Solution of Multi-Objective Location Model when Users Select Serving Centers on the Basis of Distance Criterion with Use of Tabu Search Algorithm

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ABSTRACT

The model provided in this research is one of the discrete location problems which are regarded as a simple graph. Vertexes of this graph show that demand points and potential points for construction of serving centers. In addition, edges of this graph specify possible routes between nodes. In this model, it is assumed that if a serving center is constructed in one of the nodes, demand of that node should not refer this serving center and be able to receive service from serving center. This assumption holds true in competitive markets. Dominant queue system in each serving system was considered as M/M/1. The proposed model in this research sought to minimize costs of construction and closing of centers, waiting time of users in queue; average idle time of service and increase expected coverage of the demands by these centers. In order to solve the model, Tabu search algorithm was used and its parameters were regulated with use of Design of Experiment (DOE).

KEYWORDS: Queuing, MCLP; Tabu Search; Design of Experiment.

1. INTRODUCTION

Location study is one of the key actions in construction of industrial or servicing units and attention to this important fact plays role in success of centers. These studies are so important that these studies are done again on active centers leading to change of industrial unit place. Location problems are varied in which special goals are followed. In order to achieve goal of each problem, we should use special method for solving that problem and ensure accuracy of information during studies.

2. Location problems types

Location problems are varied and for this reason, they have classified these problems in different ways but location analysis problems are included in one of the following classes:

1- P-Median problem (Webber problem): such problems are used for location of p center in P place and minimize cost criterion. If P=1, the problem will be MP-1. Cost may be expressed in terms of time, money, and number of travel, total distance or any other scale. Because the goal is to minimize cost in such problems, they are raised as Mini Sum or Webber problem.

2- P-Center problem: these problems are used to determine p place in order to minimize the maximum distance of each center to demand point which has been determined for serving that demanded point. Such problems are used for settlement of emergency services, ambulance services and police centers in society. In these problems, the number of centers is predetermined. These problems are divided into two groups. Convex P-center which limits the problem to a set of candidate places for settlement of centers and absolute P-Center which can be settled at any place.

3- unlimited facilities location problems : these problems are regarded as Mini Sum but these cost problems include fixed cost and fixed cost depends on the place in which place is located. The number of centers which should be settled is not predetermined but they are defined in such a manner that they minimize cost. Because facility of each ceteris unlimited in such problems, it will not be profitable to allocate a demand to more than one supply points.

4- Unlimited facility location problem (UFLP): these problems are like UFLP problems but facility of each center is limited in these problems. Optimal answer may be such that a user is referred to more than one supply center. In fact, facility of the center may be finished after allocation of user to a center after fulfilling some demand of the user and we may have to allocate him to other centers which have more costs in order to meet remaining demand of user. Although allocation of one user to a center causes the lowest cost, we have to allocate total demand of that user to other centers.

5- Exponential allocation problems: in these problems, n center is like n machine so that they are located in n place to minimize total cost. If we have four machines which we want to locate, four possible combinations will be available. For problem of 20 machines, there may be 20 possible answers which will require about 2×108
assessments which are difficult even for ADSL computers. For this reason, these problems will be regarded as very complex problems (Dileep, 2001) and they are difficult to solve. Some elements play important role in classification of location problems. In fact, location problems can be divided into different classes such as P-Median problem with demand limitation and P-Median without demand limitation. For this reason, we should pay attention to some elements such as all kinds of new centers available centers, interaction of the available and new centers, specifications of answer space, distance, combination with other problems, demand, facility, type of centers, certain or probable data, execution, variety of product and target function.

3. introduction of model

The model presented in this research is one of the discrete location problems. It is regarded as a simple graph and vertexes of this graph indicate demand point and potential point for construction of service center. In addition, edges of this graph specify route between nodes. In this model, in this model, it is assumed that if a serving center is constructed in one of the nodes, demand of that node should not refer this serving center and be able to receive service from serving center. This assumption holds true in competitive markets. In this model, different rates of demand of each point refer to service center for receiving service and cost between two points of demand and service center is only regarded on the basis of necessary time for the demand to reach service center. Such cost is sued for obtaining probable selection of a service center with a demand point. This probability is obtained on the basis of (logit function) (McFadden, 1974). This function was used by Mr. Marianov in an article in 2007. In this model, there is limited service quality due to limited budget; total number of servers like location emergency models with this difference that more than one service center in any demand point. The example for such problem relates to a competitive market in which some plants are active and a new plant intends to construct some some sale centers for capturing market. In this market, demand points are two classes: the first class is demand point or points in which new servers can be located and sale center has not been constructed or the number of sale center is not enough in this demand point. The second class is demand point in which there are not sale center in these points. In this article, waiting time of user in queue, total idle time of the servers, increased coverage of demands and construction servers closing cost are studied as 4 independent target functions. As result, the provided model is modified multi-objective model and this proposed model seeks to find answer which minimizes waiting time of user in queue, average idle of all servers and server opening and closing cost leading to increased coverage of demands. Since the provided model in this research is a nonlinear model or integer variables and solution time increases with increase of the problem size, TS algorithm is used and algorithm parameters have been regulated with use of DOE technique.

4. Problem Modeling

\( m \): Total number of server.
\( n \): Number of potential facility nodes (\(|N| + |N'| = n\)).

\( N \): Set of points in which new service centers are opened.
\( N' \): Set of points in which new service centers are opened.
\( \bar{p} \): Maximum number of servers can be opened. (\( \bar{p} \leq n \))
\( c_{ij} \): accessing cost matrix from customer \(i\) to facility node \(j\), \(i=1,...,n\) and \(j=1,...,m\).
\( t_{ij} \): Time matrix from customer \(i\) to facility node \(j\), \(i=1,...,n\) and \(j=1,...,m\).
\( \mu \): Common service rate for each server.
\( \gamma_j \): Demand rate at open facility \(j\), \(j=1,...,m\).
\( w_{ij} \): Expected waiting time of customers assigned to facility node \(j\), \(j=1,...,m\).
\( \rho_j \): Utilization factor at open facility \(j\), \(j=1,...,m\).
\( r \): The upper bound on the server utilization rate at each open facility node
\( f_{o_j} \): Fixed cost at each open facility node \(j\), \(j=1,...,m\).
\( f_{c_j} \): The cost to close facility node, \(j=1,...,m\).
\( \Pi_{ij} \): The probability of the server being idle at open facility (idle probability) \(j\), \(j=1,...,n\)
\( p_{ij} \): The probability of a user at customer node \(i\) choosing to go to the facility at node \(j\).
where \( y_j \) is the variable which has zero and natural numbers and its value indicates the number of service center which is constructed in node \( j \). now, we obtain \( P_{ij} \) with use of parameters mentioned above and on the basis of logit function,

\[
P_{ij} = \frac{y_j e^{-\varepsilon \eta_{ij}}}{\sum_{k \in N} y_k e^{-\varepsilon \eta_{ik}} + \sum_{k \in N} y_k e^{-\varepsilon \eta_{kj}}}.
\]

\[
\varepsilon = \frac{\pi}{6 \sqrt{\sigma}}
\]

In order to calculate \( \eta_{ij} \), two factors of cost of the user access to opened centers and time spent by the user to reach the opened centers. User may select centers which he can reach in short term but with high cost and this is not good and there should be balance between travel time and travel cost and user selects the centers in which travel time and access cost are balanced.

\[
\eta_{ij} = \alpha t_{ij} + (1 - \alpha)c_{ij}, \quad \varepsilon = \frac{\pi}{6 \sqrt{\sigma}}, \quad \alpha = [0,1]
\]

Where, \( \sigma \) indicates standard deviation in taste of the users. In the above relation, if \( \varepsilon \) receives large value, it indicates that users in each demand point refer to service centers opened in that point for receiving service and there is decrease of \( \varepsilon \) i.e. distribution of the user’s choice. Users tend to refer to other opened centers in their node. In this case, each user can refer to service center with probability of \( P_{ij} \).

4-1. Definition of model

Model provided in this research includes four target functions and some constraints as follows:

Waiting time in queue equals to \( w_j = 1/(\mu - \beta_j) \) and with regard to the fact that entry rate of the users to each center equals to \( \gamma_j \) and entry rate of the users to each center equals to \( \beta_j = \sum_{i=1}^{m} \gamma_j P_{ij} \) on the basis of probability \( (p_{ij}) \) of the ith user reference to jth service center, we can calculate total waiting time of the user in queue \( Z_i = \sum_{j=1}^{n} \gamma_j w_j P_{ij} = \sum_{j=1}^{n} \frac{\beta_j}{\mu - \beta_j} \). Therefore, the first target function equals to total waiting time of the user in queue and with regard to the fact that efficiency of each server equals to \( \rho_j = \gamma_j / \mu \) and on the basis of probability \( (p_{ij}) \) of the ith user reference to jth service center, efficiency equals to \( \rho_j = \gamma_j P_{ij} / \mu \) and idle time of each server equals to \( \Pi_{0,j} = 1 - \rho_j = 1 - (\beta_j / \mu) \), therefore, we can determine total idle time of servers equaling to \( Z_z = \sum_{j=1}^{n} (1 - (\beta_j / \mu)) \). The third target function equals to total opening costs \( (\sum_{j=1}^{n} \gamma_j f o_j) \) and closing costs \( (\sum_{j=1}^{n} (1 - \gamma_j) pfc_j) \). The fourth target function seeks to open service center in demand points and allocate these centers to demand points so that it maximizes expected coverage of the demands in these centers. All of the above four target functions in one model seek to find the best place for location of servers and their allocation to the users so that opening costs, waiting time of users in queue and average idle time of service can be minimized and expected coverage of the demands can be maximized.

Constraint \( \sum_{j=1}^{n} y_{ij} \leq \overline{p} \) controls total number of server’s location not to exceed predetermined value. Constraint \( \rho_j \leq r \) guarantees that productivity rate of each server doesn’t exceed \( r \) value so that the system can be stable. Constraint \( y_{ij} \in \{0,1\} \) considers \( y_{ij} \) as binary (zero and one). In order to achieve these goals, mathematical model will be as follows:
Tabu search algorithm

Tabu search algorithm was used for the first time by Glover (1989-1990) to solve Travelling salesman problem and its high efficiency caused to raise it as a solution in other problems such as layout problems. For example, Rolland et al (1996) tabu search algorithm to solve layout problem p. Filho and Galvao (1998) and Ghosh (2003) and Sun (2006) applied this algorithm to solve servers layout classic problem and Mariano et al (2007) used this algorithm to obtain maximum market share by assuming ranking of servers on the basis of distance and waiting time of user and Berman (2007) for layout model in network with multipurpose travels. Arostegui et al (2006) showed that tabu search algorithm has better efficiency than other metaheuristic methods have. This method can be used for directing the processes which use a set of motions for transfer from one answer to another answer. This method uses different memory structures (to utilize search information), different conditions for limiting and freeing search process (included in tabu constraints and idea condition) and memory functions in different time intervals for reinforcing the characteristics which are desirable experimentally and search direction to new areas. Tabu search assumes that we can call a solution smart when this solution can integrate adaptive memory and search. Adaptive memory means application of methods which can search solution space efficiently. Tabu search is compared with memory free methods which are inspired by physics and biology. Answer search means utilization of good answers characteristics when we search new spaces. Emphasis on answer search in tabu search results from this hypothesis that a bad strategic choice gives more information in comparison to a good stochastic choice. Tabu search algorithm starts with a feasible answer. Suitable selection of this primary answer can be effective on convergence of algorithm. Then algorithm considers it as current solution and produces all possible neighbors of current solution with use of neighborhood structure. With regard to large problems, specified number of neighbor answers is selected stochastically. After production of each neighbor answer, specifications or motion which causes that neighborhood are compared with tabu list. If these specifications are available in tabu list, the produced neighbor answer will be excluded and in case specification or changes which cause neighborhood are not included in tabu list, the produced neighbor will be accepted. After production of specified number of neighbor answer, the best produced neighbors are selected in terms of target function and then tabu list is updated. It means that specifications of new solution are brought to tabu list and included on top of the tabu list. Here, tabu list has been defined as limitation of list length and uses FIFO list. It means that after new specifications are included in tabu list, one of the specifications is released automatically on top of the list and then current solution is compared with the best solution until this stage and replaces it in case it is better than the best solution. Generally, this algorithm uses consecutive iterations for

\[
\text{Min } Z_1 = \sum_{j=1}^{n} \frac{\beta_j}{\mu - \beta_j}
\]

\[
\text{Min } Z_2 = \sum_{j=1}^{n} \left(1 - \frac{\beta_j}{\mu}\right)
\]

\[
\text{Min } Z_3 = \sum_{j=1}^{n} (1 - y_j) p_{fc} + \sum_{j=1}^{n} y_j f_1
\]

\[
\text{Min } Z_4 = \sum_{j=1}^{n} \sum_{i=1}^{n} y_j p_{\alpha}
\]

Subject to:

\[
\sum_{j=1}^{n} y_j \leq p
\]

\[
\rho_j \leq r \quad j = 1, \ldots, n
\]

\[
\sum_{j=1}^{n} p_{\alpha} = 1
\]

\[
y_j \in \{0, 1\} \quad j = 1, \ldots, n
\]

\[
\beta_j = \sum_{j=1}^{n} y_j p_{\alpha} \quad j = 1, \ldots, n
\]

\[
w_j = 1/\mu - \beta_j \quad j = 1, \ldots, n
\]

\[
p_{ij} = \frac{y_j e^{-\epsilon \eta_{ij}}}{\sum_{k \in N} y_k e^{-\epsilon \eta_{ik}}} + \sum_{k \in N} y_k e^{-\epsilon \eta_{ik}}
\]

\[
\eta_{ij} = \alpha t_{ij} + (1 - \alpha) c_{ij} \quad \epsilon = \frac{\pi}{6\sqrt{\sigma}} \quad \alpha = [0, 1]
\]
finding optimal answer. In any iteration, the current answer is moved with the process toward another answer. This motion can be expressed in different ways creating new answers. Set of new answer is called neighborhood. Then the best answer in this neighborhood is selected as candidate for movement toward it and a list with determined length is used as tabu list in which the obtained answers or specifications are kept. This list is a short term memory which doesn’t allow reselection of the previous answers up to definite iterations. General criteria for implementation of tabu search algorithm include tabu list and stop criterion. In tabu search algorithm, the next answers should not equal to the obtained answers. For this purpose, a list with definite length has been designed in which the obtained answers or their specifications are saved. Length of the tabu list is determined with regard to view of algorithm designer and its value is not considered large or small because search is time consuming due to long tabu list and neighborhood is limited due to high prohibitions. In case of short tabu list, circle will be created. For new movements with the old recorded specifications and updating tabu list, there are different ways in which entrance and exit order is considered and we can refer to entry of data to top of the list and exit of the first list data (FIFO) or exit of the first data and entry of new data to the bottom of list (LIFO). In this algorithm, in case the number of iteration reaches one presupposed number or target function doesn’t change in some consecutive repetitions and remains fixed, it will stop.

![Flowchart of the tabu search algorithm](image)

**5-1. steps of tabu search algorithm in this research**

Step 1: introducing and initialization of the problem parameters and determination of tabu list length

Step 2: production of a feasible initial answer

Step 3: if repetition numerator equals to the maximum determined repetition, go to step 11, otherwise, continue.
Step 4: length of the created tabu list is longer than length of tabu, continue, otherwise, go to step 6.
Step 5: update length of tabu list in FIFO method.
Step 6: production of neighborhood
Step 7: if the obtained target function is better than the target function of previous stage (that is value of new target function is higher than target functions of the previous stages), continue, otherwise, go to step 9.
Step 8: accept neighborhood answers and continue.
Step 9: in case the obtained target function equals to target function of the previous stage, add one number to repetition numerator, otherwise, consider repetition numerator as zero.
Step 10: adding movements to tabu list, go to step 3.
Step 11: show the best answer among target functions and stop algorithm.
Flowchart of the tabu search algorithm is given as above:

6. Regulating tabu search algorithm
Design of experiments (DOE) is one of the strongest techniques of quality improvement and increase of productivity. In this method, some changes are made in the process or system through some tests to study their effects on performance or process or system response to them. Design of experiments is systematic alteration of some variables in which effect of these alteration is assessed and the obtained results are implemented. Design of experiments methods are widely applied in many systems. In engineering world, design of experiments is extraordinary tools for correction of production process. It is widely applied in expansion of new processes. Application of primary techniques for design of experiments in expansion of process can give the following results. They play major role in engineering activities in which new products are developed and the present products are corrected. ANOVA is used for identification of parameters effective on response. Marginal mean diagrams are used for each one of the parameters. Usual way includes study of diagrams and selection of top parameters. Top parameters are the ones which have larger effects. Some levels of these parameters which have the best result are considered as optimal levels and response variable is calculated in these levels. Parameters of the proposed model include the number of users (m), the number of opened places (n) and proposed tabu search algorithm parameters include stop criterion of tabu search algorithm (SC) and length of tabu search list (TL). After design of tabu search algorithm, parameters of the tabu search algorithm should be set with use of DOE technique. Some experiments are designed (Montgomery DC (2009) ) and solved with tabu search algorithm and parameters of tabu search algorithm have been set in such a manner that algorithm stop criterion (SC=20) and tabu list length (TL=2) should be considered for small size problems and algorithm stop criterion (SC=28) and tabu list length (TL=12) should be considered for medium size problems and algorithm stop criterion (SC=31) and tabu list length (TL=16) should be considered for large size problems. The proposed mathematical model was solved in two different ways, one precise solution and another one with tabu search algorithm with set parameters for problems of different sizes and it was specified that tabu search algorithm proposed in this research was more efficient than the precise solution was. On the other hand, answer of both methods is equal but although answer of both methods is equal, precise solution time is longer than that of tabu search algorithm solution. In this research, MATLAB software has been used for programming tabu search algorithm and Minitab software has been used for design of experiments.

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<th>Problem Size</th>
<th>Time(s)</th>
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<td>Lingo</td>
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![Computation time graph]

TS
Lingo
7. Conclusion and future research

The model proposed in this research is one of the discrete location problems. The problem is considered as simple graph of which vertexes indicate demand points and potential points for opening service centers. In addition, edges of this graph specify possible routes between the nodes. Dominant queue system in each service center is considered M/M/1. The location model proposed in this research seeks to decrease opening and closing cost, waiting time of users in queue; average idle time of service and increase expected coverage of the demands by these centers. Because the presented model is part of NP-hard problems, TS algorithm has been used to solve the proposed model and algorithm parameters have been set with use of DOE technique. The proposed model is set in two different ways, one precise solution and another one with tabu search algorithm with set parameters for problems of different sizes and it was specified that tabu search algorithm proposed in this research was more efficient than solution with Lingo software. In the future research, we can use fuzzy parameters for entrance of users and serving rate of the centers instead of certain parameters and we can use other queue models instead of M/M/1 queue model. In the future research, we can use other metaheuristic algorithms instead of TS or improve metaheuristic algorithm applied in this research.

8. REFERENCES


