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Thermodynamic Study of Turbofan Engine in Off-Design Conditions

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ABSTRACT

In this paper Turbofan engine performance graphs including thrust, thrust specific fuel consumption and thermal, propulsive and the overall efficiencies, also the graphs of turbofan components such as the high pressure and low pressure compressor pressure ratio, exit temperature pressure from high pressure compressor, combustor inlet temperature, corrected inlet mass flow rate of compressor and fan and bypass ratio, that is controlled by the engine control system were drawn based on inlet Mach number and various flight heights. Graphs have been analyzed and the effect of each of the aforementioned parameters was observed on performance graphs. Also, in order to yield more accurate solutions, the method of generating performance graphs has been modified.
KEYWORDS: Turbofan, Off-Design ,Mach number.

1. INTRODUCTION

Motor thermodynamic analysis includes assessing changes in thermodynamics of operating fluid passing into engine. This study can be divided into two entirely distinct groups: On-design analysis (Parametric Cycle Analysis) and Off-Design Analysis (engine Performance Analysis). In On-Design Analysis, the geometry of engine is not considered and in studying the performance diagram related to this analysis, each point represents a different engine. It is said that On-Design Analysis examines a Rubber Engine (1). To estimate engine performance in different air conditions, a modeling approach is needed that is capable of describing the behavior of engine components in the absence of On-Design conditions. In the late sixties, it was found sufficient to optimize the engine in a specific working point. But today, mainly for economic reasons, existence of an off-design model is necessary in early stages of design. (2). When a gas turbine engine is designed and built, its degree of freedom depends on available technology, the demands of designer and the engine's main applications (3). When the engine is installed on an aircraft, its efficiency changes with throttle setting and flying conditions and is limited by engine control system (4). The goal of all Off-Design models is to calculate fluid state in different locations of main stream in engine. Using these results, thrust, fuel consumption and all major parameters of engine components can be derived. Cohen and et al in (5) explain that performance properties of individual components of the engine can be obtained from actual test. When engine components are assembled, the performance range of each component is reduced in engine. When the engine is operating in steady speed or equilibrium, depending on the type of gas turbine, equilibrium running zone can be drawn for a range of different speeds on the compressor characteristic curve until equilibrium running line or zone is obtained. When the running condition is determined, running curves of thrust or specific fuel consumption can be achieved. Oates in (4) Explains that Off-Design Analysis can be done in two ways. The first considered case is that efficiencies of components are unknown; therefore they must be estimated as a function of Performance conditions. This method is used to calculate the primary estimate of the engine's performance. In the second case, components of engine have been built and tested; therefore performance characteristics of the engine components are available. Thus, combination performance of compressor, combustion chamber and turbine are predictable. These combined characteristics, called pumping characteristics, can be used to predict the overall performance of engine. Thus, methods of combining each of the components characteristics in order to obtain pumping characteristics are provided.Once this phase has been completed, operating characteristics will be obtained. For simplicity, the example of a turbojet engine is presented. First, simple gas generator equation is obtained and after having been connected to nuzzle, final equations are generated. Also Walsh acquires performance graphs by using β Lines and Referred Performance Charts (3). Mattingly in (6) and (1) analyzes engine performance by replacing constant values obtained in function of engine pressure ratio and temperature ratio, in a state of Off-Design, with values of same function at design point. As will be explained further in this study, recent analysis will be used for analysis of turbofan engines in a state of Off-Design. Suggestions for improving the results are also presented. Zero-dimensional model is used in this analysis. These models, with regard to their simplicity and self-explanatory nature which is independent of the exact geometry of the engine, are among the most commonly used models in the world of turbo-machinery (2).

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2. Solving Methods and Hypotheses

Describing and predicting the engine performance in different flight conditions and throttle settings is done using relationships which are based on the application of mass, energy, momentum and entropy consideration to the steady flow of perfect gas, in a steady state operating point of engine. Also if the relationship between temperature ratio and pressure ratio, in a steady state operating point, is equal to constant value, the constant value can be evaluated at a reference (design) condition that is expressed by the subtitles R. Thus:

$$f(\tau,\pi)=cte=f(\tau_R,\pi_R) \tag{1}$$

This method that is based on the replacement of constant values of pressure ratio and temperature ratio functions in a state of an off-design, with the same function in condition of design point, is alternatively used in problem analysis. Sea-Level Static conditions are considered as design point conditions for values of the gas turbine engine variables (6) and (1). In turbofan engines the considered case is that high-pressure and low-pressure turbine inlet will choke as sonic stream in inlet guide vanes in turbine will be supersonic in the first stage of the rotor and this leads to a more productive power in the turbine. Furthermore, we assume the throat areas where choking in high pressure and low-pressure turbine entrance nozzle occures are constant. this turbines is called Fixed-area Turbine (FAT). These hypotheses are valid in a wide operating range for modern gas turbine engines (6). As well, based on Reference hypotheses (6), pressure ratio of combustion chamber (π_b) , main exit nozzle (π_n) and bypass exit nozzle (π_{fn}) and efficiency of components towards design point values do not change. Turbine cooling and the leakage effects are ignored and no power is removed from turbine to drive accessories. Gas in upstream and downstream of the combustion chamber is considered calorically perfect.

3. Off-design equations of turbofan

Figure 1 shows a turbofan engine. Turbine and compressor are divided into two parts of low pressure and high pressure. High-pressure turbine drives the high-pressure compressor through High Pressure Spool.also Low-pressure turbine drives fan and low pressure compressor through Low Pressure Spool.

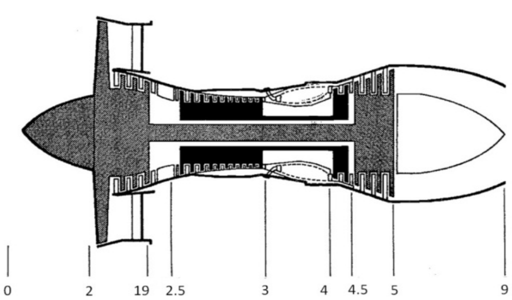

Figure 1) turbofan engine (6)

Mass flow passing through the engine core and fan are $\dot{m}_C$ and $\dot{m}_F$ respectively. The ratio of passing mass flow from the fan to passing mass flow from the core is introduced as Bypass Ratio and is shown with α thus:

$$\alpha=\frac{\dot{m}_F}{\dot{m}_C} \tag{2}$$

The mass flow parameter is a function of Mach number and is defined as follows:

$$MFP\equiv\frac{\dot{m}\sqrt{T_t}}{P_tA},\ \frac{\dot{m}}{A}=\rho V=\frac{PV}{RT}=\frac{V}{\sqrt{\gamma g_c RT}}\times\frac{P\sqrt{\gamma g_c}}{\sqrt{RT}}=M\sqrt{\frac{\gamma g_c}{R}}\frac{P}{\sqrt{T}}$$

$$MFP(M)\equiv M\sqrt{\frac{\gamma g_c}{R}}\frac{P/P_t}{\sqrt{T/T_t}} \tag{3}$$

Also Mass flow ratio of air to fuel in the combustion chamber is defined as follows:

$$f=\frac{\dot{m}_{fuel}}{\dot{m}_{air}} \tag{4}$$

Since the exhaust nozzles have fixed areas, this gas turbine has four independent variables including T_0,M_0,T_{t4},P_0. Also 11 dependent variables are obtained as follows:

High pressure Turbine - inlet mass flow rate of low pressure turbine and high pressure is equal to each other. Therefore:

$$\dot{m}_4 = \frac{P_{t4}}{\sqrt{T_{t4}}} A_4 MFP\left(M_4\right) = \dot{m}_{4.5} = \frac{P_{t4.5}}{\sqrt{T_{t4.5}}} A_{4.5} MFP\left(M_{4.5}\right)$$

Assuming constant area and choking in these sections cause $\dot{m}_4$ and $\dot{m}_{4.5}$ to have constant values:

$$\frac{P_{t4.5}}{P_{t4}} \times \frac{\sqrt{T_{t4}}}{\sqrt{T_{t4.5}}} = \frac{\pi_{tH}}{\sqrt{\tau_{tH}}} = cte$$

Also, according to the constant value of η_{tH} ‘ π_{tH} and τ_{tH} will have fix values.

Low-pressure turbine - inlet mass flow rate of low-pressure turbine is equal to outlet mass flow rate of the exit nozzle:

$$\dot{m}_{4.5} = \frac{P_{t4.5}}{\sqrt{T_{t4.5}}} A_{4.5} MFP(M_{4.5}) = \dot{m}_9 = \frac{P_{t9}}{\sqrt{T_{t9}}} A_9 MFP(M_9) \rightarrow$$

$$\rightarrow \frac{P_{t9}}{P_{t5}} \times \frac{P_{t5}}{P_{t4.5}} \times \frac{\sqrt{T_{t4.5}}}{\sqrt{T_{t5}}} \times \frac{\sqrt{T_{t5}}}{\sqrt{T_{t9}}} MFP(M_9) = \frac{A_{4.5}}{A_9} MFP(M_{4.5})$$

According to the constant value of $\frac{P_{t5}}{P_{t9}}$ and $\frac{A_{4.5}}{A_9}$, choking in the low pressure turbine inlet, and equity $T_{t9} = T_{t5}$, the above equation with respect to what was expressed in equation 1, can be simplified into the following equations:

$$\frac{\pi_{tL}}{\sqrt{\tau_{tL}}} MFP\left(M_9\right) = \frac{P_{t5}}{P_{t9}} \times \frac{A_{4.5}}{A_9} MFP\left(M_{4.5}\right) = cte \rightarrow \frac{\pi_{tL}}{\sqrt{\tau_{tL}}} MFP\left(M_9\right) = \frac{\pi_{tLR}}{\sqrt{\tau_{tLR}}} MFP\left(M_{9R}\right)$$

$$\pi_{tL} = \pi_{tLR} \sqrt{\frac{\tau_{tL}}{\tau_{tLR}}} \frac{MFP\left(M_{9R}\right)}{MFP\left(M_9\right)} \tag{5}$$

$$\tau_{tL} = 1 - \eta_{tL}\left(1 - \pi_{tL}^{\gamma_t - 1/\gamma_t}\right) \tag{6}$$

High pressure compressor - according to power balance between turbine and the high-pressure compressor:

$$\eta_{mH} \dot{m}_4 c_{pt}\left(T_{t4} - T_{t4.5}\right) = \dot{m}_{2.5} c_{pc}\left(T_{t3} - T_{t2.5}\right) \rightarrow \eta_{mH} \frac{\dot{m}_4}{\dot{m}_{2.5}} c_{pt} T_{t4}\left(1 - \tau_{tH}\right) = c_{pc} T_{t2.5}\left(\tau_{cH} - 1\right)$$

$$\frac{\dot{m}_4}{\dot{m}_{2.5}} = \frac{\dot{m}_{2.5} + \dot{m}_f}{\dot{m}_{2.5}} = 1 + \frac{\dot{m}_f}{\dot{m}_{2.5}} = 1 + f \quad , \quad T_{t2.5} = \frac{T_{t2.5}}{T_{t2}} \times \frac{T_{t2}}{T_{t0}} \times \frac{T_{t0}}{T_0} \times T_0 = \tau_f \times 1 \times \tau_r \times T_0 = \tau_r \tau_f T_0$$

$$\tau_r \tau_f \frac{T_0}{T_{t4}}\left(\tau_{cH} - 1\right) = \eta_{mH}(1 + f)(1 - \tau_{tH}) \frac{c_{pt}}{c_{pc}} = cte = \tau_{rR} \tau_{fR} \frac{T_{0R}}{T_{t4R}}\left(\tau_{cH} - 1\right)_R$$

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R} \frac{(\tau_r \tau_f)_R}{\tau_r \tau_f} (\tau_{cH} - 1)_R \tag{7}$$

Also

$$\pi_{cH} = \left[1 + \eta_{cH}\left(\tau_{cH} - 1\right)\right]^{\gamma_c/(\gamma_c - 1)} \tag{8}$$

Low pressure compressors – because the low pressure compressor and fan are on a spool, increasing low-pressure compressor enthalpy is proportional with increasing fan enthalpy therefore:

$$T_{t2.5} - T_{t2} = K\left(T_{t19} - T_{t2}\right) \rightarrow \tau_{cL} - 1 = K\left(\tau_f - 1\right) \rightarrow \frac{\tau_{cL} - 1}{\tau_f - 1} = K = \frac{\tau_{cLR} - 1}{\tau_{fR} - 1}$$

And ratios of temperature and pressure of the low pressure compressor are obtained:

$$\tau_{cL} = 1 + \left(\tau_f - 1\right)\frac{\tau_{cLR} - 1}{\tau_{fR} - 1} \tag{9}$$

$$\pi_{cL} = \left[1 + \eta_{cL}\left(\tau_{cL} - 1\right)\right]^{\gamma_c/(\gamma_c - 1)} \tag{10}$$

From the power balance between the low-pressure turbine, the fan and low pressure compressor we have:

$$\dot{m}_F c_{pc}(T_{t19} - T_{t2}) + \dot{m}_C c_{pc}(T_{t2.5} - T_{t2}) = \eta_{mL}\dot{m}_{4.5} c_{pt}(T_{t4.5} - T_{t5})$$

$$\dot{m}_{4.5} = \dot{m}_C + \dot{m}_F = \dot{m}_C(1+f) \quad , \quad \dot{m}_C = \alpha\,\dot{m}_C \quad , \quad T_{t2} = T_{t0} \quad , \quad T_{t4.5} = \frac{T_{t4.5}}{T_{t4}} \times \frac{T_{t4}}{T_0} \times T_0 = \tau_{tH}\tau_\lambda T_0$$

$$\alpha \times c_{pc}T_{t2}\left(\frac{T_{t19}}{T_{t2}} - 1\right) + c_{pc}T_{t2}\left(\frac{T_{t2.5}}{T_{t2}} - 1\right) = \eta_{mL}c_{pt}\tau_{tH}\tau_\lambda T_0(1+f)\left(1 - \frac{T_{t5}}{T_{t4.5}}\right) \rightarrow$$

$$\rightarrow \frac{T_{t2}\left[\alpha\left(\frac{T_{t19}}{T_{t2}} - 1\right) + \left(\frac{T_{t2.5}}{T_{t2}} - 1\right)\right]}{\tau_\lambda T_0} = \eta_{mL}\tau_{tH}\frac{c_{pt}}{c_{pc}}(1+f)\left(1 - \frac{T_{t5}}{T_{t4.5}}\right) \rightarrow$$

$$\rightarrow \frac{\tau_r\left(\alpha(\tau_f - 1) + (\tau_{cL} - 1)\right)}{\tau_\lambda(1 - \tau_{tL})} = \eta_{mL}\tau_{tH}\frac{c_{pt}}{c_{pc}}(1+f) = cte = \frac{\tau_{rR}\left(\alpha(\tau_f - 1) + (\tau_{cL} - 1)\right)_R}{\tau_{\lambda R}(1 - \tau_{tL})_R}$$

$$\tau_f = 1 + (\tau_{fR} - 1)\left[\frac{1 - \tau_{tL}}{(1 - \tau_{tL})_R}\;\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\;\frac{\tau_{cLR} - 1 + \alpha_R(\tau_{fR} - 1)}{\tau_{cLR} - 1 + \alpha(\tau_{fR} - 1)}\right] \tag{11}$$

and also:

$$\pi_f = \left[1 + \eta_f\left(\tau_f - 1\right)\right]^{\gamma_c/(\gamma_c - 1)} \tag{12}$$

Bypass ratio - in engine core, according to occurrence of choking in high-pressure turbine enterance, fixed inlet area of turbine and constante value of fuel to air ratio, and replacing a constant value with reference values, we have:

$$\dot{m}_C = \dot{m}_4 - \dot{m}_f = \frac{\dot{m}_4}{1+f} = \frac{P_{t4}A_4}{\sqrt{T_{t4}}}\frac{MFP(M_4)}{1+f} \rightarrow \frac{\dot{m}_C\sqrt{T_{t4}}}{P_{t4}} = \frac{A_4 MFP(M_4)}{1+f} = cte = \frac{\dot{m}_{CR}\sqrt{T_{t4R}}}{P_{t4R}}$$

$$\frac{\dot{m}_C}{\dot{m}_{CR}} = \frac{P_{t4}}{P_{t4R}}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{13}$$

Similarly for passing flow from bypass duct we have:

$$\dot{m}_F = \frac{P_{t19}A_{19}}{\sqrt{T_{t19}}}MFP(M_{19}) \rightarrow \frac{\dot{m}_F\sqrt{T_{t19}}}{P_{t19}MFP(M_{19})} = \frac{1}{A_{19}} = cte = \frac{\dot{m}_{FR}\sqrt{T_{t19R}}}{P_{t19R}MFP(M_{19R})}$$

$$\frac{\dot{m}_F}{\dot{m}_{FR}} = \frac{P_{t19}}{P_{t19R}}\sqrt{\frac{T_{t19R}}{T_{t19}}}\frac{MFP(M_{19})}{MFP(M_{19R})} \tag{14}$$

By replacing relations 2-16 and 2-17 both in relation related to bypass ratio:

$$\alpha = \frac{\dot{m}_F}{\dot{m}_C} = \frac{\dot{m}_{FR}}{\dot{m}_{CR}} \times \frac{P_{t19}}{P_{t4}} \times \frac{P_{t4R}}{P_{t19R}} \times \sqrt{\frac{T_{t19R}}{T_{t4R}}} \times \sqrt{\frac{T_{t4}}{T_{t19}}} \times \frac{MFP(M_{19})}{MFP(M_{19R})}$$

According to:

$$P_{t19} = \frac{P_{t19}}{P_{t2}} \times P_{t2} = \pi_{fn} \times \pi_f \times P_{t2} \quad , \quad P_{t4} = \frac{P_{t4}}{P_{t3}} \times \frac{P_{t3}}{P_{t2.5}} \times \frac{P_{t2.5}}{P_{t2}} \times P_{t2} = \pi_b \times \pi_{cH} \times \pi_{cL} \times P_{t2}$$

$$\frac{P_{t19}}{P_{t4}} = \frac{\pi_{fn}\pi_f}{\pi_b\pi_{cH}\pi_{cL}}$$

Also consider that $T_{t19} = T_{t2.5}$

$$\frac{T_{t4}}{T_{t19}} = \frac{T_{t4}}{T_0} \times \frac{T_0}{T_{t0}} \times \frac{T_{t0}}{T_{t2.5}} \times \frac{T_{2.5}}{T_{t19}} = \tau_\lambda \times \frac{1}{\tau_r} \times \frac{1}{\tau_f} \times 1 = \frac{\tau_\lambda}{\tau_r \tau_f}$$

$$\alpha = \alpha_R \times \frac{\pi_{fn}\pi_f}{\pi_b \pi_{cH} \pi_{cL}} \times \frac{P_{t4R}}{P_{t19R}} \times \sqrt{\frac{T_{t19R}}{T_{t4R}}} \times \sqrt{\frac{\tau_\lambda}{\tau_r \tau_f}} \times \frac{MFP(M_{19})}{MFP(M_{19R})} \rightarrow$$

$$\rightarrow \frac{\alpha\, \pi_{cH}\pi_{cL}}{\pi_f MFP(M_{19})}\sqrt{\frac{\tau_r \tau_f}{\tau_\lambda}} = \alpha_R \times \frac{\pi_{fn}}{\pi_b} \times \frac{P_{t4R}}{P_{t19R}} \times \sqrt{\frac{T_{t19R}}{T_{t4R}}} \times \frac{1}{MFP(M_{19R})} = cte = \frac{\alpha_R\, \pi_{cHR}\pi_{cLR}}{\pi_{fR} MFP(M_{19R})}\sqrt{\left[\frac{\tau_r \tau_f}{\tau_\lambda}\right]_R}$$

$$\alpha = \alpha_R \frac{\pi_{cHR} \times {\pi_{cLR}}/{\pi_{fR}}}{\pi_{cH} \times {\pi_{cL}}/{\pi_f}} \sqrt{\frac{\tau_\lambda/(\tau_r \tau_f)}{[\tau_\lambda/(\tau_r \tau_f)]_R}} \frac{MFP(M_{19})}{MFP(M_{19R})} \tag{15}$$

Mass flow rate of the engine:

$$\dot{m}_0 = (1+\alpha)\dot{m}_C = (1+\alpha)\frac{P_{t4}}{\sqrt{T_{t4}}}\frac{A_4 MFP(M_4)}{1+f} =$$

$$= (1+\alpha)\frac{P_{t4}}{P_{t3}} \times \frac{P_{t3}}{P_{t2.5}} \times \frac{P_{t2.5}}{P_{t2}} \times \frac{P_{t2}}{P_{t0}} \times \frac{P_{t0}}{P_0} \times P_0 \times \frac{A_4}{\sqrt{T_{t4}}} \times \frac{MFP(M_4)}{1+f} =$$

$$= (1+\alpha)\pi_b \pi_{cH} \pi_{cL} \pi_d \pi_r P_0 \frac{A_4}{\sqrt{T_{t4}}} \times \frac{MFP(M_4)}{1+f} \rightarrow$$

$$\rightarrow \frac{\dot{m}_0 \sqrt{T_{t4}}}{(1+\alpha)P_0 \pi_{cH} \pi_{cL} \pi_d \pi_r} = \frac{A_4 \pi_b MFP(M_4)}{(1+f)} = cte = \frac{\dot{m}_{0R}\sqrt{T_{t4R}}}{(1+\alpha)_R (P_0 \pi_{cH} \pi_{cL} \pi_d \pi_r)_R}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{1+\alpha}{1+\alpha_R} \times \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{16}$$

the outlet mach number at the main channel:

$$M_9^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right] \tag{17}$$

If the outlet of the main channel is not choking $\frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$ otherwise $\frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\frac{\gamma_t}{\gamma_t - 1}}$

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_f \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_n$$

exit Mach number in bypass channel:

$$M_{19}^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c - 1)/\gamma_c} - 1\right] \tag{18}$$

If the outlet of the main channel is not choking $\frac{P_{t19}}{P_{19}} = \frac{P_{t19}}{P_0}$ Otherwise $\frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_t + 1}{2}\right)^{\frac{\gamma_t}{\gamma_t - 1}}$

$$\frac{P_{t19}}{P_0} = \pi_r \pi_d \pi_f \pi_{fn}$$

According to above obtained equations, $\pi_{tL}, \tau_{tL}, \alpha, \tau_{cL}, \pi_{cL}, \pi_{cH}, \tau_{cH}, \pi_f, \tau_f, M_9, M_{19}$ are the dependent variables of turbofan engine, which constitute a system consisting of 11 equations. These equations should be solved by Iterative approach to obtain values.

4. design variables, objective functions and constraints

Among input parameters, the three parameters including flying height (h_0), inlet Mach number (M_0), and combuster exit temperature (T_{t4}), are considered as design variables. As seen in the following relation, inlet air mass into the engine is a function of inlet mach number and altitude of airplane.

$$\dot{m}_i = \frac{P_{ti}}{\sqrt{T_{ti}}} A_i \times MFP(M_i) \tag{19}$$

Combuster exit temperature (T_{t4}) is also one of the factors that the engine efficiency is sensitive to its change. As shown in references (1) and (6), Pumping characteristic of a gas generator is a function of T_{t4}.The most important outlet parameters assumed as objective functions in turbofan engine include Thrust, Thrust Specific Fuel Consumption, Propulsive efficiency, Thermal efficiency, and Overall Efficiency (1). These functions for turbofan engine are as follow (6):

$$F = \dot{m}_0 \left(\frac{F}{\dot{m}_0} \right) \tag{20}$$

$$\begin{aligned} \frac{F}{\dot{m}_\circ} &= \frac{1}{1+\alpha} a_0 \left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f) \times \frac{R_t T_9 / T_0}{R_c V_9 / a_0} \frac{1 - P_0 / P_9}{\gamma_c} \right] \\ &+ \frac{\alpha}{1+\alpha} a_0 \times \left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19} / T_0}{V_{19} / a_0} \frac{1 - P_0 / P_{19}}{\gamma_c} \right) \end{aligned} \tag{21}$$

$$S = \frac{f}{(1+\alpha) F / \dot{m}_0} \tag{22}$$

$$\eta_P = \frac{2M_0 \left[(1+f) V_9 / a_0 + \alpha \left(V_{19} / a_0 \right) - (1+\alpha) M_0 \right]}{(1+f)\left(V_9 / a_0 \right)^2 + \alpha \left(V_{19} / a_0 \right)^2 - (1+\alpha) M_0^2} \tag{23}$$

$$\eta_T = \frac{a_0^2 \left[(1+f)(V_9 / a_0)^2 + \alpha (V_{19} / a_0)^2 - (1+\alpha) M_0^2 \right]}{2 f h_{PR}} \tag{24}$$

$$\eta_o = \eta_P \eta_T \tag{25}$$

Engine control must keep high-pressure and low-pressure compressor pressure ratio, compressor exit temperature, corrected mass flow rate of fan and compressor and bypass ratio from exceeding specifiec value. Compressor exit temperature as a function of compressor total temperature as well as the corrected mass flow rate as a function of inlet mass flow rate will be calculated as follows:

$$T_{t3} = \left(T_{t3} / T_{t2.5} \right) \times \left(T_{t2.5} / T_{t2} \right) \times \left(T_{t0} / T_0 \right) \times T_0 = \tau_{cH} \times \tau_{cL} \times \tau_r \times T_0 = \tau_c \times \tau_r \times T_0 \tag{26}$$

$$\dot{m}_{ci} = \frac{\dot{m}_i \sqrt{\theta_i}}{\delta_i} \quad \Rightarrow \quad \dot{m}_{c2} = \frac{\dot{m}_2 \sqrt{\dfrac{T_{t2}}{288.13}}}{\dfrac{P_{t2}}{101.3}} = \frac{\dot{m}_0 \sqrt{\dfrac{T_{t0}}{288.13}}}{\dfrac{P_{t0} \times \pi_d}{101.3}} = \frac{\dot{m}_0 \sqrt{\dfrac{T_0 \times T_r}{288.13}}}{\dfrac{P_0 \times \pi_r \times \pi_d}{101.3}} \tag{27}$$

5) Solving equations

The obtained equations in the third part must be solved by iterative method. Reference values of engine are considered as the initial guess.

$$\tau_{tL} = \tau_{tLR} \quad , \quad \tau_f = \tau_{fR} \quad , \quad \pi_{cL} = \pi_{cLR} \quad , \quad \pi_{tL} = \pi \tag{28}$$

With this assumption, functions of the number (7) to (18) are calculated, this process is repeated until the difference between the values obtained from bypass ratio (α), low pressure turbine temperature ratio (τ_{tL}), high pressure compressor temperature ratio (τ_{cH}) , low pressure temperature ratio (τ_{cL}) and fan temperature ratio (τ_f), are less than 0.0001 from the corresponding values of Previous step. If the mentioned condition is established, solutions obtained as converged solutions are used to calculate the values of performance in relations (20) to (25) . above methods can be used for drawing off-design performance graphs .On the other hand, the engine control system controls the various components performance of gas turbine so that the combuster exit temperature (T_{t4}), high-pressure and low-pressure compressor ratio (π_{cH}, π_{cL}), fan pressure ratio (π_f), the compressor exit temperature (T_{t3}), corrected mass flow rate of compressor $(\dot{m}_{c2})$ and bypass ratio (α) do not exceed from permissible values. Some of these limitations including exit temperature of combuster and exit temperature of the compressor are due to limited temperature tolerance of the parts.It also controls mass flow rate in compressor and compressor pressure ratio of low and high pressure compressor to prevent the occurrence of surge and stall in the engine.Controlling the above-Mentioned parameters within the desired range is done by reducing (T_{t4}).Figure 2 shows the flowchart of drawing performance charts of turbofan engine.

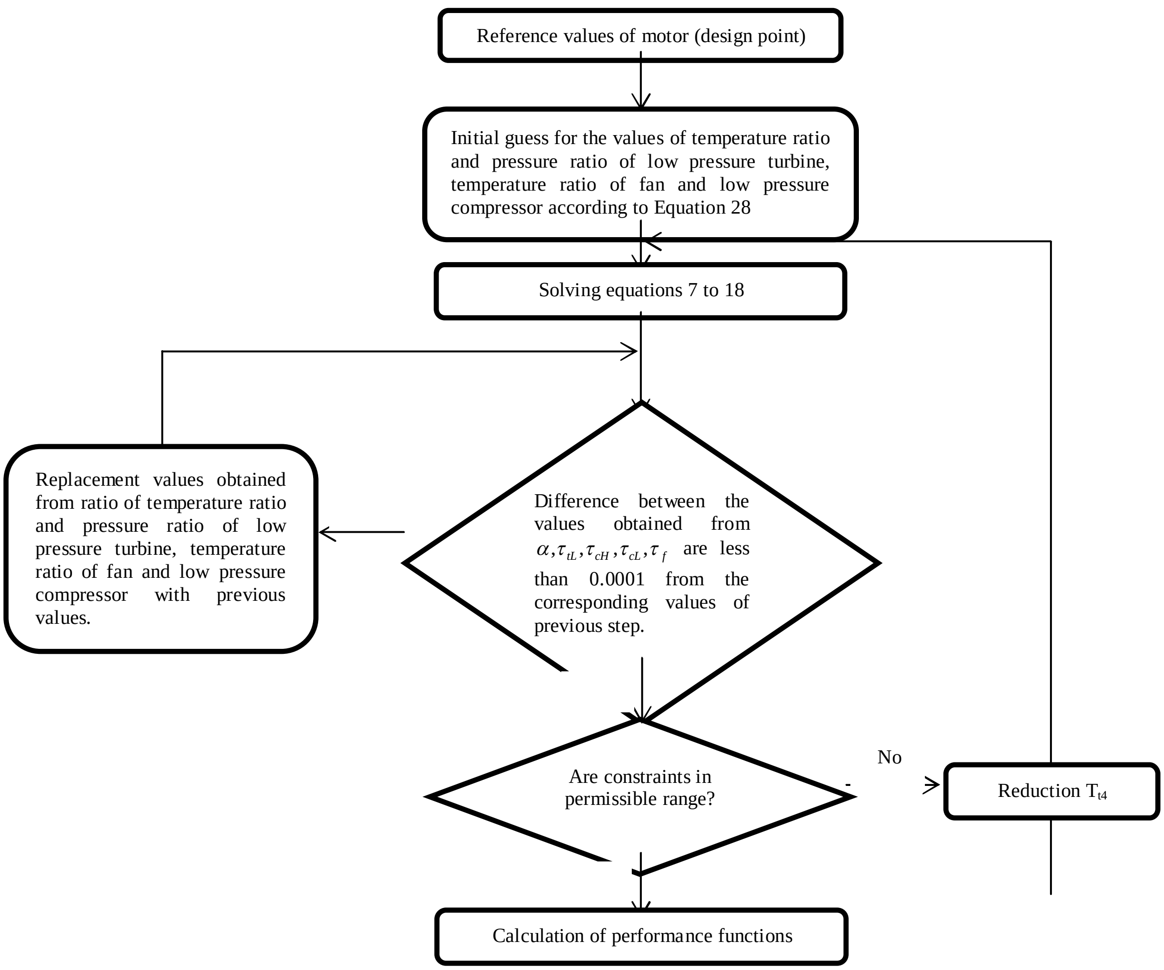

Figure 2) Flowchart of drawing performance diagrams of turbofan engine

6. Performance charts

In this part, performance charts of a turbofan engine in off-design condition are obtained. Referred values of engine (design point) in sea level and zero Mach number are as follow:

$\alpha=8$ $\pi_f=2$ $\pi_{cL}=4$ $\pi_{cH}=8$ $T_{t4}=1890K$ $\dot{m}_0=760\,kg/\sec$ $h_{PR}=42800\,kJ/kg$
$\gamma_c=1.4$ $\gamma_t=1.3$ $C_{pc}=1.004$ $C_{pt}=1.239$ $\tau_{tH}=0.7580$ $\eta_{tH}=0.9625$ $\tau_{tL}=0.7262$
$\eta_{tL}=0.9636$ $\eta_{cH}=\eta_{cL}=0.8512$ $\eta_f=0.8815$ $\eta_{mH}=0.9915$ $\eta_{mL}=0.997$ $\eta_b=0.99$
$\pi_b=0.96$ $\pi_n=\pi_{fn}=\pi_{d\max}=0.99$

The compressor pressure ratio is limited to 32. Also maximum exit temperature of the combuster and compressor are 1890 and 890 Kelvin respectively. In this case, bypass ratio, fan and the compressor pressure ratio, combuster and compressor exit temperatures and corrected mass flow rate of compressor and the fan are controlled by engine controller. Turbofan engine performance charts based on variables of flight speed, at different altitude are drawn and presented. In figure 3 changes in the thrust is drawn versus mach number at various flight heights. As can be seen, as altitude increases, flight speed change has less effect on the thrust. As in $h=12$ thrust is almost constant. Also in $h=0$ and $h=1.5$ an increase in flight speed led to a significant decrease in the thrust force. For a closer look, three heights of $h=0$, $h=3$ and $h=9$ are being investigated.

Investigation in $h=0\ km$

In Figures 9 and 10 corrected mass flow of the compressor and fan are drawn respectively. $\dot{m}_{cC}$ (figure 9) will reduce from Design amount $85.3\,kg/\sec$ in zero mach number to $57.77\,kg/\sec$ in mach number one. Similary $\dot{m}_{cF}$ (figure 10) will reduce from Design amount $682.38\,kg/\sec$ to $564.93\,kg/\sec$. In both charts of Figures 9 and 10 it can be seen that the amount of $\dot{m}_{cC}$ and $\dot{m}_{cF}$ are below the critical values. In Figure 8, with an increase in flight Mach number, $\dot{m}_0$ increases from amount designed in zero Mach number $(760\,kg/\sec)$, to amount $1060\,kg/\sec$. Figure12 shows changes of fan pressure ratio based on Mach number. Fan pressure ratio decreased from a value of 2 in zero Mach number to value 1.598 in Mach number one. Figure 15 shows the Compressor pressure ratio decreases with increasing Mach number. In figures 13 and 14, these changes are presented separately for the low-pressure compressor and high pressure compressor. In Figure 15, with increasing Mach number from zero to one, compressor pressure ratio decrease at the rate of 57% from the amount of 32. Similarly this decreasing in the low-pressure compressor and in high pressure compressor is 87% and 66% (figure 13 and 14). The incremental change of Bypass ratio from its design point in zero Mach number to 10.31 in Mach number one is shown in Figure 11. Also Incremental Changes of compressor exit temperature (T_{t3}) versus increases in flight speed is shown in Figure 16. These changes start from $884K$ in Zero Mach number and continue to Mach number 0.44. From Mach number 0.44 to Mach number one, the maximum increase in temperature would be controlled at $890K$. This control is done by reducing the exit temperature of the Combustor (T_{t4}).Therefore, as seen in Figure 17 ,in Mach number range between 0 and 0.44, T_{t4} remains in design amount of $1890K$ and then is reduced to maintain T_{t3} at its maximum value and this continues until Mach number one. At this point T_{t4} reaches $1817K$.In Figure 18 the core ratio of inlet pressure to exit pressure (P_0/P_9), from value of 0.91 in zero Mach number reaches to 0.8353 in Mach number one . This graph has break in a flight Mach number of 0.44, which is due to control T_{t3}. In figure 19, bypass inlet to exit pressure (P_0/P_{19}) changes rates, increases by increasing the Mach number. Also this function decrease from value of 0.966 in zero Mach number to value of 0.639 in Mach number one.

Investigation in $h=3\ km$

As it can be seen in figure 3, the chart has a breakpoint in flight Mach number 0.6. Also, it is seen that thrust decreases with an increase in Mach number. The breakpoint is due to the fact that all of constraints functions in figure 9 to 14 have reached their design value in Mach number 0.6, thus, by decreasing T_{t4} (Figure 17) they are maintained at their design value. As seen in Figure 17, T_{t4} in Flight Mach numbers less than 0.6 is less than its design value. Also, in Mach numbers higher than 0.76 the value of T_{t4} decrease from its maximum value. This is due to the fact that in this range, function of T_{t3} (Figure 16) has reached its maximum value. As can be seen in figure 16, T_{t3} increases with increase in flight Mach number. The mentioned figure has two breakpoints, in flight mach 0.6, due to a decrease in the amount of T_{t4} in order to control constraints and in flight Mach number 0.76, as T_{t3} has reached its maximum value. As can be seen in Figures 18 and 19, two functions P_0/P_9 and P_0/P_{19} decrease with increase in Mach number values and have break points in Mach number 0.6.

Investigation in $h = 9\ km$

As can be seen in Figure 3, in all Mach number values, thrust is nearly constant. There is no break in the curve. This is due to the fact that all constraints have reached the values of design extrema (figures 9-14) and have been controlled as T_{t4} decreases from value 1890K (Figure 17). Therefore, in Figure 17, the T_{t4} graph for all values of flight Mach numbers is less than 1890K and It is evident that it increases as Flight Mach number increases. As seen in Figure 16, values of T_{t3} for all flight Mach number range is less than the maximum values and the changes trend is rising with increasing flight Mach number.

Know we'll examine other objective functions. Figure 4 shows changes of specific fuel consumption in terms of the Mach number. As can be seen, specific fuel consumption will be increased with constant rate with increasing of Mach number. Figure 5 shows the change of thermal efficiency with flight mach Number at various heights. It is seen that thermal efficiency decreases when Mach number increases. Also, in range of flight Mach values 0 to 0.2 thermal efficiency remains constant in all flight heights. Figure 6 shows increasing change of thrust efficiency by flight Mach number. As it is seen, in flight mach values 0 to 0.35 thrust efficiency doesn't change in various heights. This difference reaches its maximum amount in Mach number one. Figure 7 shows changes graph of overall efficiency in terms of Mach number values. As can be seen, in zero kilometers height, the overall efficiency increases as Mach number increases and reaches its maximum(0.2283) in Mach number 0.63 and then decreases. Also in the range of flight mach values 0 to 0.2, thrust efficiency is not affected by change of height.

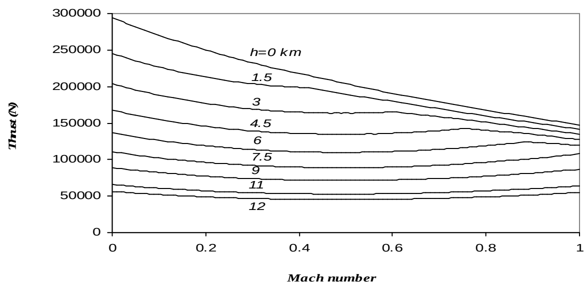

Figure 3) thrust based on flight Mach number at various heights

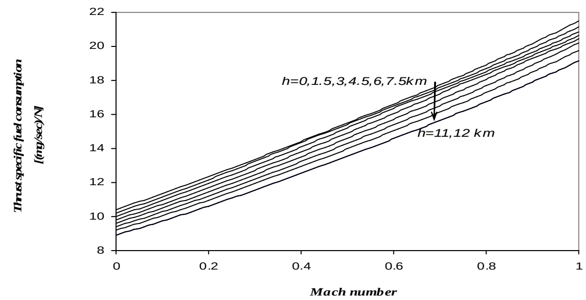

Figure 4) Thrust Specific Fuel Consumption based on flight Mach number at various heights

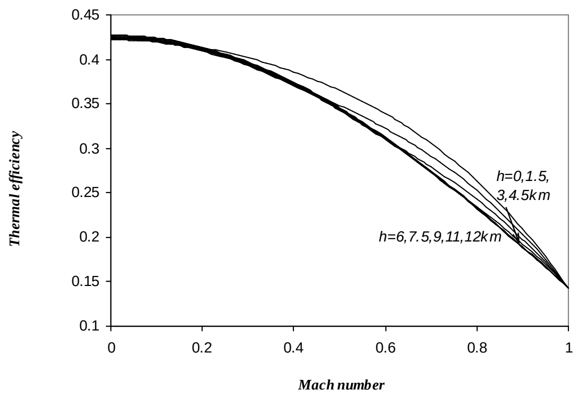

Figure 5) Thermal efficiency based on flight Mach number at various heights

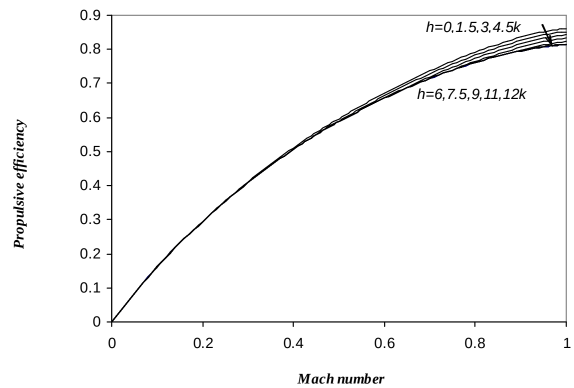

Figure 6) propulsive efficiency based on flight Mach number at various heights

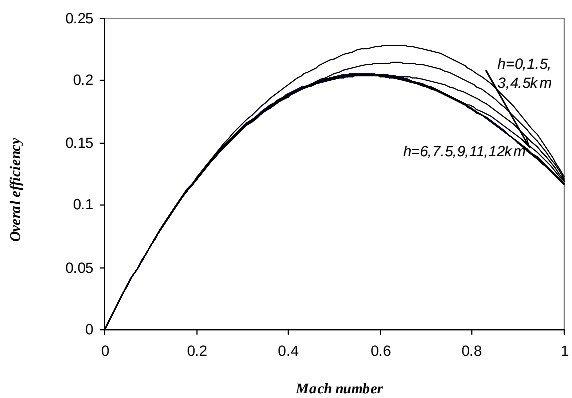

Figure 7) overall efficiency based on flight Mach number at various heights

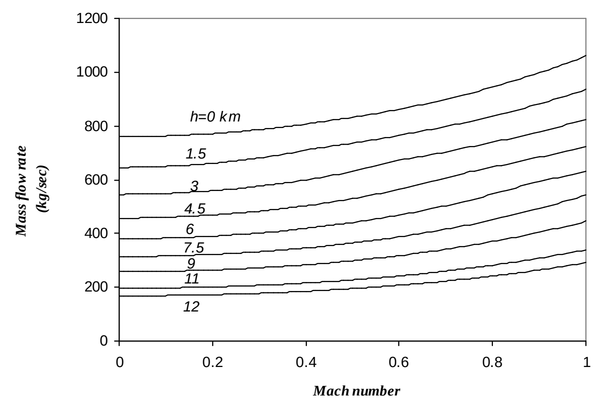

Figure 8) mass flow rate based on flight Mach number at various heights

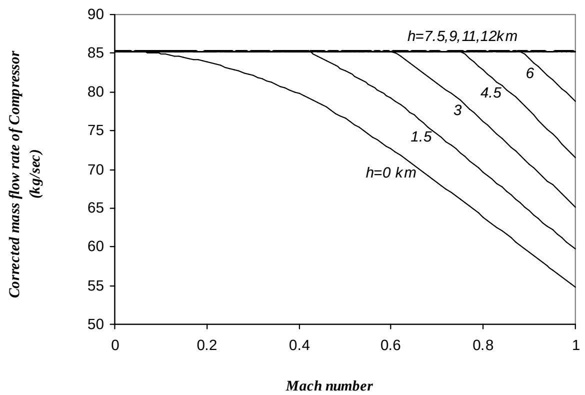

Figure 9) corrected mass flow rate of compressor based on flight Mach number at various heights

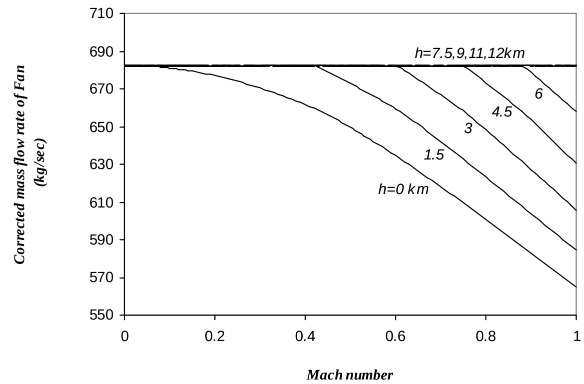

Figure 10) corrected mass flow rate of fan based on flight Mach number at various heights

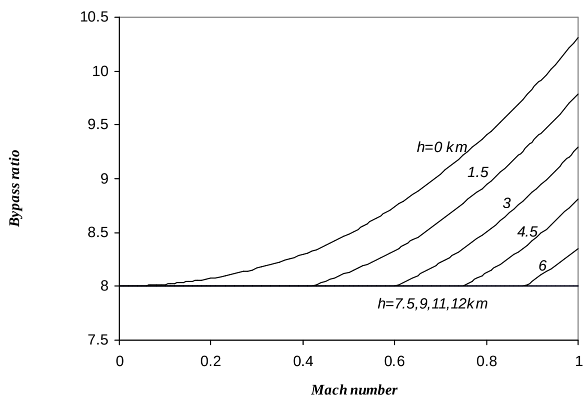

Figure 11) bypass ratio based on flight Mach number at various heights

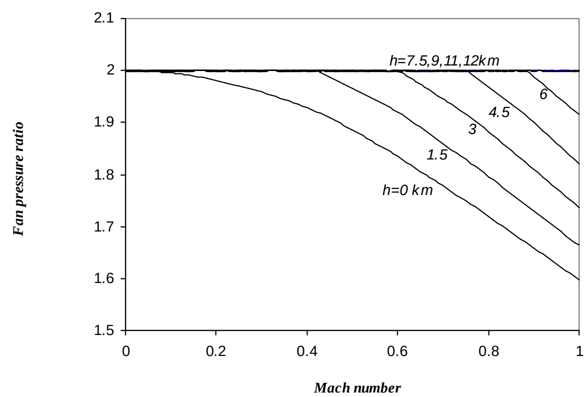

Figure 12) fan pressure ratio based on flight Mach number at various heights

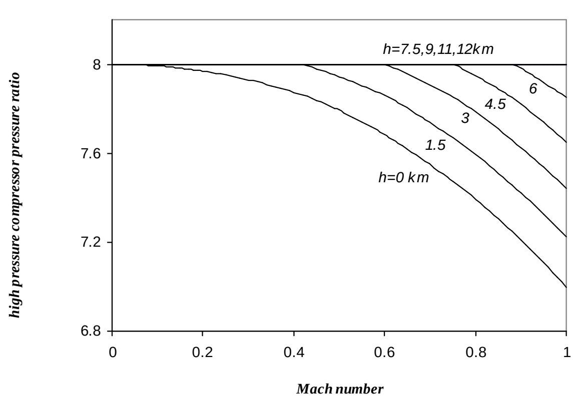

Figure 13) high pressure compressor pressure ratio based on flight Mach number at various heights

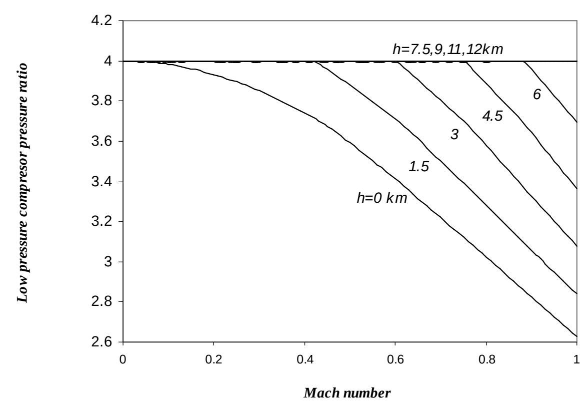

Figure 14) Low pressure compressor pressure based on flight Mach number at various heights

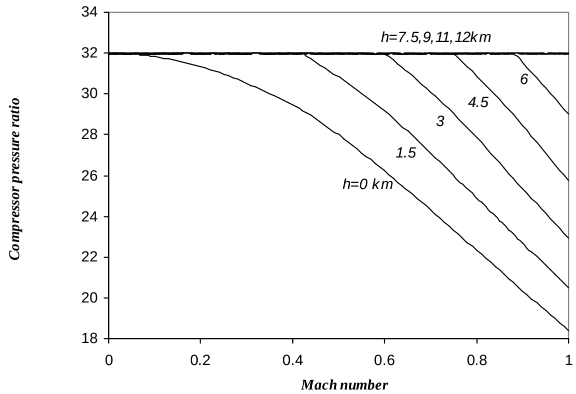

Figure 15) Compressor pressure ratio based on flight Mach number at various heights

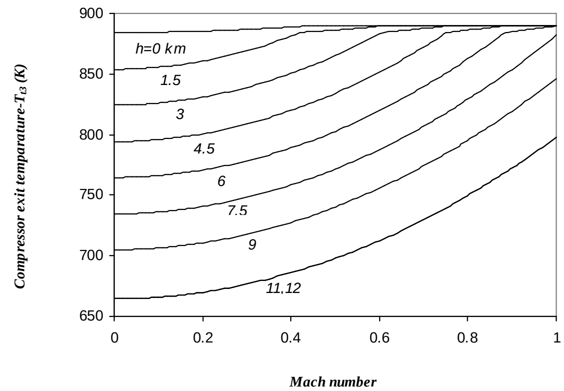

Figure 16) Compressor exit temperature based on flight Mach number at various heights

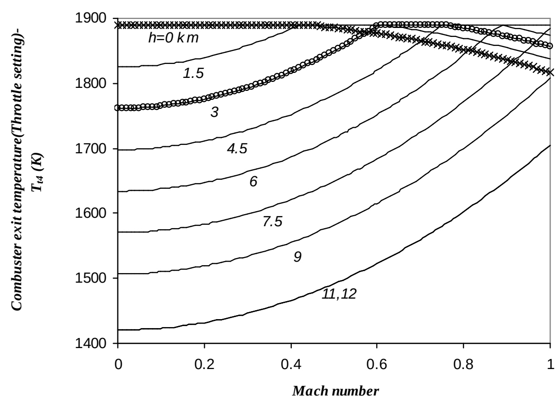

Figure 17) Combuster exit temperature (throttle setting) based on flight Mach number at various heights

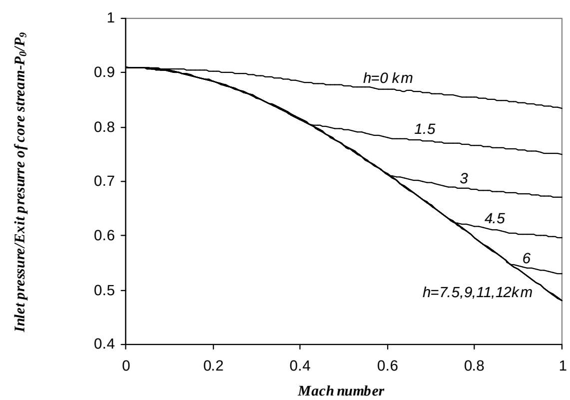

Figure 18) Inlet to exit pressure of core stream based on flight Mach number at various heights

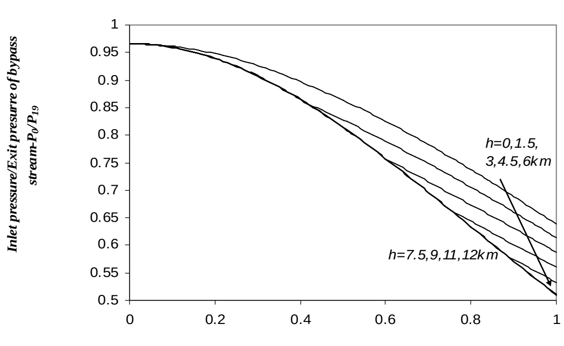

Figure 19) Inlet to exit pressure of bypass stream based on flight Mach number at various heights

7) Corrective suggestion

As illustrated in part 4, to obtain the performance graph in various flight conditions, equations set 7 to 18 should be solved respectively for turbofan engines by iterative method. As suggested In reference (6), this repetition continues until the difference between the temperature ratio of the low pressure turbine, in two consecutive steps is less than 0.0001. However, as can be seen in table 1, temperature ratio and pressure ratio values of fan obtained from this method are not applicable to relation (12). Because as seen in relation (5), when $M_9 = M_{9R} = 1$, the new value of pressure ratio will not change to its previous state and when is substituted in the relation (6), the same values will be produced for temperature ratio and the circle will stop. Therefore, according to part 5 and the flowchart in figure 2, in this study repetition continues until the difference between values of bypass ratio (α), low pressure turbine temperature ratio (τ_{tL}), high pressure compressor temperature ratio (τ_{cH}), the low pressure compressor temperature ratio (τ_{cL}) and fan temperature ratio (τ_f), from corresponding values of the previous step will be less than 0.0001. Fan pressure and temperature ratio values obtained from two methods are shown in Table (1). The values obtained from the modified method are applicable to relation (12). In this section, engine referred values (design point) in sea level and zero Mach number are as follow:

$$\alpha = 5 \quad \pi_f = 2 \quad \pi_{cL} = 4 \quad \pi_{cH} = 5 \quad T_{t4} = 1777.778K \quad \dot{m}_0 = 45.3597\,kg/\sec$$
$$h_{PR} = 42798.4\,kJ/kg \quad \gamma_c = 1.4 \quad \gamma_t = 1.3 \quad C_{pc} = 1.004 \quad C_{pt} = 1.239 \quad \tau_{tH} = 0.8901$$
$$\eta_{tH} = 0.906 \quad \tau_{tL} = 0.7436 \quad \eta_{tL} = 0.914 \quad \eta_{cH} = \eta_{cL} = 0.8791 \quad \eta_f = 0.8898$$
$$\eta_{mH} = \eta_{mL} = 0.99 \quad \eta_b = 0.995 \quad \pi_b = 0.96 \quad \pi_n = \pi_{fn} = 0.98 \quad \pi_{d\max} = 0.97$$

Table 1: various values of fan temperature and pressure ratio (τ_f, π_f) obtained from research and referred proposed method (6), at various flight Mach

$(h = 0km, T_{t4} = 1777.778K)$	*M=0*	*M=0.2*	*M=0.4*	*M=0.6*	*M=0.8*	*M=1*
τ_f **Results of the present study:**	*1.2461*	*1.2429*	*1.2334*	*1.2189*	*1.201*	*1.1813*
π_f **Results of the present study:**	*2*	*1.9833*	*1.9357*	*1.8645*	*1.7787*	*1.6877*
π_f **Results of relation (2-23) based on obtained** τ_f **from the present study.**	*2*	*1.9833*	*1.9357*	*1.8644*	*1.7787*	*1.6877*
τ_f **Results of referred proposed method (6):**	*1.2461*	*1.2429*	*1.2337*	*1.2195*	*1.2019*	*1.1826*
π_f **Results of referred proposed method (6):**	*2*	*1.9857*	*1.9449*	*1.8835*	*1.8094*	*1.7301*
π_f **Results of relation (2-23) based on obtained** τ_f **from referred proposed method (6):**	*2*	*1.9836*	*1.937*	*1.8672*	*1.7831*	*1.6938*

In Continue ,present results will be compared with the results obtained from PERF* software uploaded by Jack Mattingly- the writer of (6) and the (1) - on his personal website (7). This program calculates thrust and fuel specific consumption for turbofan engine.In figures 20 to 24, the results obtained from the PERF software and this study are compared.As it can be seen, Slight differences in terms of flight Mach variable are seen in higher mach numbers, which are resulted by method of solving equations which is examined in this section.

* Performance Analysis of Gas Turbine Engines-Version 3.11

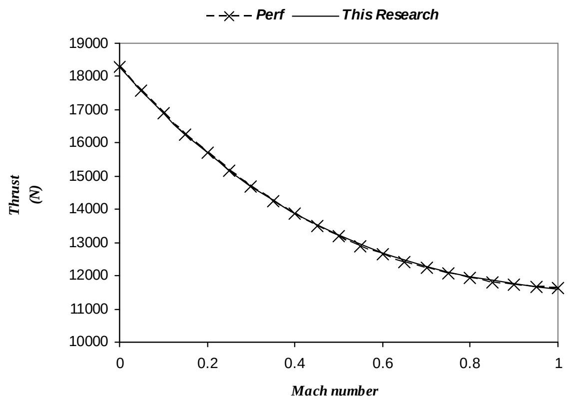

Figure 20) Thrust based on Mach number: Comparison of results obtained of this research and the results obtained of. Perf software

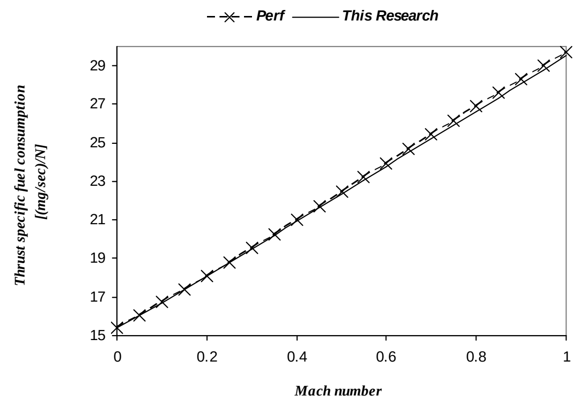

Figure 21) Specific Fuel Consumption based on Mach number: Comparison of results obtained of this research and the results obtained of. Perf software

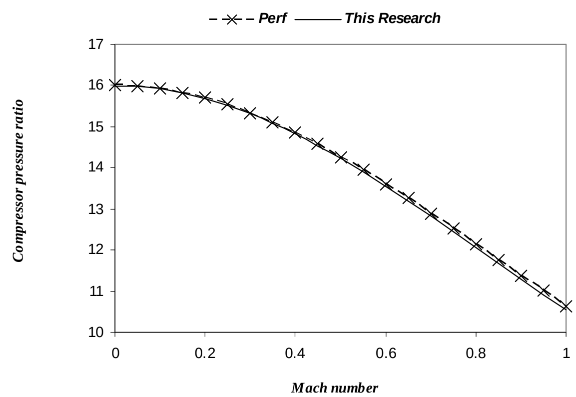

Figure 24) Compressor Pressure ratio based on Mach number: Comparison of results obtained of this research and the results obtained of. Perf software

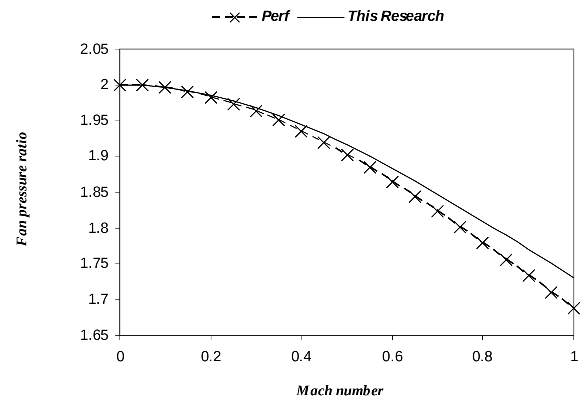

Figure 23)Fan pressure ratio based on Mach number: Comparison of results obtained of this research and the results obtained of. Perf software

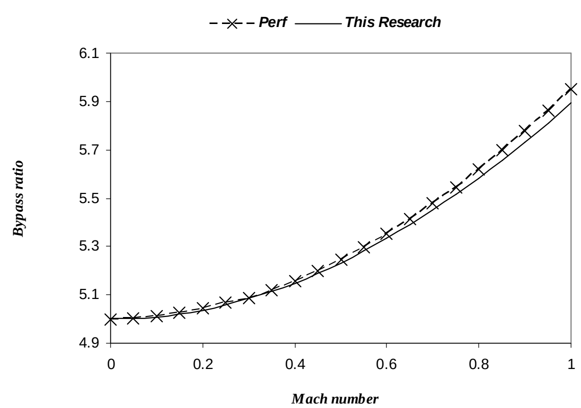

Figure 24) bypass ratio based on Mach number: Comparison of results obtained of this research and the results obtained of. Perf software

8. RESULT

Various methods examined to check off-design performance of turbofan engine,that have been generally addressed in various references. By investigating these methods to obtain performance graphs of turbofan engines, the method presented in references (1) and the (6) were used and also solution algorithm for more accurate solutions obtained were corrected. Turbofan engine performance graphs as well as the constraints functions that are controlled by the controller based on flight Mach number and in various heights were accurately drawn and discussed.

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