# New Method for Determination of Driver Sight Distance from Roadside Obstacles on Horizontal Curves 

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#### Abstract

In this paper several equations for calculating the distance of inner curve edge from road side obstacles and clear zone area in the horizontal curves is presented based on the horizontal curve equation which is divided into two different states of circular curve and transition curve. The objectives were to provide new and also accurate method for determining the obstacles distances from inner edge of the curve and also to determine the clear zone area from road side obstacles on the length of horizontal curves in order to increase safety level of driving around horizontal curves. The method that is used in this paper is based the mathematical theory of horizontal curve equations for determining the boundary and area of the clear zone of obstacles on horizontal curves. The results indicate that the theory and the derived functions and also the length of curve are effective with in the inner curve edge distance from the obstacles and also the clear zone area.


KEY WORDS: Horizontal Curve, Transition Curve, Clear zone, Curve Equation

## 1. INTRODUCTION

One of the items that should be considered in geometric design of a road way is to provide adequate stopping sight distance on horizontal curves. To prevent reduction of the sight distance in horizontal curves around an arc in which obstacles exist, there is a need for a clear zone area that if not provided would imperil the lives of road users. Stopping sight distance in horizontal curves is not a straight line, but it is circular curve (AASHTO,2001). In the current graphical methods, if the beginning and the end of sight distance on the length of a curve are connected together by the chords we could obtain the providing sight margin by plotting envelop of these chords. The removal limit of obstacles perpendicular to the length of circular curve is a constant transition curve. The transition curve is starts at the point of contact with straight line and gradually extends into the strait of $\mathbf{M}$ at the contact point with circular curve (AASHTO,2001). The existing equations for calculating the distance of obstacles from driver' sight that provides adequate sight distance may be divided into two segments, as follows:

$$
S \prec L
$$

$$
\begin{equation*}
m=R\left[1-\cos \left(\frac{28.65 S}{R}\right)\right] \tag{1}
\end{equation*}
$$

$$
S \succ L
$$

$$
\begin{equation*}
m=R(1-\cos \alpha)+\left(\frac{S}{2}-\left(\frac{\pi R \alpha}{180}\right)\right) \sin \alpha \tag{2}
\end{equation*}
$$

In equations 1 and $2, \mathbf{m}$ is the distance from edge of the closest obstacle to center line of the inner lane, $\mathbf{R}$ is the radius of the circular curve, $\mathbf{S}$ is the stopping sight distance and $\boldsymbol{\alpha}$ is central angle of circular curve. Relevant research in the field of sight distance for elimination of the obstacles from horizontal curves, we could refer to David et al, (2001), who proposed an algorithm based on sight distance considerations within the horizontal curves that the clear zone border approximately is characterized on the length of curve by line pieces. In the proposed algorithm, error rates in contrast to the graphical methods are almost close to zero. David and Aida (2003) proposed another algorithm in which they indicated that lines which connect the beginning and the end of the sight distance are parabolic-shape. In another study, Aida and David (2007) provided a theory for envelop of curves by drawing line segment that the results of the proposed theory was more reasonable than other theories from time and accuracy of calculation point of view.

[^0]In general, overview of the past researches indicates that the methods are based on the graphical solution with no mathematical relations to determine the area of clear zone. In this paper exact mathematical solutions are proposed that can be used to calculate the safe right of way distance from obstacles on the road side and also the area of clear zone accurately.

## 2. Determining Equations of Distance and Area of the Clear Zone of Sight Obstacles

### 2.1. Determination of the Distance from Inner Transition Curve Edge

In fig. 1, the curve with the equation $\mathrm{Y}=\mathrm{F}(\mathrm{x}, \mathrm{y})$ is transition curve with radius of curvature $\mathbf{r}$ at any point, and the curve $\mathrm{Y}^{\prime}=\mathrm{F}^{\prime}(\mathrm{x}, \mathrm{y})$ is clearance limit of sight distance from obstacles, if distance of any arbitrary point with coordinate ( $\mathrm{x}^{\prime}$, $y^{\prime}$ ) on the curve of clearance limit of sight distance from obstacles to its match point on the transition curve with coordinate $(\mathrm{x}, \mathrm{y})$ is $\Delta \mathbf{m}$, then the curvature radius of clearance limit curve is $\mathbf{r}-\Delta \mathbf{m}$ at that point.


Fig. 1 The distance of transition curve to the curve of clearing sight obstacles
If it assumes that the curve $f^{\prime}(x, y)$ is the rotated curve $f(x, y)$ and also this rotation is around $z$ - axis and with angle $\alpha$ and anti-clockwise (positive) is taken, then the point with coordinate ( $\mathrm{x}, \mathrm{y}$ ) will be become the point with coordinate ( $x^{\prime}, y^{\prime}$ ) [6,7]:

$$
\left[\begin{array}{l}
X^{\prime} \\
Y^{\prime}
\end{array}\right]=\left[\begin{array}{cc}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{array}\right]\left[\begin{array}{l}
X \\
Y
\end{array}\right]
$$

where:

$$
\begin{align*}
X^{\prime} & =x \cos \alpha+y \sin \alpha \\
Y^{\prime} & =-x \sin \alpha+y \cos \alpha \tag{3}
\end{align*}
$$

Where $\mathbf{x}$ and $\mathbf{y}$ are coordinate of the point on the transition curve, and $\mathrm{X}^{\prime}$ and $\mathrm{Y}^{\prime}$ are coordinate of the match point on the rotated curve (removal limit of obstacles). If the distance $\Delta \mathbf{m}$ is equal to the length of line segment at any point, then one may calculated it using equation 4 :

$$
\begin{equation*}
\Delta m=\left[\left(x^{2}+y^{2}\right)(2-2 \cos \alpha)\right]^{1 / 2} \tag{4}
\end{equation*}
$$

Where $\Delta \mathbf{m}$ is distance of any arbitrary point on the inner edge of transition curve from the curve that provides sight margin, and $\boldsymbol{\alpha}$ is the angle between those two curves. By considering general equation of transition curves $\left(L_{S} . R_{C}=r . L=c t e\right)$ and substituting $X=L$ and also $Y=f(x)=f(L)$ equation 4, relation 5 is derived:

$$
\begin{equation*}
\Delta m=\left[\left(L^{2}+f^{2}(L)(2-2 \cos \alpha)\right)\right]^{1 / 2} \tag{5}
\end{equation*}
$$

Where $\mathbf{L}$ is the length of transition curve, $\mathbf{f}(\mathbf{L})$ is the functional value of the transition curve at any point , and $\boldsymbol{\alpha}$ is intersection angle between the curves. To investigate the validity of the derived function, its boundary conditions should be checked. The boundary condition at the starting point of transition curve is the length of curve which is equal to zero and the functional value should be zero for that value, this is true for relation $5\left(\lim _{L \rightarrow 0} \Delta m=0\right)$.
By assuming $\Delta \mathbf{m}$ is $\mathbf{M s}$ at the end point of transition curve, one could calculate the final relation $\Delta \mathbf{m}$. The boundary conditions at the end point of connection arc are as follows:

$$
\begin{aligned}
& \Delta m=M_{S}=W / 2 \\
& L=L_{S}
\end{aligned}
$$

Where $\mathbf{W}$ is the width of way, and $\mathbf{L s}$ is the length of transition curve, by substituting boundary condition of the end point of transition curve into equation 5 we obtain:

$$
\begin{equation*}
M_{S}=\left[\left(L_{S}{ }^{2}+f^{2}\left(L_{S}\right)(2-2 \cos \alpha)\right)\right]^{1 / 2} \tag{6}
\end{equation*}
$$

One could calculate by using equation 6 the intersection angle between the two curves $(\alpha)$ in terms of equation 7 :

$$
\begin{equation*}
\alpha=\cos ^{-1}\left[1-\frac{M_{S}{ }^{2}}{2 L_{S}{ }^{2}+2 f^{2}\left(L_{S}\right)}\right] \tag{7}
\end{equation*}
$$

By substituting the obtained values for $(2-2 \cos \alpha)$ into equation (6), the final equation for determining $\Delta \mathrm{m}$ is:

$$
\begin{equation*}
\Delta m=\left[L^{2}+f^{2}(L)\left(\frac{M_{S}{ }^{2}}{L_{S}^{2}+f^{2}\left(L_{S}\right)}\right)\right]^{1 / 2} \tag{8}
\end{equation*}
$$

This equation gives the distance of inner edge of transition curve from the curve of sight limit at any point. Also, this equation for $\mathbf{L}=\mathbf{L s}$ results in:

$$
\Delta \mathrm{m}=\mathrm{Ms}
$$

### 2.2. Calculation of the Area of Clear Sight Obstacles on the Length of Transition Curve

In fig 1 , one can calculate the area between the two curves integration of equation (9):

$$
\begin{equation*}
A=\int\left(F^{\prime}(x, y)-F(x, y)\right) d x \tag{9}
\end{equation*}
$$

In terms of equation (3) $\mathbf{Y}=\mathbf{f}(\mathbf{x}, \mathbf{y}), \mathbf{Y}^{\prime}=\mathbf{f}^{\prime}(\mathbf{x}, \mathbf{y})=-\mathbf{x} \sin \boldsymbol{\alpha}+\mathbf{y} \cos \boldsymbol{\alpha}$, if this value is substituted in equation (11), the area between those two curves from equation (10) is given:

$$
\begin{equation*}
A=\int-x \sin \alpha d x+\int y(\cos \alpha-1) d x \tag{10}
\end{equation*}
$$

$$
X=L
$$

By assuming: $\quad Y=f(X)=f(L)$

And by substituting into equation (10), the area between those two curves would be obtained from equation (11):

$$
\begin{equation*}
A=-\sin \alpha \int_{0}^{L} L d L+(\cos \alpha-1) \int_{0}^{L} f(L) d L \tag{11}
\end{equation*}
$$

By substituting integral bounds into equation (11):

$$
\begin{equation*}
A=\frac{-\sin \alpha L^{2}}{2}+(\cos \alpha-1) \int_{0}^{L} f(L) d L-\int_{0}^{L} f(L) d L \tag{12}
\end{equation*}
$$

In equation (12), $\mathbf{A}$ is the area of clear zone of sight distance from obstacles at any point of transition curve, $\mathbf{L}$ is the length of transition curve at any point.

### 2.3. Calculation of the Area of Clear Sight Distance from obstacles within Circular Curves

In fig. 2, the curve $\mathbf{Y}=\mathbf{F}(\mathbf{x})$ is circular curve with radius R and the curve $\mathbf{Y}^{\prime}=\mathbf{F}^{\prime}(\mathbf{x})$ is the sight distance margin curve (clearance obstacles limit). If the maximum distance of circular curve from the curve that provides sight distance margin is in the middle of curve and equal to $\mathbf{M s}$, then the radius of curve $\mathbf{Y}^{\prime}$ is $\mathbf{R - M s}$.


Fig. 2 Distance of the edge of the inner curve from obstacles within circular curve
By considering half of the length of curve in accordance with Fig. 3, one could determine distance and the area between the curves.


Fig. 3 Area of clearing obstacles within circular curve
The equation from section 2.1 could be written into following form:

$$
\begin{equation*}
\Delta m^{2}=\left(x^{2}+y^{2}\right)(2-2 \cos \alpha) \tag{13}
\end{equation*}
$$

By assuming,

$$
\mathrm{X}=\mathrm{R} \cdot \sin \Delta \quad \mathrm{Y}=\mathrm{f}(\mathrm{R} \cdot \sin \Delta)
$$

Where $\Delta$ is the central angle of circular curve, and $\mathbf{R}$ is the radius of circular curve. (Recent equation is obtained from the extension of circle equation.) Also, boundary conditions ( $\mathbf{\Delta M}=\mathbf{M s}$ and $\Delta=\mathbf{L} / \mathbf{R}$ ) are substituted into equation 13, in terms of equation 14 , one could calculate the angle between these two curves $(\alpha)$ :

$$
\begin{equation*}
\alpha=\cos ^{-1}\left[1-\frac{M_{S}{ }^{2}}{2 R^{2} \sin ^{2} \Delta+2 f^{2}(R \sin \Delta)}\right] \tag{14}
\end{equation*}
$$

The area between the two circular curve is calculated by integrating in terms of $y$ in accordance with equation 15:

$$
\begin{equation*}
A=\int_{0}^{y}\left(x-x^{\prime}\right) d y \tag{15}
\end{equation*}
$$

By substituting values of $x$ ' into equation 3 of section 2.1, equation 16 is obtained:

$$
\begin{equation*}
A=\int_{0}^{y} x(1-\cos \alpha) d y-\int_{0}^{y} y \sin \alpha d y \tag{16}
\end{equation*}
$$

By substituting following assumptions into equation 16 :

$$
\begin{gather*}
X=R \cdot \sin \Delta \\
Y=f(R \cdot \sin \Delta) \\
d y=d f(R \cdot \sin \Delta) \cdot R \cdot \cos \Delta \cdot d \Delta \\
A=(R \cdot \sin \alpha)\left(R \cdot \int_{0}^{y} \sin \Delta f(R \cdot \sin \Delta) \cos \Delta d \Delta-\int_{0}^{y} f^{2}(R \cdot \sin \Delta) \cos \Delta d \Delta\right) \tag{17}
\end{gather*}
$$

## 3. SENSITIVITY ANALYSIS

In this section, sensitivity is performed by using derived equations. For this purpose we consider the change in $\Delta \mathbf{m}$ and $\mathbf{A}$ by considering the change in effective parameters. Equation 8 indicates that there is linear relationship between the considered point and the value of curve function in terms of the length of transition curve to the considered point. Also, it has inverse relation with the length of transition curve and the value of function in terms of the length of transition curve. In the equation 12, the area of clear zone of sight distance from obstacles is proportional to the length of curve that the area of clearance zone increases as the length of curve increases.

Equation 14 indicates that the angle between two curves ( $\boldsymbol{\alpha}$ ) decreases as the radius and central angle of circular arc increase. Also, equation 17 indicates that the area of clear zone (A) increases as the radius of circular curve increases.

## 4. Conclusions

This paper presented mathematical theories of calculus is used to calculate the area of clear zone of sight distance from obstacles. The proposed method is more accurate than graphical methods and its error rate minimal. The results of study indicate that the length of transition curve and the value of function in terms of the transition length are of utmost important parameters to determine the border and the area of clear zone of sight distance from obstacles. The angle between two curves decreases as the transition length increases, and so $\Delta \mathbf{m}$ decreases as the angle $\boldsymbol{\alpha}$ decrease. It can be concluded that the equations for the distance of the inner edge of the curve form sight distance to the obstacles decreases as the length of transition curve increases. Hence these equations may also be used to determine adequate distance from the road side to location for posting traffic signs and alignments.

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