m \quad (4)$$

$$W_{q_j} - \frac{\rho_j^2}{\sum_{i=1}^p \lambda_i [1 - \rho_j]} = 0 \quad j = 1, \dots, m \quad (5)$$

$$M_{\min} \leq \sum_{j=1}^m y_{jk} \leq M_{\max} \quad k = 1, \dots, c \quad (6)$$

$$\rho_j \leq 1 \quad j = 1, \dots, m \quad (7)$$

$$x_{ik}, y_{jk} \in \{0,1\}, \rho_j \geq 0 \quad (8)$$

The objective function of the proposed model is the total sum of expected idleness costs for machines in cells and expected total cost of part waiting in the queue of machines. Eq.(2) guarantees that each part is assigned to a single cell. Eq.(3) guarantees that each machine is assigned to a single cell. Eq.(4) computes utilization factor or the probability that each machine is busy.(if the part needs to be operated on the machine and also the part and the machine are located together in a same cell, arrival rate for a part is considered in summation for total arrival rate of each machine ).Eq.(5) computes average time of parts in the queue of a machine. Eq.(6) specifies maximum and minimum of machines permitted in any cell. Eq.(7) guarantees that the utilization factor must be less than one . Eq.(8) determines the type of variables .

## 2.4 LINEARIZATION :

The proposed model is a nonlinear Mixed-integer programming model .therefore we reformulate the model as Mixed-integer linear programming model with define the new variables  $z_{ijk}$  to replace the  $x_{jk} \cdot y_{ik}$  .

### 2.4.1 THE PROPOSED MODEL INEARIZATION:

$$\min z = \sum_{j=1}^m [1 - \rho_j] \times v_j + \sum_{i=1}^p \sum_{j=1}^m \sum_{k=1}^c W_{q_j} u_{ij} a_{ij} z_{ijk} \quad (1)$$

s.t: (2)(3)(5)(6)(7)(8)

$$z_{ijk} \leq x_{ik} \quad \forall i, j, k \quad (10)$$

$$z_{ijk} \leq y_{jk} \quad \forall i, j, k \quad (11)$$

$$x_{ik} + y_{jk} - z_{ijk} \leq 1 \quad \forall i, j, k \quad (12)$$

Also constraint (4) is changed as follows :

$$\rho_j - \sum_{k=1}^c \frac{\sum_{i=1}^p \lambda_i a_{ij} z_{ijk}}{\mu_j} = 0$$

## 3. DISCUSSION OF RESULTS

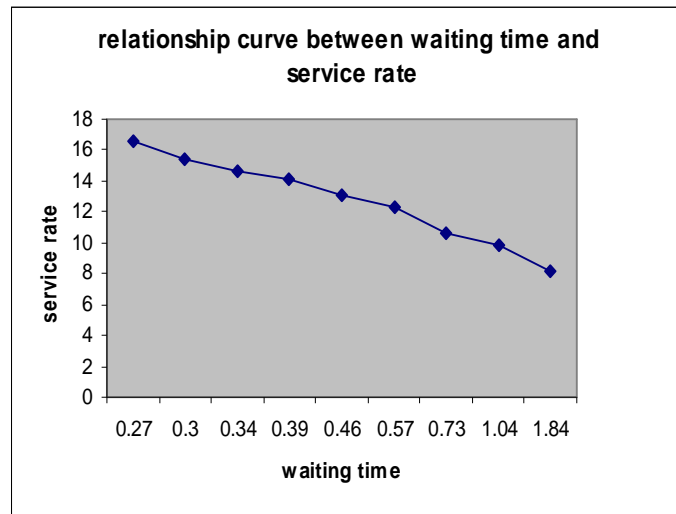
To verify the performance of the proposed model we generate a random example and solve by branch-and-bound (B&B)method under the LINGO 8.0 software on a personal computer with 3 GHZ and 512 MB RAM.we suppose there are 10 parts and 3 machines and 2 cells,to design new manufacturing cells,then solved model for 10 times times where all parameters applied in the model are fix,expect waiting time are obtained.

**Table 1 effectiveness of queuing approach in a CMS problem**

Pro.No	Waiting Time%	Service rate
1	0.244	17.03
2	0.269	16.57
3	0.300	15.40
4	0.339	14.60
5	0.391	14.08
6	0.461	13.10
7	0.565	12.24
8	0.729	10.60
9	1.04	9.86
10	1.84	8.10

As it can be seen in Table 1,if waiting cost increases,waiting time for parts decreases,too.In other words,by increasing waiting cost,in orther to minimize total cost for problem,the model must decrease the waiting time for parts in queue of each machine.if we suppose arrival rate is fixed by decreasing of waiting time,service rate(and out put of system)increases.

Note:by increasing waiting cost,utilization factor decreases,but this is normal,because,based on paradox aims of servicing levels,managers usually like the queuing system in which no customers is in queue and servers always are busy,too.the major point for analyst is,If the analyst tends to have minimum waiting customers in queue, he admit the little utilization for the servers [14].



**Figure2.relationship between waiting time and service rate**

Figure 2 shows relation between waiting time of parts in queue of machines and service rate.also queuing measurement (waiting time).as it can shown If the less waiting time is needed, the service rate of machine should decrease in order to minimize the costs. so,it can be found that queuing measurements are suitable tools to analysis and optimize a CMS problem and it has important effect in determining machine cells and part family in CMS.

### Conclusions

We have developed nonlinear mixed\_integer programming model of CMS with two queueing system parameters 1)Idleness cost 2)waiting time

The objective is to minimize the total costs of the idleness cost for machines and cost of part waiting in the queue of machines.the nonlinear formulation of the proposed model was linearized using some auxiliary variables .the performance and effectiveness of the model is illustrated by example and showed relationship between waiting time and service rate and finally effectiveness of queueing system in optimal form of workcell and part families.some guide lines for future researches can be outlined as follows:

- reformulated model with finit capacity of length of queue and servers.
- Optimizing he other queue measurements such as average queue length.
- Optimizing the other model to determine no of servers in order to minimization of total cost.

### REFERENCES

1. Wemmerlow,u.,Hyer,n1989.,cellular manufacturing in the us industry:A survey of users.International journal of production research 27(9), 1511-1530,
2. Ballakur, A. ,1985 An Investigation of Part Family/Machine Group Formation in Designinga Cellular Manufacturing System. PhD thesis, University of Wisconsin, Madison, WI.
3. Askin, R., Selim, H. and Vakharia, A. , 1997 “A Methodology for Designing Flexible CellularManufacturing Systems”. IIE Transactions, vol29, no 7. pp 599–610.
4. Anan mungwattana ,2000. Design of cellular manufacturing systems for Dynamic and uncertain production requirements with presence of routing Flexibility. PhD thesis, Virginia Polytechnic Institute and State University, September 1.
5. Benjaafar,s.sheikhzadeh,m. Design of flexible plant layouts IIE Transaction,To appear
6. Chen, M. ,1998. A Mathematical programming model for system reconfiguration in a Dynamic cellular manufacturing environment. Annals of operation research vol.77, no.1.pp.109-128.

7. M.saidi-mehrabad, V.r. Ghezavati ,2009 .,Designing cellular Manufacturing systems under uncertainty, journal of uncertain systems, Vol.3, No.4, pp.315-320
8. Snyder, L. V, 2006, Facility location under uncertainty: A review, IIE Transactions, Vol.38, pp.537-554
9. Ghezavati, V.R., Jabal-Ameli, M.S., and A. Makui; ,2009. A new heuristic method for distribution networks considering service constraint and coverage radius, Expert systems with Applications, vol.36, no.3, pp.5620-5629.
10. Heragu, S., Facilities Design, PWS Publishing Company, Boston, MA, pp.316, 1997.
11. Wayne .J. Winston, 2007, operation research Applications and Algorithms, 4th ed., Thomson Indiana.
12. Frederick . s. Hillier, Gerald. J. Lieberman , 2007 Introduction to Operations research, 8th ed., Tata McGraw-Hill, New Delhi.
13. Hamdy. A. Taha 2006., An Introduction Operations research, 8th Prentice-Hall India.
14. Iravani. S. M, 1991. Queue Systems, science and technology, Tehran, Iran.