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Seepage Analysis through Earth Dam Based on Finite Difference Method

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ABSTRACT

In the present paper, the finite difference method (FDM), the five-point approximation technique, has been presented to deal with seepage problem in earth dams. The grid system, with computational boundary being coincident with the physical boundary, was numerically obtained by solving Laplace equation. The method was applied to analyze the steady seepage in an earth dam. In this study, three different grid types were considered, and the results were compared with ones obtained by analyzing with Geostudio 2007 software. It showed that by choosing small enough increments, the results are satisfactory.
KEYWORDS: Seepage, Earth dam, Finite difference method, Geostudio.

1. INTRODUCTION

Problems concerning movement of water through soil as porous medium are very broad. In most projects of Civil engineering especially in water and soil trends, irrigation and Oil engineering, the fluid flow is discussed in a porous medium and this topic is one of the proposed problems in all hydraulic structures and especially in earth dams and embankments.

Water moving in a permeable soil media produces a force on the volume of soil which is proportional to hydraulic gradient in desired direction and is called the seepage force. Determination of the seepage force is required to calculate stability of dams and other hydraulic structures [1]. Another phenomena that is raised about dams, is piping. This phenomenon is an erosion process that may occur in the body and also under foundation of the dam. It occurs in places that there is concentrated seepage and output hydraulic gradient exceeds the critical hydraulic gradient. In other word, resistance force of soil against erosion is less than destructive force of seepage at that point. This may cause to sudden destruction of the dam [2].

Analyzing the water flow through a permeable media was first begun in 1856 by introducing Darcy's law. Then it was shown that water flow in isotropic permeable media can be discussed by Darcy's law and this law forms foundations of studies of water seepage in permeable soil media. The Laplace equation which governs water seepage cannot be solved analytically, except for cases with very simple and special boundary conditions. Therefore, researchers have invoked to empirical, graphical and recently numerical methods [3]. Recent developments in computer science have advanced the use of numerical techniques in seepage problems, and new works show the capability of these techniques [4-7].

The finite difference approximation method is a convenient method used to solve the Laplace equation which governs water seepage through soil media. In this paper, the five-point approximation method is applied to deal with steady seepage analysis in homogeneous isotropic medium.

2 Governing Equations

The 2-D governing equation describing the water flow through a porous medium in steady state and obeying the Darcy's law cab be written as:

$$\frac{\partial}{\partial \mathrm{x}}(K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial z}(K_z \frac{\partial h}{\partial z}) = 0 \tag{1}$$

where K_x and K_z are hydraulic conductivities in the horizontal and vertical directions and h is the water head. If the soil is homogeneous with respect to the hydraulic conductivity, Equation 1 simplifies to

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0 \tag{2}$$

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3 The Finite Difference Method

The finite difference method is a numerical procedure used to solve a partial differential equation by discretizing the continuous physical domain into a discrete finite difference grid, approximating the individual exact partial derivatives the partial differential equation by algebraic finite difference approximations, substituting these approximations into the partial differential equation to obtain an algebraic finite difference equation, and solving the resulting algebraic equations for the dependent variable [8]. The finite difference approximations developed by writing Taylor series for the depended variable at several neighboring grid points using grid point (i,j) as the base point, and combining these Taylor series to solve for the desired partial derivatives.

Replacing the derivatives in the 2-D seepage equation, Equation 2, by the second-order centered-difference approximations at grid point (i,j), yields

$$K_{x_{i,j}}\left(\frac{h_{i+1,j}-2h_{i,j}+h_{i-1,j}}{\Delta x^2}\right)+K_{z_{i,j}}\left(\frac{h_{i,j+1}-2h_{i,j}+h_{i,j-1}}{\Delta z^2}\right)=0 \qquad (3)$$

Equation 3 is a second-order centered-difference approximation of Equation 2. Equation 3 can written as

$$h_{i+1,j}+\beta^2 h_{i,j+1}+h_{i-1,j}+\beta^2 h_{i,j-1}-2(1+\beta^2)h_{i,j}=0 \qquad (4)$$

where $\beta=\frac{\Delta x}{\Delta z}\sqrt{\frac{K_z}{K_x}}$. Equation 4 is called The Five-point approximation of the Laplace equation. Solving Equation 4 for $h_{i,j}$ yields

$$\mathrm{h}_{\mathrm{i,j}}=\frac{h_{i+1,j}+\beta^2 h_{i,j+1}+h_{i-1,j}+\beta^2 h_{i,j-1}}{2(1+\beta^2)} \qquad (5)$$

The finite difference stencil for the five-pint method is shown in Figure 1.

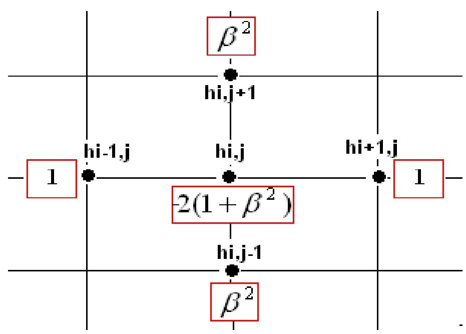

Figure 1. Finite difference stencil for the five-pint method.

4 Boundary Conditions

Boundary conditions are required at the boundaries of solution domain. The boundary conditions governing the solution of the Laplace equation in seepage analysis through earth dam could be divided into the following types.

Dirichlet boundary condition: In this case the function values are specified on boundaries. In seepage analysis through the earth dam, the Dirichlet boundary condition is dominant when the water head is specified on boundaries, for example upstream and downstream of the dam or on Phreatic surface (saturated line).

Neumann boundary condition: In this case the function derivative values are specified on boundaries. For example in seepage analysis, at the last flow line (impervious layer), hydraulic gradient is equal to zero in vertical direction.

Using a second-order centered-difference approximation for the derivative boundary condition yields

$$\left.\frac{\partial \mathrm{h}}{\partial \mathrm{z}}\right|_{i,j} = \frac{h_{i.j+1} - h_{i,j-1}}{2\Delta z} + 0(\Delta z^2) \tag{6}$$

Substituting Equation 6 into Equation 4 yields

$$h_{i+1,j} + 2\beta^2 h_{i,j+1} + h_{i-1,j} - 2(1+\beta^2)h_{i,j} = 2\beta^2 \left.\frac{\partial h}{\partial z}\right|_{i,j} \Delta z \tag{7}$$

Equation 7 applies at derivative boundary conditions to determine water head values.

5 Example of Practice

Figure 2 shows a definition sketch for an isotropic and homogeneous earth dam with a toe drain at downstream built up on an impervious horizontal base. The hydraulic conductivity of materials from which the dam is constructed is equal to $3.3*10^{-6}\, m/s$.

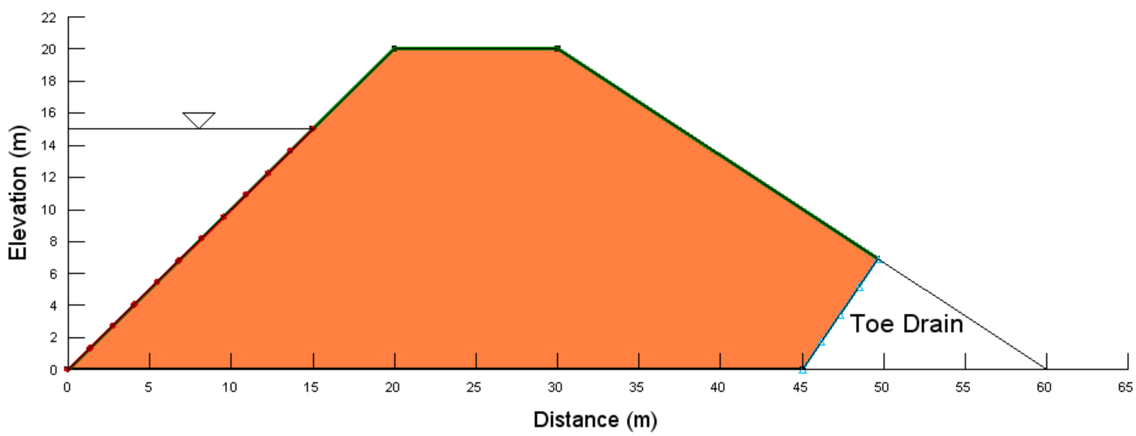

Figure 2. Geotechnical section of the dam.

6 Solution and Results

The first flow line (Phreatic surface) is calculated according to Casagrande's method, and the closed solution domain is created in xz space. The finite difference grids are created for different spacings Δx and Δz. The water head values are calculated at grid points by solving governing equations (Equations 5 and 7) simultaneously according to boundary conditions.

The preceding earth dam was modeled and analyzed with Geostudio 2007 software. Figure 3 shows the model analyzed with Geostudio software.

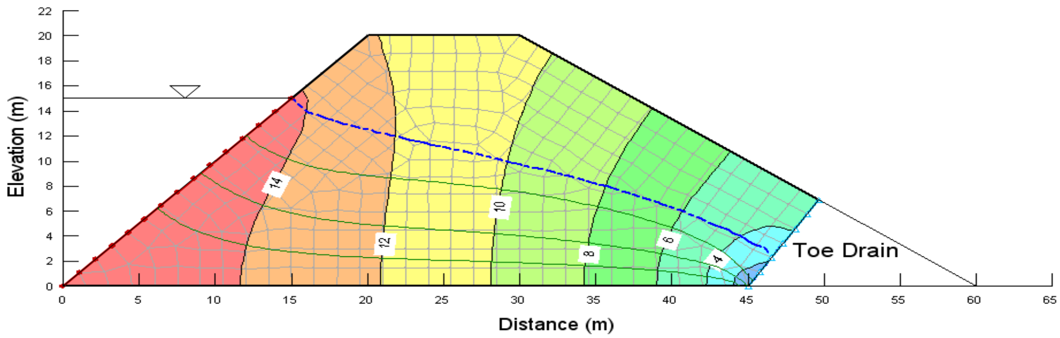

Figure 3. Presentation of the model analyzed with Geostudio software.

The water head values obtained by the five-point approximation method for different grid sizes, Δx and Δz, are summarized in Tables 1 and 3. These values were compared with the results obtained by Geostudio software.

Table 1. Water head values for Δx= 5m and Δz=3m.

Distance (m)	Elevation (m)	Five-point Method	Geostudio Software	difference
5	0	14.813	13.92	0.893
5	3	14.906	13.98	0.926
10	0	14.254	13.14	1.114
10	3	14.329	13.48	0.849
10	6	14.649	13.54	1.109
15	0	13.355	12.85	0.505
15	6	13.572	12.93	0.642
15	12	14.182	13.54	0.642
20	0	12.2	11.43	0.77
20	3	12.228	11.49	0.738
20	6	12.297	11.49	0.807
20	9	12.449	11.63	0.819
25	0	10.879	10.05	0.829
25	6	10.97	10.12	0.85
25	9	11.042	10.19	0.852
30	0	9.411	8.72	0.691
30	3	9.449	8.78	0.669
30	6	9.558	8.93	0.628
35	0	7.71	7.02	0.69
35	3	7.79	7.03	0.76
35	6	8.003	7.18	0.823
40	0	5.488	4.62	0.868
40	3	5.741	4.78	0.961
45	3	3.429	2.76	0.669

Table 2. Water head values for Δx= 2.5m and Δz=2m.

Distance (m)	Elevation (m)	Five-point Method	Geostudio Software	difference
5	0	14.813	14.17	0.643
5	3	14.906	14.28	0.626
10	0	14.254	13.71	0.544
10	3	14.329	13.76	0.569
10	6	14.649	13.94	0.709
15	0	13.355	13.03	0.325
15	6	13.572	13.11	0.462
15	12	14.182	13.87	0.312
20	0	12.2	11.76	0.44
20	3	12.228	11.79	0.438
20	6	12.297	11.83	0.467
20	9	12.449	11.94	0.509
25	0	10.879	10.39	0.489
25	6	10.97	10.47	0.5
25	9	11.042	10.61	0.432
30	0	9.411	9.04	0.371
30	3	9.449	9.12	0.329
30	6	9.558	8.23	1.328
35	0	7.71	7.24	0.47
35	3	7.79	7.28	0.51
35	6	8.003	7.54	0.463
40	0	5.488	4.93	0.558
40	3	5.741	5.32	0.421
45	3	3.429	3.08	0.349

Table 3. Water head values for Δx= 1m and Δz=1m.

Distance (m)	Elevation (m)	Five-point Method	Geostudio Software	difference
5	0	14.813	14.67	0.143
5	3	14.906	14.71	0.196
10	0	14.254	14.11	0.144
10	3	14.329	14.22	0.109
10	6	14.649	14.41	0.239
15	0	13.355	13.2	0.155
15	6	13.572	13.32	0.252
15	12	14.182	14.02	0.162
20	0	12.2	11.98	0.22
20	3	12.228	12.05	0.178
20	6	12.297	12.11	0.187
20	9	12.449	12.33	0.119
25	0	10.879	10.68	0.199
25	6	10.97	10.83	0.14
25	9	11.042	10.94	0.102
30	0	9.411	9.28	0.131
30	3	9.449	9.37	0.079
30	6	9.558	9.49	0.068
35	0	7.71	7.65	0.06
35	3	7.79	7.71	0.08
35	6	8.003	7.88	0.123
40	0	5.488	5.31	0.178
40	3	5.741	5.64	0.101
45	3	3.429	3.31	0.119

5 Conclusions

Seepage analysis is necessary for earth dam design. Irregular seepage through the earth dam may be thread to the integrity and stability of the structure and could lead to the failure of the dam. In this paper, the five-point method based on the finite difference approximations is presented to deal with the steady seepage through an earth dam. An isotropic and homogeneous earth dam with a toe drain at downstream built up on an impervious horizontal base is considered in this paper. The solution closed domain is created, and the grid system is numerically obtained with computed boundary coinciding with the physical boundary. The solution of seepage problem by the five-point method is compared with the solution obtained by Geostudio software. It shows that with small enough Δx and Δz, the results are satisfactory. It is to be mentioned that some other numerical methods, including finite volume method, finite element method and FEM, have been utilized before to deal with seepage problems. These methods are more complicated compared with the finite difference method. The evaluation of capability of the five-point method for seepage analysis through an isotropic and homogeneous earth dam proved to be successful. However, it is suggested that this method applied when the medium is inhomogeneous.

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