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Selection of the Optimal Portfolio Investment in Stock Market with a Hybrid Approach of Hierarchical Analysis (AHP) and Grey Theory Analysis (GRA)

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ABSTRACT

Investors are always considering selecting a set of stocks in financial markets, which have more profit and less risk. In classical model of investment, the main issue was distributing capital for stuck buying. There are so many methods were proposed to select stocks and in this paper, for selecting four (4) stocks in different industries, we combined analytical hierarchy process (AHP) and Grey theory. In this, the four industries in the field of machinery and equipment, insurance, pharmaceutical and investment companies were used as the sample. Also samples were selected by judgmental. For weighting the attributes, AHP was applied and then, by Grey theory, and the achieved weights by AHP, the industries were prioritized. This new approach can be useful to investors as a means for selecting a basket of stocks that has the best performance.
KEYWORDS: Stock markets, portfolio, gray relation analysis (GRA), Analytical Hierarchy Process (AHP).

1. INTRODUCTION

Achievement of continuous and long-time economic growth needs optimal equipment and specialization of resources in the national economic level; also this important point can't be simply gotten without financial markets especially by the extensive and efficient investment markets. In a perfect economy, efficient financial system plays principle role in suitable distribution of investment and financial resources. Individuals and organizations that have lack of financial resources face with individuals and organizations having surplus financial resources in financial markets. Note that word of markets has been used plural, because it means that financial markets are very variant and different in which every one includes many organizations and individuals.

One of the principle aims of economic analysis is to forecast accurately economic variables and to help executives for making accurate decision according forecasted cases. Financial markets are among systems being very different from other systems, because it involves complex mechanism and feedback. Principally, financial markets are uncertain environment in which people are busy with exchange and risk trade. If the future can be predicted, then there would be no risk or it would be reduced a lot. In fact we are going to know what happen in financial market (Tang et.al.2002).

By combination of Analytic Hierarchy Process (AHP) and gray theory (GR) in this research having been a little focused and ignored in previous studies; we are going to introduce a comprehensive model for optimal selection of portfolio. The aforesaid criterions were extracted from experts, professors, and brokers' points of views. In this research we firstly paid to theoretical framework of the research and then we attended to analysis of data.

2. Theoretical frame of research

2.1. Optimization models of portfolio

Classic model of portfolio is allocating cash assets of individuals for investment in the financial markets (Gondzio & Grothey, 2007; Ince & Trafalis, 2006 ;Markowitz & Arnott, 1952; Wu & Chang, 2007).

Markowitz (1987) introduced the first and most important role in optimization of portfolio including the famous formula of mean-variance. Among other methods, this formula shows that analytic unit should be all portfolios not stocks singly for all investors. Risk of single stock can not be attended without concerning all portfolios. Here, stock risk is equal with its covariance and other portfolio sections (Miller and Merton, 1999).

Traditional model of portfolio can be formalized as the following:

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$$MinZ : -\lambda(\sum_{i=1}^{N} X_i E_i) + \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j C_{ij}$$

$$ST: \quad \sum_{i=1}^{N} X_i = 1 \qquad \textbf{Eq (1)}$$

$$\lambda, X_i \geq 0$$

That
λ = degree of risk aversion for investor
Ei=expected income of i^th plan
X_i= part of budget that has been invested in i_{th} plan
Xj= part of budget that has been invested in i_{th} plan
Cij= cov(I, j) investing covariance of i with j investment

Studying Markowitz model, Jia & Dyer (1996) understood characteristic of this model can't practically sated investors' need. Also, Mean – Variance function can't be as the best instrument of measuring risk for investors. Hence, other limitations such as sale and purchase stock, capacity of portfolio should be entered in this model so that it is changed from linear programming to quadratic programming and its solution will be so difficult.

If we have many investment plans, we would face accounting complexity. So, many heuristic methods have been presented to promote and solve the aforesaid model. Many researchers, Loraschi and others (1995) by Genetic Algorithm method, Rolland (1996) by Tabu Search method, Tanaka & Guo (1999) by Quadratic Programming paid to advance and promote Markowitz model. Inuiguchi and Ramik (2000) attended to their comparisons by reviewing linear programming method and probable methods.

2.2 Analytic hierarchy process

Analytic hierarchy process is one of the most famous multi-criteria decision making techniques that were firstly presented by L. Thomas Sa'ati. He constructed it in order to allocate rare resources and programming needs for military. AHP has been turned to one of the most functional methods of multi criteria decision making and has been used to solve non-structural problems in different realms of interests and human being needs like politics, economics, social sciences and management.

Analytic hierarchy process is really a method of analyzing complex decision making problems with different criteria and turning it to a hierarchy tree and prioritizing group decision makings from experts' decision makers and measures consistency of their judge (Saaty, 1980, 1994; Tung andTang, 1998; Lee et al., 1999; Macharis et al., 2004).

Analytic hierarchy process includes principle that will be point them out:

Principle 1: Reciprocal Condition

If element of A equals n on element of B, element of A on element of B will be equaled 1/n on the element of B.

This principle causes that pairwised comparisons of decision maker (DM) for a matrix of n*n needs comparison of *n(n-1)/*2.

Principle 2: Homogeneity

Element of A should be comparative and homogeneity with element of B. in other words, priority of A on B can not be infinitely or zero.

Principle 3: Dependency

Every element of hierarchy is depended on the element of its higher level and this dependency continues linearly to the highest level.

Principle 4: Expectation

When a change is occurred in the structure of hierarchy, assistive process will be fulfilled again. While AHP is used as the decision making instrument, group should provide a suitable hierarchy tree stating problems being under studying. Hierarchy of decision making is a tree that includes different levels concerning studying problem and show comparison factors and assessing competitive alternatives in decision making. Especially, the first level of every tree shows aim of decision making. The last level of every tree shows alternatives that are compared with each other and are compete with each other for selection. Other level (middle) shows criterion that are criterion of comparing alternatives.

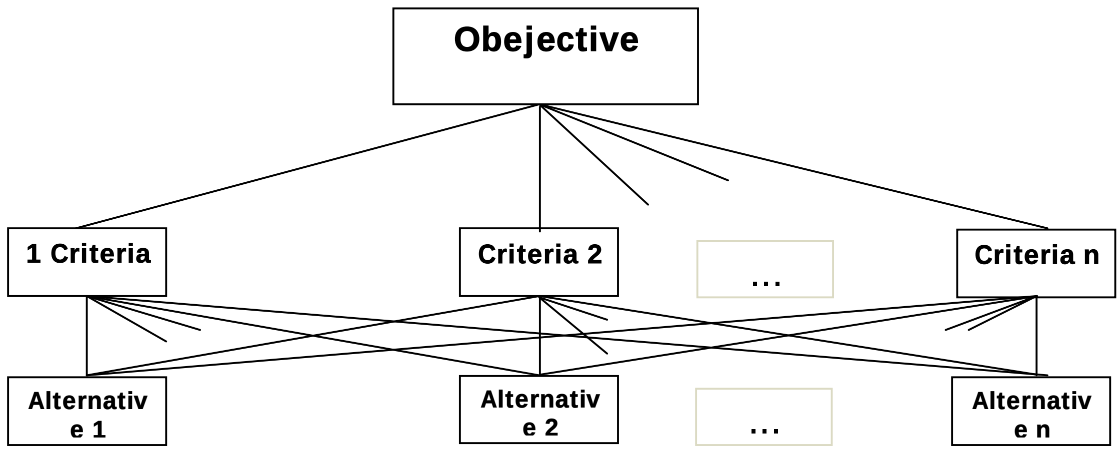

Figure 1: Model of hierarchy structure

In fact, frame of hierarchy analytic process is fulfilled on the basis of matrix and local weighting vectors that designs comparative importance of criteria (Dong et al., 2008). Equation 2 shows this subject.

$$AW = \lambda_{Max} W \qquad \textbf{Eq (2)}$$

That A is pairwised comparisons matrix; W is eigenvalue vector and λ_{Max} is the biggest number of eigenvaluevector for matrix of A.

Also the following relations are used to determine consistency rate.

$$\textbf{C.I} = \frac{\lambda_{Max} - n}{n-1} \qquad \textbf{Eq (3)}$$

$$\textbf{C.R} = \frac{C.I}{R.I} \qquad \textbf{Eq (4)}$$

If we have C.R $\leq$1, then it shows consistency of expert individuals' decisions.

3. Gray relational theory

Deng (1982) founded context of gray relations based on the theory of systems. This method consults togetherness among components of one system and reference series (Deng, 1988; Huang et al., 2008). This theory is used to solve ambiguous problems and the problems having disconnected and incomplete data. It provides satisfactory and popular outputs by a little data and with many changes in criteria.

Gray theory, like fuzzy theory is an effective mathematic model to solve indefinite and ambiguous problems. This theory is used in many fields and has been utilized in the field of solving multi criterion decision making problems named gray relational analysis. Gray relational analysis being one of gray relation is used for solving complex relations between factors and variations for solving problems. Theory of Gray systems is an algorithm that analyzes non-logical relations of one system members with a reference member and it includes capability of solving multi criterion decision making problems.

Steps of gray theory are as the following:

3.1 Calculate gray relational grade

If X_o be reference of k criteria, then we would have X_1, X_2… X_N (Wu et, al. 2010).

$$X_0 = \{X_0(1), X_0(2), \ldots\ldots, X_0(j), \ldots..X_0(k)\}$$
$$X_1 = \{X_1(1), X_1(2), \ldots\ldots.., X_1(j), \ldots..X_1(k)\}$$
.

.

$$X_i = \{X_0 = \{X_0(1), X_0(2), \ldots\ldots, X_0(j), \ldots..X_0(k)\}$$
.

.

$$X_N = \{X_i(1), X_i(2), \ldots\ldots.., X_i(j), \ldots..X_i(k)\}$$

Grey Relational Coefficient is achieved by difference between X_i series and reference series of X_0 for $k_{th.}$

$$\gamma_{0i} = \frac{\min_i \min_k |X_0(k) - X_i(k)| + \xi \max_i \max_k |X_0(k) - X_i(k)|}{|X_0(k) - X_i(k)| + \xi \max_i \max_k |X_0(k) - X_i(k)|} \quad \textbf{Eq (5)}$$

That the aforesaid formula can be rewritten for the simplicity in understanding as the following:

$$\gamma_{0i} = \frac{\Delta \min + \xi \Delta \max}{\Delta X_{0i} + \xi \Delta \max} \quad \textbf{Eq (6)}$$

If ΔX_{0i} is absolute value between X_0 and $X_{i,}$, then we have $|X_0(k) - X_i(k)| = \Delta X_{0i}$

And these relations are $\Delta \max$ =$\max_i \max_j \Delta X_{0i}$ (k) and for ΔX_{0i} $\Delta \min = \min_i \min_k$

Also, ξ is the distinguishing coefficient is 0.5. The following formula is used for accounting gray relational grade.

$$\Gamma_{0i} = \sum_{j=1}^{k} w_j \gamma_{0i} \quad \textbf{Eq (7)}$$

That W_j is the weight of k_{th} that $W_j = \frac{1}{k}$ can be used in stead of it.

Normalized research data should be used before calculating gray relational coefficient; therefore the following two methods are used for this matter (Hsia and Wu, 1997):

$$x_i^*(j) = \frac{x_i(j) - \min_j x_i(j)}{\max_j x_i(j) - \min_j x_i(j)}. \quad \textbf{Eq (8)}$$

$$x_i^*(j) = \frac{\max_j x_i(j) - x_i(j)}{\max_j x_i(j) - \min_j x_i(j)}. \quad \textbf{Eq (9)}$$

That we use equation 8 in this paper.

4. Constructing model and research methodology

Research methodology is a collection of roles, instruments, valid and systematic methods in order to assess facts, passives' discovery and achievement to difficult solutions. Adopting scientific research methodology is the only way of achievement to acceptable and scientific accomplishments. After reviewing literature and fulfilling researches, a collection of criteria were gathered.

Concerning pervious performances of companies were assessed in this research, in the other words, historical information of companies was used, therefore, this research is classes as chronology. Figure 2 shows algorithm of this research.

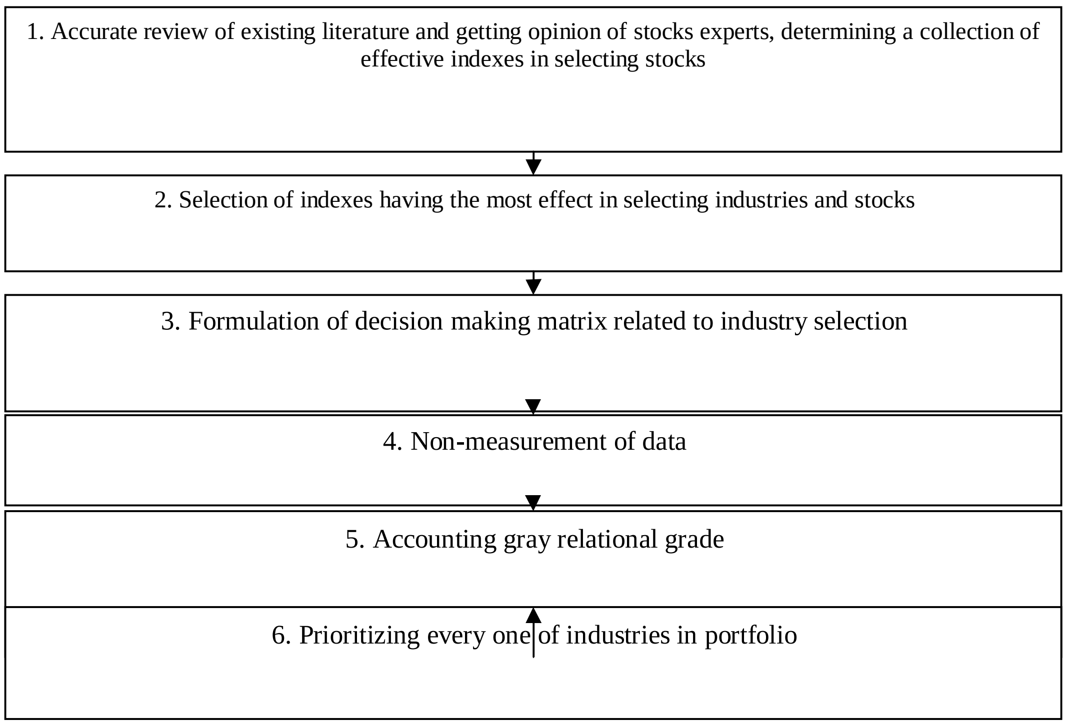

Figure 2: steps of research fulfillment

Criteria were concerned in for assessment of selecting stocks, 19 cases were collected on the basis of financial experts' opinions and university professors and stock exchange actives. Table 1 shows weights of criteria achieved by pairwised comparisons questioners.

Table 1: criteria and weights of importance

Index importance	*(index) C*
0.08	*Stock price*
0.08	*DPS*
0.08	*EPS*
0.08	*Firm management**
0.08	*Operating income ratio*
0.05	*technology**
0.07	*P/E*
0.04	*Firm size*
0.03	*capital*
0.04	*EVA(economic value added)*
0.04	*beta*
0.04	*Current ratio*
0.04	*Quick ratio*
0.04	*Inventory turnover*
0.03	*Weighted average index*
0.04	*Market rumors**
0.03	*International factors**
0.03	*Government policies**

One of incomparable characteristics of this research is the existence of qualitative criteria being assigned with star mark in the table 1. For example, one of criteria being market rampant includes significant proportion in price, demand and supply waves and influences a lot on investors' decisions; this case has been neglected in the pervious researches. Table 2 shows concerned industries for investing.

Table 2: studied companies

Machines and equipments
Assurance industry
Medicine industry
Industry of machines and equipments
Investing companies

The third step is the construction of decision matrix. Table 3 shows decision matrix of criteria through AHP. Figure 3 shows structure of research model.

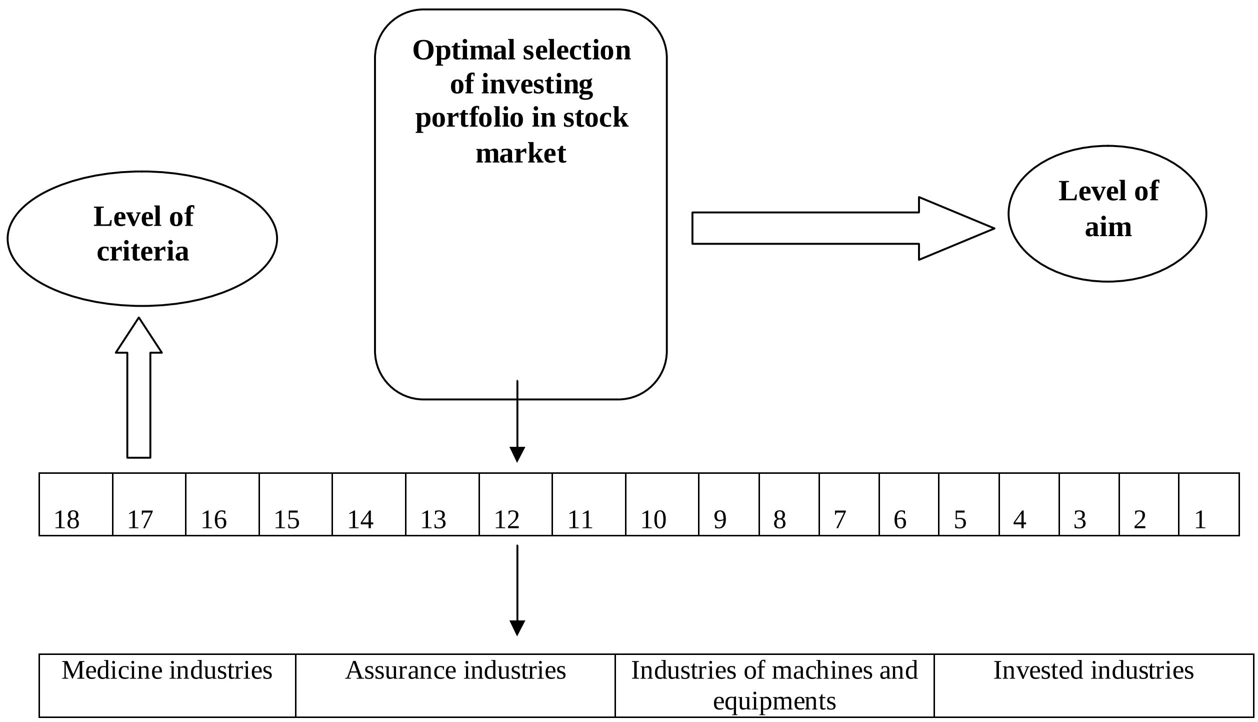

Figure 3: structure of research model

Table 3: decision matrix for criteria and industries

INDUSTRY SELECTION	INDEX																	
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18
Medical	0.16	0.27	0.23	0.28	0.21	0.15	0.24	0.18	0.17	0.16	0.16	0.21	0.21	0.21	0.50	0.28	0.44	0.36
Machines	0.15	0.26	0.25	0.30	0.20	0.14	0.15	0.16	0.18	0.31	0.31	0.20	0.20	0.45	0.16	0.30	0.19	0.22
Insurance	0.25	0.12	0.27	0.16	0.44	0.45	0.44	0.51	0.43	0.21	0.21	0.44	0.44	0.17	0.19	0.22	0.13	0.16
Investing Organizations	0.45	0.36	0.25	0.27	0.15	0.26	0.17	0.15	0.22	0.33	0.33	0.15	0.15	0.17	0.14	0.20	0.24	0.26

Regarding the a foresaid matrix and equation 8, we normalized data of table 3 and table 4.

Table 4: normalized data

Reference Index (X_0) / **Industry Selection**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18
Medical	0.02	0.33	0.22	0.36	0.16	0.0	0.25	0.08	0.05	0.02	0.16	0.16	0.16	0.16	0.97	0.22	0.80	0.52
Machines	0.05	0.70	0.68	0.94	0.23	0.0	0.05	0.11	0.23	0.29	0.29	0.35	0.35	1.8	0.11	0.11	0.29	0.47
Insurance	0.33	0.0	0.38	0.1	0.82	0.84	0.82	1.0	0.79	0.23	0.23	0.82	0.82	0.12	0.17	0.25	0.12	0.1
Investing Organizations	1.0	0.74	0.42	0.48	0.14	0.45	0.2	0.14	0.34	0.65	0.65	0.14	0.4	0.2	0.11	0.28	0.4	0.45

After achievement of normalized matrix, we should get gray relational coefficient through equation 5; but previously, number of ΔX_{0i} should be distinguished.

Table 5: calculating numbers of ΔX_{0i}

Reference Index)X₀ /(**Industry Selection**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18
Medical	0.98	0.77	0.78	0.64	0.84	1.0	0.75	0.92	0.95	0.98	0.84	0.84	0.84	0.84	0.03	0.88	0.2	0.58
Machines	0.95	0.3	032	0.06	0.77	1.0	0.95	0.89	0.77	0.71	0.71	0.65	0.65	0.8	0.89	0.89	0.71	0.53
Insurance	0.77	1.0	0.62	0.9	0.18	0.16	0.18	0.0	0.21	0.77	0.77	0.18	0.18	0.88	0.83	0.75	088	0.9
Investing Organizations	0.0	0.26	0.58	0.52	0.86	0.55	0.8	0.86	0.66	0.35	0.35	0.86	0.6	0.8	0.89	0.72	0.6	0.55

Table 6: accounting numbers of γ_{0i}

Reference Index)X₀ /(**Industry Selection**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18
Medical	0.40	0.46	0.47	0.56	0.32	0.29	0.34	0.29	0.3	0.3	0.32	0.32	0.32	0.32	0.58	0.31	0.59	0.37
Machines	0.33	0.61	0.6	0.88	0.38	0.32	0.33	0.35	0.38	0.4	0.4	0.42	0.42	0.37	0.35	0.35	0.4	0.47
Insurance	0.39	0.33	0.44	0.35	0.73	0.75	0.73	1.0	0.7	0.39	0.39	0.73	0.73	0.73	0.37	0.4	0.36	0.35
Investing Organizations	1.0	0.63	0.43	0.46	0.34	0.45	0.37	0.34	0.4	0.56	0.56	0.4	0.42	0.37	0.33	0.38	0.42	0.45

We should attain gray relational coefficient though equation 6 in the next step, coefficients of 4 industries have been achieved in table 6.

Now, gray relational coefficient can be calculated by numbers of γ_{0i} that final results and rank of every one of these industries have been distinguished in table 7. Concerning our characters are four industries, and relations of $W_j = \frac{1}{k}$, weight of every one of these industries is 0.25.

Table 7: gray relational coefficient of Γ_{0i}

Industries	Γ_{0i}	**Rank**
Medical	0.13	3-4
Machines	0.13	3-4
Insurance	0.14	2
Investing Organizations	0.25	1

Concerning table 7, medicine industry with 0.25 coefficients is selected as the best alternative for investing on machines and equipments industries; also investing on assurance industry is regularly located in the next rank.

5. Conclusion and discussion

Since 1950, optimal selection of portfolio has been attended by many researchers. Though Markowitz model was a new invention in solving such matters, but it couldn't be suitable for easy matters. Hence, many researches were fulfilled to advance and improve his model. Many presented models were concerned qualitative criteria for selection of portfolio, whereas most of factors like management of company, market rampant and etc effect so much on market portfolio.

This research steps improvably in related matters of selecting portfolio by Utilization of a compound of AHP and gray theory and concerning incertitude and entering qualitative variables like market rampant and company management. In fact, this research used a new strategy and technique in selecting portfolio.

The main aim of this article was to use a new method towards the best selection of industry for investing in securities exchange market. The model of this research includes 18 principle criteria that were achieved by universities professors and exchanges experts. After extracting effective criteria in exchange market, weights of every criterion were achieved by pairwised comparisons matrix. Then gray theory was used in order to select the best industry in portfolio and reduce uncertainty and incertitude in financial markets.

It is suggested to concern the following matters in future researches:

1. To advance and improve the research model through adding portfolios of every industry in the aforesaid research
2. Whereas AHP method is independency of the aforesaid levels and there is not any dependency of relations between criteria, but in fact there is relations between factors. It is suggested to use ANP method and assess results by the aforesaid research
3. To increase reliability of this research by adding numbers of industries and their relative portfolios.

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